Platform Design for Costly Learning *

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October 2018

Abstract

This paper studies the optimal design of a platform to incentivize its users to collectively acquire costly information about the quality of a product (or a service). A constant flow of users arrive in sequential order. Each user observes information disclosed by the platform and may acquire a costly private signal about the product quality before making his purchase decision. The platform receives users’ feedback about the product quality. It is shown that if the platform learns about the product quality through negative feedback, it is optimal for the platform to release no information early on to induce user exploration, and publish a list of potentially good products at a later point in time, once and for all. On the other hand, if the platform learns about the product quality through positive feedback, it is optimal for the platform to continuously flag projects to be good for an extended period of time right after the product is released. Welfare comparison with several different benchmarks are discussed.

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*I thank Benjamin Brooks, Doron Ravid, Elliot Lipnowski, Emir Kamenica, Roger Myerson, and Philip Reny for valuable comments and discussion. All errors remain my own.

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1 Introduction

Consumers reply extensively on other people’s past purchase decisions to learn about the quality of a product. For example, readers deciding which book to buy often consult the list of best sellers, restaurant-goers look at the length of queues at the entrance of restaurants to judge the quality of the food, and investors assess the prospect of equity-investment on the basis of the firm’s prior capital raising activities.

With the rapid development of digital technology, firms are now equipped with the capability to delicately design how information about purchase histories is communicated to consumers. Different existing platforms have adopted different policies regarding the disclosure of past purchases. For two extreme cases, *Ebay* fully reveals detailed history of past purchases, including the timing as well as the user ID of the purchaser; while *Amazon* does not disclose any information regarding purchase history. Other platforms adopt policies somewhere in between.

Some recent efforts have been devoted to understanding the consequences of such practices. For example, the European Commission published the “Communication on Online Platforms” on 25 May 2016, with several follow up pilot initiatives. One of the programs is aimed at understanding how algorithms adopted by online platforms “shape, filter or personalize the information flows that they intermediate”. A consensus is far from being reached, and the regulatory framework of these practices is at its infant stage.

Under this background, our research focuses on understanding one particular aspect of online information flows, i.e. how purchase histories are communicated to consumers. We will study the optimal way for a firms to disclose purchase histories, and how it impacts market outcome.

To motivate our model and results, we will briefly discuss the strategy adopted by Bookings.com, a travel metasearch engine for lodging reservations. As shown in Figure 1, on the page displaying information of a hotel on Bookings.com, besides conventional items such as price and location, one additional piece of information is being highlighted: “Booked 5 times in the last 6 hours”. This is the piece of information that we study in this paper. We seek to understand what is the optimal way for the platforms (or sellers)

\(^1\) *Online Platforms and the Digital Single Market Opportunities and Challenges for Europe*, European Commission, May 25 2016
to disclose past purchases. We also want to understand the welfare consequences of such practice, and whether any policy intervention is necessary in these situations.

There are plenty of ways that Bookings.com could disclose information about past purchases. They could reveal the timing of the last purchase, the number of purchases in the past 7 days, the ratio of the number of purchases to the total number of people who viewed the page, etc. In fact, Bookings.com are dynamically altering the information being disclosed based on the evolving sales volume. By tracking the information disclosed on Bookings.com, we summarize the disclosure policy of Bookings.com as consisting of two phases:

1. For hotels that haven’t accumulated enough transactions, Bookings.com does not disclose any information regarding past purchases

2. For hotels that have accumulated sufficient amount of transactions, Bookings.com discloses some vague information regarding past purchases

As we will show in the paper, this “delayed-garbling” approach of disclosing past purchases, as adopted by Bookings.com, is optimal for the platforms (or sellers) to maximize profit. The exact form of the optimal mechanism depends on the information structure of the consumers’ private signals. In particular, if the signal reveals bad news more quickly than it does for good news, the two-phase “delayed-garbling” approach above is optimal. Otherwise, a generalized three-phase “delayed garbling” with a interim phase of delicate information provision, that balances short-term profit and long-term information gathering, is optimal. In either of these cases, the optimal disclosure mechanism will induce
a Pareto improvement for the seller and the buyers, as compared with a setting where no information of past purchases is disclosed. Therefore, absent other appealing reasons, there is no normative justification for policy intervention in such practices.

The rest of the paper is organized as follows. Section 2 reviews the relevant literature. Section 3 introduces the model. Section 5 characterizes the optimal mechanism under exogenous price. Section 6 conducts welfare analysis. Section 7.2 analyze the case for endogenous price. Section 10 concludes.

2 Literature Review

This paper is most closely related to the recent literature which studies the design of online platforms. In particular, Glazer et al. (2015) study the socially optimal information design in a sequential learning setting. In terms of the model set-up, my paper shares the same feature as theirs by allowing the designer to implement any mechanism to disclose information about past actions. The main difference is that, in my paper, the designer’s (i.e., the seller’s) objective is to maximize profit rather than agents’ (i.e., the consumers’) welfare. Kremer et al. (2014) considers a related problem of information design where realized payoffs, instead of actions, can be disclosed to future agents. Vellodi (2018) considers the design of ratings platforms that aggregate reviews of firms for consumers, and showed that upper censorship in the form of random deletion of reviews of good firms is socially optimal. (Che and Horner, 2018) studies how to optimally design the recommendation system to organize social learning on a product.

These papers aim to characterize the socially optimal mechanism to reduce inefficiencies in their respective settings, assuming the existence of a social planner who can implement the mechanism. My paper is thematically related to these papers, but, instead of studying social optimal mechanism, we study the optimal information provision by a monopolist who maximizes profit, and considers the welfare consequences.

There are a few papers studying how to design signals directly about product quality rather than histories of purchases. (Bergemann et al., 2015) characterizes the possible outcomes in a model where the buyer is fully informed and the seller observes a signal about the buyer’s valuation. They show that additional information to the seller may
benefit the buyer, contrary to intuition. (Szentes and Roesler, 2017) characterizes buyer-optimal learning in environments where the monopolist best-responds to the buyer’s signal structure. They show that the buyer-optimal signal generates efficient trade. (Condorelli and Szentes, 2016) also consider a bilateral trade model. In contrast, the distribution of the buyer’s valuation is not given exogenously. Instead, the buyer chooses her value-distribution supported on a compact interval and perfectly observes its realization. The seller observes the buyer’s distribution but not her valuation and sets a price. The authors show that the equilibrium distribution generates a unit-elastic demand and trade always occurs.

In all these papers, the focus is on analyzing how a direct signal to either the buyer or the seller may affect the welfare, which differs from my model, which analyzes the indirect channel through which consumers learn about product quality from actions of other consumers. Also, in all these models, if the seller is allowed to “design” the signal structure, the buyer will be worse off. However, in my model, it will be shown that even through the seller is selecting a signal structure to his own benefit, it will not hurt the buyer.

There is a strand of literature which studies dynamic monopoly and herding that formalizes how purchase history influences consumer behavior. These papers serve as benchmarks without endogenous information provision. (Ottaviani, 1996) gave a first formulation of the problem and derived implications for learning and welfare. (Bose et al., 2005) generalizes the model and the results contained in (Ottaviani, 1996), and provides a full characterization of the seller’s optimal strategy, comprising the short as well as the long run, for the specific case where buyers have binary signals. (Bose et al., 2006) study the general case with a finite number of signal realizations, and provide a full characterization of the solution for the case with symmetric binary signals. These papers features inefficiencies caused by consumer herding when they face fixed prices.

There is also a literature in behavioral economics which studies the psychological effect of information on behavior. For example, it is argued that firms intentionally disclose information about sales to scare consumers into hasty purchases in fear of the product being sold out. My model focuses on rational agents, and should be viewed as complementary to this literature.
3 Model

There is a long-lived seller who can produce one unit of a homogeneous product in each period with a marginal cost normalized to zero, and can sell it at an exogenous price $p$. Each period, one short-lived consumer arrives, and makes purchase decision $a_t \in \{0, 1\}$ of whether to purchase the product ($a_t = 1$) or not ($a_t = 0$). In what follows, we will simply refer to the consumer arrive in period $t$ as “consumer $t$”. Consumers have a common value for the product $v \in \{0, 1\}$ which is ex ante unknown to the seller and the consumers.

All agents are risk-neutral, and share a common prior that $v = 1$ with probability $\mu_0$. The seller does not value the product and maximizes expected discounted profit with discount factor $\delta \in (0, 1)$. Before the first consumer arrives, the seller commits to a disclosure mechanism $m = \{m_t\}_{t=1}^{\infty}$ where $m_t$ maps $H_{t-1}$, the set of histories up till time $t-1$, to a randomization over signal realizations in $M$, the set of all possible signal realizations. In each period $t$, the signal $m_t$ is realized according to the randomization specified by the mapping given the realized history. In terms of notation, we use $m$ to represent the entire disclosure mechanism. We use $m_t$ to represent the experiment (or mapping) in time $t$ as well as the random variable corresponding to the mapping, taking equilibrium strategies as given.

When consumer $t$ arrives, he will first observe the realized signal $m_t$ disclosed by the seller about past purchases of the product by consumers $1, 2, ..., t-1$. After observing $m_t$ and before making his purchasing decision, consumer $t$ can pay a cost $c > 0$ to observe an additional private signal $s_t \in \{h, l\}$ about the quality of the product.

The private signal is assumed to be generated independently according to the following signal structure:

$$Pr(s_t = h|v = 1) = q_1 \quad \text{and} \quad Pr(s_t = l|v = 0) = q_2,$$

where $q_1, q_2 \in [0, 1]$. Without loss of generality, we assume $q_1 \geq 1 - q_2$.\footnote{If it is violated, we can just switch the names of the signal realizations $h$ and $l$ to make the inequality hold.} The parameters $q_1, q_2$ govern how quickly good news and bad news get revealed. For example, a signal structure with $q_1 = 1$ and $q_2 < 1$, is such that a good product generates a high signal with probability one, but a low quality product also generate a high signal with positive probability less than one. In this case, the signal structure features “perfect bad news”,
in the sense that once a signal \( l \) is observed, it is perfectly revealed that the product is of bad quality, i.e. \( v = 0 \). On other hand, if \( q_1 < 1 \) and \( q_2 = 1 \), the signal structure features “perfect good news”.

We assume that signals \( s_t \) are independent across different periods, and are independent of \( m_t \). Therefore, each consumer \( t \) has access to two signals: the endogenous signal of past purchases \( m_t \) disclosed by the seller, and the exogenous signal of quality \( s_t \) privately observed by consumer \( t \).

In the paper, we study the pure strategy perfect Bayesian equilibria of this game between the seller and the consumers \( t \in \mathbb{N} \).

Since the signal space \( M \) is unrestricted, the set of all possible mechanisms is potentially very large. However, as is standard in (Kamenica and Gentzkow, 2011), it is without loss of generality to focus on mechanisms which directly reveal the posterior belief that the signal \( m_t \) induces. Similarly, the only payoff-relevant information contained in \( s_t \) is the posterior belief of \( v = 1 \) it induces. Thus, it is without loss of generalized to normalize the signal realizations of both \( s_t \) and \( m_t \) to the corresponding posterior beliefs. Hence, the signal realizations \( \{h, l\} \) of \( s_n \) is such that

\[
\begin{align*}
    h &= \frac{\mu_0 q_1}{\mu_0 q_1 + (1 - \mu_0)(1 - q_2)}, \\
    l &= \frac{\mu_0(1 - q_1)}{\mu_0(1 - q_1) + (1 - \mu_0)q_2},
\end{align*}
\]

and the possible signal realizations of \( m_t \) are \( M = [0, 1] \).

4 Consumer Decision

Consumer \( t \)’s information set is \( \{m, m_t, s_t\} \), where \( m \) is the disclosure mechanism chosen by the seller, \( m_t \) is the signal about past purchases realized according to the disclosure mechanism, and \( s_t \) is his private signal about the product. Consumer \( t \)’s strategy is a mapping from his possible information sets to purchase decisions.

According to Bayes rule, upon observing \( m, m_t \) and \( s_t \), consumer \( t \)’s posterior belief of \( v = 1 \) is

\[
q(s_t, m_t) = \frac{\mu_0 \cdot \mathbb{P}(s_t|\theta = 1) \cdot \mathbb{P}(m_t|\theta = 1)}{\mu_0 \cdot \mathbb{P}(s_t|\theta = 1) \cdot \mathbb{P}(m_t|\theta = 1) + (1 - \mu_0) \cdot \mathbb{P}(s_t|\theta = 0) \cdot \mathbb{P}(m_t|\theta = 0)}
\]
Thus, we have

$$\frac{q(s_t, m_t)}{1 - q(s_t, m_t)} = \frac{\mu_0 \cdot \mathbb{P}(s_t|\theta = 1)}{\mu_0 \cdot \mathbb{P}(s_t|\theta = 1) + (1 - \mu_0) \cdot \mathbb{P}(s_t|\theta = 0)} \cdot \frac{\mathbb{P}(m_t|\theta = 1)}{\mathbb{P}(m_t|\theta = 1) + (1 - \mu_0) \cdot \mathbb{P}(m_t|\theta = 0)}$$

Due to the normalization that $s_t = \mathbb{P}(\theta = 1|s_t)$ and $m_t = \mathbb{P}(\theta = 1|m_t)$, we have

$$s_t = \frac{\mu_0 \cdot \mathbb{P}(s_t|\theta = 1)}{\mu_0 \cdot \mathbb{P}(s_t|\theta = 1) + (1 - \mu_0) \cdot \mathbb{P}(s_t|\theta = 0)}$$

$$m_t = \frac{\mu_0 \cdot \mathbb{P}(m_t|\theta = 1)}{\mu_0 \cdot \mathbb{P}(m_t|\theta = 1) + (1 - \mu_0) \cdot \mathbb{P}(m_t|\theta = 0)}$$

Hence,

$$\frac{\mathbb{P}(s_t|\theta = 1)}{\mathbb{P}(s_t|\theta = 0)} = \frac{s_t}{1 - s_t} \cdot \frac{1 - \mu_0}{\mu_0}$$

$$\frac{\mathbb{P}(m_t|\theta = 1)}{\mathbb{P}(m_t|\theta = 0)} = \frac{m_t}{1 - m_t} \cdot \frac{1 - \mu_0}{\mu_0}$$

Therefore,

$$q(s_t, m_t) = \frac{(1 - \mu_0) s_t m_t}{(1 - \mu_0) s_t m_t + \mu_0 (1 - s_t) (1 - m_t)}$$

Consumer $t$’s expected payoff from purchasing the product is $q(s_t, m_t) - p$. Thus, he strictly prefer to purchase the product if and only if $q(s_t, m_t) > p$.

Let $\mu_b$ and $\mu_n$ be the cutoffs such that the consumer will choose to acquire a signal and follow it if and only if $m_t \in [\mu_n, \mu_b]$, i.e. $[\mu_0 q_1 + (1 - \mu_b) (1 - q_2)] [q(h, \mu_b) - p] - c = \mu_b - p$ and $[\mu_n q_1 + (1 - \mu_n) (1 - q_2)] [q(h, \mu_n) - p] - c = 0$. For example, when $q_1 = 1$, we have

$$\mu_n = \frac{p + c - pq}{1 - pq}$$

$$\mu_b = \frac{pq - c}{pq}$$

Then, consumer $t$’s purchase decision can be summarized as follows.

**Lemma 4.1.** Given $m_t$, it is optimal for consumer $t$ to follow the private signal $s_t$ in deciding whether to purchase the product if $m_t \in [\mu_n, \mu_b]$, i.e. it is optimal for him to purchase the product if and only if $s_t = h$. It’s optimal for him to ignore the private signal and always purchase the product if $m_t \geq \mu_b$. It’s optimal for him to ignore the private signal and always not purchase the product if $m_t \leq \mu_n$. 8
5 Optimal Disclosure Mechanism

In this section, we will start by discussing the relevant histories that the seller can disclose, and then characterize the set of feasible direct mechanisms that the seller can implement by disclosing histories. Finally, we will characterize the optimal disclosure mechanism for the seller.

Before disclosing information to the consumer arriving in period \( t \), a history in \( H_{t-1} \) that the seller may observe consists of past signals \( \{m_i\}_{i=1}^{t-1} \) sent to earlier consumers, and past purchase decisions \( \{a_i\}_{i=1}^{t-1} \) of those consumers. For example, before the first consumer arrives, the seller observes no useful information. Thus regardless of the message \( m_1 \), consumer 1 will have a belief \( \mu_0 \) before observing \( s_1 \). Hence, as argued previously, it is without loss of generality to assume that \( m_1 = \mu_0 \) with probability 1. Since we assumed that \( \mu_0 \in [\mu_n, \mu_b] \), the first consumer with \( m_1 = \mu_0 \) will follow his private signal \( s_1 \) in determining whether to purchase the product. Hence, at the end of period 1, and before the second consumer arrives, there are two possible realizations of histories that the seller may observe:

\[
H_1 = \{(m_1 = \mu_0, a_1 = 1), (m_1 = \mu_0, a_1 = 0)\}.
\]

From an ex ante perspective, the history \( h_1 = (m_1 = \mu_0, a_1 = 1) \) realizes with probability \( \mu_0 q_1 + (1 - \mu_0)(1 - q_2) \), and the history \( h_1' = (m_1 = \mu_0, a_1 = 0) \) realizes with probability \( \mu_0 (1 - q_1) + (1 - \mu_0) q_2 \). \( H_1 = \{h_1, h_1'\} \) is the state space of the experiment \( m_2 \). Again, since the only payoff-relevant information contained in histories are the posterior belief of the state \( \theta = 1 \) it induces, we can, without loss of generality, take the set of posterior beliefs induced by histories as the state space. We use \( V_t \) to denote the set of posterior beliefs induced by histories in \( H_t \). For example, according to Bayes rule, \( V_1 = \{l, h\} \). In general, for \( t > 1 \), the set \( V_t \) has fewer elements than \( H_t \) because multiple histories may induce the same posterior belief. Let \( V = \cup_t V_t \).

Let \( \nu_t(h_t) \) denote the posterior belief of \( v = 1 \) upon observing the history \( h_t \). \( \nu_t(h_t) \) evolves according to Bayes rule, i.e. \( m_t \notin (\mu_n, \mu_b) \) and consumer \( t \) ignores his private signal, and will be updated according to the purchase decision if \( m_t \in [\mu_n, \mu_b] \) and consumer \( t \) follows his private signal. Therefore,

\[
\nu(h_t) = \frac{\mu_0 q_1^{n_1(h_t)}(1 - q_1)^{n_0(h_t)}(1 - \mu_0)(1 - q_2)q_2^{n_0(h_t)}}{\mu_0 q_1^{n_1(h_t)}(1 - q_1)^{n_0(h_t)} + (1 - \mu_0)(1 - q_2)q_2^{n_0(h_t)}}
\]
where \( n_1(h_t) \) (resp. \( n_0(h_t) \)) is the number of private signals equal to \( h \) (resp. \( l \)) that have been revealed to the seller through the history. In what follows, we will suppress \( h_t \) and write \( n_1, n_2 \) whenever it does not create confusion. Since \( n_1, n_2 \) summarizes all payoff-relevant information contained in \( h_t \), we will also write \( \nu(n_1, n_2) \) and \( \nu(h_t) \) interchangeably. We should note that for any \( t \), the set \( V_t \) can contain at most \((t+1) \cdot (t+2)/2\) values, but it may contain less if some consumers ignore their private signals in making purchase decisions. As an example, for the case when \( q_1 = 1, q_2 < 1 \), we have

\[
V_t = \left\{ 0, \frac{\mu_0}{\mu_0 + (1 - \mu_0)(1 - q_2)^t}, \ldots, \frac{\mu_0}{\mu_0 + (1 - \mu_0)(1 - q_2)^t} \right\}
\]

Thus, it is equivalent to consider mechanisms \( m = \{m_t\}_{t=1}^\infty \) with \( m_t : V_t \to \Delta([0,1]) \). We will use \( k_t \in \Delta(V_t) \) to represent the random variable of the posterior belief \( \nu_t \in V_t \) upon observing histories up to time \( t \). \( k_t \) represents the seller’s knowledge of the state \( v \) in time \( t \), and poses a upper bound (in the Blackwell order) on the informativeness of the signal \( m_t \) that can be disclosed to consumer \( t \).

The seller solves the following dynamic programming problem with the initial knowledge \( k_0 \) degenerate at \( \mu_0 \):

\[
W(k) = \max_m \Pi(\hat{k}) + \delta W(\tilde{k})
\]

\[
s.t. \quad \hat{k} = \langle m | k \rangle \\
\tilde{k} = k(\mu)(1 - m([\mu_n, \mu_b] | \mu)) \\
+ \sum_{\mu' s.t. \mu' = \nu(n_1, n_0), \mu = \nu(n_1, n_0)} k(\mu') m([\mu_n, \mu_b] | \mu')(\mu_0 q_1 + (1 - \mu_0)(1 - q_2)) \\
+ \sum_{\mu' s.t. \mu' = \nu(n_1, n_0), \mu = \nu(n_1, n_0 + 1)} k(\mu') m([\mu_n, \mu_b] | \mu')(\mu_0(1 - q_1) + (1 - \mu_0)q_2)
\]

where

- \( k \in \Delta(V) \) is the knowledge of the seller at the beginning of the period
- \( m : V \to \Delta(M) \) is the experiment used to disclose signals to the current consumer about information of histories contained in \( k \)
- \( \hat{k} = \langle m | k \rangle \in \Delta([0,1]) \) is the distribution over posterior beliefs induced by the experiment \( m \) given \( k \), i.e. the current consumer’s knowledge of the state \( v \)
\[ \Pi(\hat{k}) = pk(\lbrack \mu_b, 1 \rbrack) + \sum_{\nu \in [\mu_n, \mu_b]} (\nu q_1 + (1 - \nu)(1 - q_2)) p(\nu) \] is the seller’s profit for the current period, where \( \hat{k}(\lbrack \mu_b, 1 \rbrack) \) and \( \hat{k}(\nu) \) are the probabilities that the random variable \( \hat{k} \) takes values in the respective intervals.\(^3\)

- Condition (2) governs how the evolution of the seller’s knowledge across periods. In terms of notation, \( m([\mu_n, \mu_b]|\mu') \) is the probability that a message inducing the consumer to follow his private signal is generated, conditional on the posterior belief \( \mu' \) (induced by the realized history).

If \( \mu_0 \notin [\mu_n, \mu_b] \), then the first consumer will ignore his private signal in deciding whether to purchase the product. Hence, his action will be not informative about quality of the product, and the seller has no useful information to disclose to subsequent consumers, and therefore, all subsequent consumers will behave exactly the same as the first consumer. Thus, to focus on the non-trivial cases, we assume \( \mu_0 \in [\mu_n, \mu_b] \).

### 5.1 Perfect Bad News

In this section, we characterize the optimal disclosure mechanism for the seller, when the signal structure of \( s_t \) is in the form of “perfect bad news”, i.e. \( q_1 = 1 \). We will show that the optimal disclosure mechanism in this case consists of two phases, where no information is disclosed in the first few periods, and the static Bayesian persuasion information structure is induced afterwards.

**Definition 5.1.** The “delayed garbling” disclosure mechanism is such that:

- for all \( t \leq \bar{t} \),
  \[ m_t(\mu_0|\nu_t) = 1 \text{ for all } \nu_t \in V_t; \]

- for \( t > \bar{t} \), then \( m_t \) is such that
  \[ \hat{k}_t(\mu_n) = \frac{\mu_b - \mu_0}{\mu_b - \mu_n} \text{ and } \hat{k}_t(\mu_b) = \frac{\mu_0 - \mu_n}{\mu_b - \mu_n}. \]

\(^3\)When the consumer has a posterior belief in \( [\mu_b, 1] \) he will purchase the product with probability one, generating profit \( p \) for the seller. When the consumer has a posterior belief \( \nu \in [\mu_n, \mu_b] \), he will purchase the product if and only if he observes a high signal \( s_t = h \), which happens with probability \( \nu q_1 + (1 - \nu)(1 - q_2) \), generating an expected profit \( [\nu q_1 + (1 - \nu)(1 - q_2)]p \) for the seller.
where \( \hat{k}_t = (m_t | k_{t-1}) \) and \( \bar{t} = \max\left\{ t : \frac{\mu_0 q_t^{t-1}}{\mu_0 q_t^{t-1} \cdot (1-\mu_0)(1-q_2)^{t-1}} \leq \mu_b \right\} \).

As it turns out, the “delayed garbling” disclosure mechanism is optimal under the “perfect bad news” signal structure.

**Theorem 5.1.** If \( q_1 = 1 \), the “delayed garbling” mechanism is the optimal mechanism for the seller.

Intuitively, when only a few consumers have arrived, the optimal disclosure mechanism does not reveal any information to the consumer regarding past purchases of other consumers. When sufficient number of consumers have arrived, depending on the parameter values, the optimal disclosure mechanism garbles the information in a particular way to induce the consumer to randomize between purchasing and following his private signal, or randomize between purchasing and not purchasing. One should note that the optimal mechanism has the the same qualitative feature as the disclosure policy adopted by Bookings.com.

The formal proof is relegated to the appendix. I will summarize the main intuition behind the proof here. When the seller decides how to disclose information to the consumer in a particular period, two effects matter for him: direct profit in the current period and gain in information by observing the consumer’s purchase decision if he follows his private signal. If the consumer decide to purchase in this period, the seller earns profit directly, thus the seller want the consumer to purchase as much as possible. On the other hand, if the consumer follows his private signal, the signal realization will be revealed to the seller through the consumer’s decision to purchase or not, and can be used in future periods to persuade subsequent consumers to purchase the product.

Note that the static Bayesian persuasion information structure is a mixture between \( \mu_n \) and \( \mu_b \) if and only if \( \mu_n q_1 + (1-\mu_n)(1-q_2) \geq \mu_n/\mu_b \), which is always satisfied under the assumption \( \mu_b > \mu_n \), or equivalently \( c < (1-p)pq \).

In the first \( \bar{t} \) periods, the direct profit and the information gain are both maximized by disclosing no information to the consumers: there is not yet enough information to induce the consumer to choose \( b \) (i.e. purchase the product regardless of his private signal), and thus, inducing the consumer to acquire a signal maximizes both the profit in the current period and gain of knowledge for the seller. From period \( \bar{t} + 1 \) onward, the direct
profit and the information gain may not be fully aligned anymore, because information
accumulated so far is already enough for the seller to induce the seller to purchase the
product without acquiring a signal. However, it turns out, from period \( t + 1 \) onward, if
\( q_t \) is large enough, the marginal benefit to the seller for accumulating more knowledge is
zero (formally shown in the appendix). In this regard, the seller only need to maximize
his current period profit, which involves a delicate garbling of the information as in the
static Bayesian persuasion.

6 Welfare Analysis

In this section, we will compare the consumers’ welfare under the optimal mechanism
characterized in Section 5 to the case when past purchase data are not disclosed. The
no-disclosure scenario serves a good benchmark for several reasons. First, it is the default
situation for sellers that do not operate online, and even among online sellers, only a very
small fraction disclose any information regarding past purchases. Thus, it is important to
understand the welfare consequences as information technology makes it easier for sellers
to collect and disclose such information.

As is shown below, allowing seller’s to optimally disclose data of past purchases will
increase seller’s surplus, and render consumers’ welfare exactly the same as when no
information is disclosure. Two effects on consumers’ welfare exactly offset each other: the
consumers gain by saving costs of acquiring outside information, but lose due to some
times herding on buying the product with poor quality. Overall, allowing the seller to
disclose purchase data helps achieve a Pareto improvement for the sellers and consumers.
The result is summarized in Theorem 6.1.

**Theorem 6.1.** The consumer is indifferent between the no-disclosure arrangement and
the seller’s optimal disclosure mechanism characterized in Theorem 5.1, and the seller
strictly prefers the optimal mechanism in Theorem 5.1.

**Proof.** Under no disclosure, all consumers will have a belief \( \mu_0 \in [\mu_n, \mu_b] \), and thus will all
acquire a signal. Thus, the consumer’s expected surplus is

\[
\mu_0(1 - p - c) + (1 - \mu_0)(- (1 - q)p - c) = \mu_0(1 - pq) - (1 - q)p - c
\]
Under the seller’s optimal mechanism, the consumers will not observe no useful information from the seller in the first \( t \) periods, and thus have exactly the same payoff. From period \( t+1 \) onward, the consumers will either acquire a signal or herd on purchasing the product, depending on the realized history. From period \( t \) onward, the consumer’s expected surplus is

\[
\frac{\mu_0 - \mu_n}{\mu_b - \mu_n}(\mu_b - p) + \frac{\mu_b - \mu_0}{\mu_b - \mu_n}(\mu_n(1-p) - (1-\mu_n)(1-q)p - c)
\]

We notice that, under both the no-disclosure arrangement, and the seller’s optimal mechanism, the first \( t \) consumers behave the same: they all acquire an outside signal and follows it. Thus, in order to show that the consumers’ welfare is exactly the same under no-disclosure, and under seller’s optimal mechanism, it is sufficient to show that the per-period consumer welfare after period \( t \) is exactly the same across the two settings, i.e.

\[
\left[\frac{\mu_0 - \mu_n}{\mu_b - \mu_n}(\mu_b - p) + \frac{\mu_b - \mu_0}{\mu_b - \mu_n}(\mu_n(1-p) - (1-\mu_n)(1-q)p - c)\right] = (\mu_0 pq - (1-q)p)
\]

Plugging in \( \mu_b = 1 - c/pq \) and \( \mu_n = (p + c - pq)/(1-pq) \), we have that the LHS is equal to

\[
\frac{1}{\mu_b - \mu_n} \left( (\mu_0 - \mu_n)\mu_b + (\mu_b - \mu_0)p \right) - p
\]

\[
= \frac{pq(1-pq)}{pq - p^2q - c} \left( \frac{\mu_0 - p + c - pq}{1-pq} \frac{pq - c}{pq} + \frac{pq - c - pq}{pq} - \mu_0 \right) - p
\]

\[
= \mu_0(1-pq) + \frac{pq(1-pq)}{pq - p^2q - c} \frac{pq - c - pq - p^2q - c}{pq} - p
\]

\[
= \mu_0(1-pq) - p(1-q) - c
\]

Thus, the condition is satisfied, and consumers are indifferent. By definition of that the mechanism, the seller is strictly better off than under the no disclosure. Thus, the mechanism induces a Pareto improvement.

\[\square\]

7 Pricing

7.1 Optimal Pricing

Now, we characterize the optimal price for the seller if he is allowed to set a fixed price \( p \) at \( t = 0 \) which will be posted for consumers arriving in all periods.
We first solve for the benchmark case with no disclosure. In this case, if he sets a price such that \( \mu_0 < \mu_n(p) \), he will get zero profit. If the seller sets a price such that \( \mu_n(p) \leq \mu_0 < \mu_b(p) \), he will get an expected profit of \( \frac{\delta}{1-\delta}(\mu_0 p q + (1 - q) p) \). In this case, the best he can do is to set price such that \( \mu_n(p) = \mu_0 \), i.e. \( p = \frac{\mu_0 - c}{(1 - q + pq)} \), which gives him expected profit of \( \frac{\delta}{1-\delta}(\mu_0 - c) \). On the other hand, if he sets a price such that \( \mu_0 \geq \mu_b(p) \), he gets an expected profit of \( \frac{\delta}{1-\delta} p \). The best he can get in this case is by setting \( p = \frac{c}{(1 - \mu_0) q} \) such that \( \mu_b(p) = \mu_0 \), which gives him an expected profit of \( \frac{\delta}{1-\delta} \frac{c}{(1 - \mu_0) q} \). Therefore, we know that the seller’s optimal price is \( p = \frac{\mu_0 - c}{1 - q + pq} \) if and only if

\[
\mu_0 - c \geq \frac{c}{(1 - \mu_0) q} \quad \iff \quad c \leq \frac{\mu_0 (1 - \mu_0) q}{1 - (1 - \mu_0) q}.
\]

While when the condition is violated, the optimal price is \( p = \frac{\mu_0 - c}{1 - q + pq} \). We should note that it was assumed \( \mu_0 \in [\mu_n, \mu_b] \) which implies

\[
\begin{align*}
\mu_0 < \mu_b &= 1 - \frac{c}{pq} \iff c < (1 - \mu_0) pq \\
\mu_0 > \mu_n &= \frac{p + c - pq}{1 - pq} \iff p < \frac{\mu_0 - c}{1 - (1 - \mu_0) q}
\end{align*}
\]

Thus,

\[c < \frac{\mu_0 - c}{1 - (1 - \mu_0) q} \cdot (1 - \mu_0) q < \frac{\mu_0 (1 - \mu_0) q}{1 - (1 - \mu_0) q}.
\]

So it is optimal to set price at \( p = \frac{\mu_0 - c}{1 - q + pq} \) (i.e. \( \mu_n(p) = \mu_0 \) and all consumers are induced to acquire their private signal \( s_t \)) if purchase history is not disclosed to consumers.

Now, we consider the optimal price under the seller’s optimal disclosure policy. If the seller sets a price such that \( \mu_0 \geq \mu_b(p) \), then he gets an expected profit of \( \frac{\delta}{1-\delta} p \). If he sets a price such that \( \mu_0 \in [\mu_n(p), \mu_b(p)] \), he will get an expected profit as derived in the proof of Theorem 5.1. Thus, the seller’s expected profit is

\[
\begin{align*}
\sum_{t=1}^{\infty} \delta^t (\mu_0 p + (1 - \mu_0)(1 - q) p) + \sum_{t=1}^{\infty} \delta^t \left[ \frac{\mu_0 - \mu_n}{\mu_b - \mu_n} p + \frac{\mu_b - \mu_0}{\mu_b - \mu_n} (\mu_n + (1 - \mu_n)(1 - q)) p \right] \\
= p \left\{ \frac{\delta - \delta^i}{1 - \delta} (\mu_0 + (1 - \mu_0)(1 - q)) + \frac{\delta^i - 1}{1 - \delta} \frac{(1 - p) pq - c - q(1 - p - c)[pq(1 - \mu_0) - c]}{(1 - p) pq - c} \right\}
\end{align*}
\]

Let’s consider the case when \( \delta \to 0 \), i.e. the seller only care about the long term steady state profit. In this case, the seller maximizes

\[
\Pi(p) = \frac{(1 - p) pq - c - q(1 - p - c)[pq(1 - \mu_0) - c]}{(1 - p) pq - c} \cdot p
\]

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Let $X(p) = (1 - p)pq - c - q(1 - p - c)[pq(1 - \mu_0) - c]$. We have

$$
\Pi'(p) = \frac{[X(p) + pX'(p)][(1 - p)pq - c] - X(p) pq(1 - 2p)}{[(1 - p)pq - c]^2} = \frac{X(p)(pq^2 - c) + pX'(p)(pq - pq^2 - c)}{[(1 - p)pq - c]^2}
$$

where

$$
X'(p) = q - 2pq + 2pq^2(1 - \mu_0) - qc - q^2(1 - \mu_0) + cq^2(1 - \mu_0)
$$

We have

$$
X(p)(pq^2 - c) = p^3q^2 - p^4q^2 - p^3q^3(1 - \mu_0) + \mu_0 pq^2q^3 c + p^4 q^3(1 - \mu_0)
$$

$$
- p^3q^2 c + p^3q^3c(1 - \mu_0) - p^2 q^2 c^2 - pq c + c^2 +
$$

$$
pq^2 c(1 - \mu_0) - qc^2 + pq c^2 - pq^2 c^2(1 - \mu_0) + qc^3
$$

$$
pX'(p)(pq - pq^2 - c) = p^2 q^2 - 3p^3q^2 + p^3q^3(1 - \mu_0) - p^2 q^2 c(3 - 2\mu_0) - p^2 q^3(1 - \mu_0)
$$

$$
+ p^2 q^3 c(1 - \mu_0) + 2p^4 q^2 - 2p^4 q^3(1 - \mu_0) + p^3 q^2 c - p^3 q^3 c(1 - \mu_0)
$$

$$
- pq c + 2pq c^2 + pq c^3 + pq^2 c(1 - \mu_0) - pq^2 c^2(1 - \mu_0)
$$

Thus,

$$
\Pi'(p) \cdot [(1 - p)pq - c]^2 = -2p^3q^2 - p^2 q^2 c^2 - 2pq c + c^2 + 2pq c +
$$

$$
2pq^2 c(1 - \mu_0) - qc^2 + 2pq c^2 - 2pq^2 c^2(1 - \mu_0) + qc^3
$$

$$
+ p^2 q^2 + 2p^3 q^3(1 - \mu_0) - 3p^2 q^2 c(1 - \mu_0) - p^2 q^3(1 - \mu_0)
$$

$$
+ p^2 q^3 c(1 - \mu_0) + p^4 q^2 - p^4 q^3(1 - \mu_0)
$$

$$
= p^2 q^2 (1 - (1 - \mu_0)q)(p - 1)^2
$$

$$
+ qc^3 + c^2(1 - p^2 q^2 - q + 2pq - 2pq^2)(1 - \mu_0)
$$

$$
+ c(2pq^2 - 2pq + 2pq^2)(1 - \mu_0) - 3p^2 q^2(1 - \mu_0) + pq^3(1 - \mu_0))
$$

**Theorem 7.1.** For $\delta \to 0, c \to 0$, the optimal price is $p = \frac{\mu_0 - c}{1 - q - \mu_0}$, i.e. $\mu_n(p) = \mu_0$.

**Proof.** First notice that as $\lim_{c \to 0} \Pi'(p) > 0$ for all $p$ such that $\mu_0 \in [\mu_n(p), \mu_b(p)]$. If $p$ is set too large such that $\mu_0 < \mu_n(p)$, then there will be no purchase in all periods. In the interval such that $\mu_n(p) \leq \mu_0$, as $p$ decreases, $t$ will jump downward discretely as a step.
function. If the seller cares only about profits after the first \( t \) periods, he will set \( p \) such that \( \mu_n(p) = \mu_0 \) since \( \Pi'(p) > 0 \). For \( \delta \) close to 0, it would be profitable for him to lower the price to decrease \( \bar{t} \) for even one period.

Thus, the optimal prices for both the no-disclosure regime and the seller optimal disclosure regime are the same, i.e. \( \mu_n(p) = \mu_0 \). Therefore, the argument that the seller optimal disclosure generates Pareto improvement is preserved even if the seller optimally set a fixed price.

7.2 Dynamic Pricing

In the previous sections, we assumed that the product has an exogeneously fixed price. However, in principle, it is very easy for platforms such as Bookings.com to dynamically set prices. In fact, prices on such online platforms indeed fluctuates across time. In this section, we analyze the optimal strategy for the seller when he is allowed to set prices in each period, and design the disclosure mechanism at the same time.

Consider any history \( h_t \) upon which the buyer has a posterior belief \( \mu_t \), we first characterize the optimal price that the seller should set given \( \mu_t \).

Notice first that if the price \( p \) is such that the buyer will choose \( n \) (not to purchase the product without acquiring a signal), it will result in zero profit in the current period and no information gain for the seller. This is clearly a dominated strategy for the seller. Thus, we don’t need to consider such cases.

**Lemma 7.2.** Given \( \mu_t \), if the seller set a price that induces \( b \) from the consumer, the optimal price is

\[
p_t^* = \begin{cases} 
\frac{c}{(1-\mu_t)q}, & \text{if } \mu_t \in [x^-, x^+] \\
\mu_t, & \text{if } \mu_t < x^- \text{ or } \mu_t > x^+
\end{cases}
\]

where \( x^- = \frac{1 - \sqrt{1 - \frac{x^+}{2}}}{2} \) and \( x^+ = \frac{1 + \sqrt{1 - \frac{x^-}{2}}}{2} \).

**Proof.** If the seller want to induce \( b \) from the buyer, according to Lemma ??, he can set \( p_t \) such that either \( \mu_t \geq \mu_b(p_t) \) and \( c \leq (1 - p_t)p_0q, \) or \( \mu_t \geq p_t \) and \( c \geq (1 - p_t)p_0q \). Note that \( x^-, x^+ \) are the two roots of the function \( f(x) = x^2 - x + c/q \). Thus, \( f(x) \leq 0 \) if and only if \( x \in [x^-, x^+] \). According to the conditions above, to induce \( b \), the seller can set prices such that either \( p_t \leq \frac{c}{(1-\mu_t)q} \) and \( p_t \in [x^-, x^+] \), or \( p_t \leq \mu_t \) and \( p_t \in [0, x^-] \cup [x^+, 1] \).
If $\mu_t \in [x^-, x^+]$, then we have $\mu_t(1 - \mu_t)q \geq c$ which implies $x^- \leq \frac{c}{(1 - \mu_t)q} \leq \mu_t \leq x^+$. Thus, when $\mu_t \in [x^-, x^+]$, the highest price that can induce $b$ is $\frac{c}{(1 - \mu_t)q}$. If $\mu_t \geq x^+$, we have $x^+ \leq \mu_t \leq \frac{c}{(1 - \mu_t)q}$. Thus, when $\mu_t \geq x^+$, the highest price that can induce $b$ is $\mu_t$. Similarly, when $\mu_t \leq x^-$, we have $\mu_t \leq \frac{c}{(1 - \mu_t)q} \leq x^-$, so the highest price that induces $b$ is $\mu_t$. \hfill \Box

Lemma 7.3. Given $\mu_t$, if $\mu_t < x^-$ or $\mu_t > x^+$, it is impossible for the seller to induce $e$. If $\mu_t \in [x^-, x^+]$, it is possible for the seller to induce $e$, and the optimal price is

$$p_t = \frac{\mu_t - c}{1 + \mu_t q - q}.$$

Proof. If the seller want to induce $e$ from the buyer, according to Lemma 7.2, he must set $p_t$ such that $c \leq (1 - p_t)p_t q$ and $\mu_n(p_t) \leq \mu \leq \mu_b(p_t)$, or equivalently, $c \leq (1 - p_t)p_t q$ and $p_t \in \left(\frac{c}{(1 - \mu_t)q}, \frac{\mu_t - c}{1 + \mu_t q - q}\right]$. First note that $p_t \leq \frac{\mu_t - c}{1 + \mu_t q - q}$ which implies $c \leq \mu_t(1 - \mu_t)q$. Therefore, a necessary condition for the existence of a price that induce $e$ is $\mu_t \in [x^-, x^+]$. If $\mu_t \in [x^-, x^+]$, we know that $x^- \leq \frac{c}{1 - \mu_t q} \leq \mu_t \leq x^+$. Thus, the optimal price is the lower value of $x^+$ and $\frac{\mu_t - c}{1 + \mu_t q - q}$. The argument below shows that $\frac{\mu_t - c}{1 + \mu_t q - q} \leq x^+$ when $\mu_t \in [x^-, x^+]$: a sufficient condition for if $f(\frac{\mu_t - c}{1 + \mu_t q - q}) \leq 0$ which is equivalent to

$$\frac{\mu_t - c}{1 + \mu_t q - q} \cdot \left(1 - \frac{\mu_t - c}{1 + \mu_t q - q}\right) q \geq c$$

$$\Leftrightarrow -\mu_t q^2 (1 - c)(1 - \mu_t) + \mu_t (1 - \mu_t)q + c(1 - c)q \geq 0$$

$$\Leftrightarrow [1 - q(1 - c)]\mu_t(1 - \mu_t)q + c(1 - c)q \geq 0$$

It should now be obvious that the LHS is always positive. \hfill \Box

According to the two lemmas above, we get the following proposition.

Proposition 7.4. Assume that $(1 - c)^2 - 4c/q > 0$, i.e. $c$ is small enough. Given $\mu_t$, the seller’s optimal price is

$$p_t = \begin{cases} 
\frac{\mu_t - c}{1 + \mu_t q - q}, & \text{if } \mu_t \in [a^-, a^+] \\
\frac{c}{(1 - \mu_t)q}, & \text{if } \mu_t \in [x^-, a^-] \cup [a^+, x^+] \\
\mu_t, & \text{if } \mu_t \in [0, x^-] \cup [x^+, 1] 
\end{cases}$$

where $a^- = \frac{1 + c - \sqrt{(1 - c)^2 - 4c/\mu_t q}}{2}$ and $a^+ = \frac{1 + c + \sqrt{(1 - c)^2 - 4c/\mu_t q}}{2}$.

Proof. It is clear that when $\mu_t \notin [x^-, x^+]$, the seller’s optimal price is $\mu_t$. If $\mu_t \in [x^-, x^+]$, the seller will choose between $p_t^* = \frac{c}{(1 - \mu_t)q}$ which generates profit $\frac{c}{(1 - \mu_t)q}$, and $\tilde{p}_t = \frac{\mu_t - c}{1 + \mu_t q - q}$.
which generates profit
\[
\frac{\mu_t - c}{1 + \mu_t q - q} \cdot (\mu_t + (1 - \mu_t)(1 - q)) = \mu_t - c.
\]
Thus, the seller prefers \(p_t^*\) over \(\hat{p}_t\) if and only if
\[
\frac{c}{(1 - \mu_t)q} \geq \mu_t - c \\
\iff (\mu_t - c)(1 - \mu_t)q \leq c
\]
which is again a quadratic inequality. Denote the roots of the inequality as \(a^-, a^+\) where
\[
a^- = \frac{1 + c - \sqrt{(1 - c)^2 - 4c/q}}{2} \quad \text{and} \quad a^+ = \frac{1 + c + \sqrt{(1 - c)^2 - 4c/q}}{2}.
\]
We have that
\[
a^- = \frac{1 + c - \sqrt{(1 - c)^2 - 4c/q}}{2} > \frac{1 - \sqrt{1 - 4c/q}}{2} = x^-
\]
Next, we show that \(a^+ < x^+\). Note that
\[
(2x^+ - 1)^2 = 1 - 4c/q \\
(2a^+ - 1)^2 = \left(c + \sqrt{(1 - c)^2 - 4c/q}\right)^2 \\
= c^2 + (1 - c)^2 - 4c/q + 2c\sqrt{(1 - c)^2 - 4c/q} \\
= (1 - 4c/q) + 2c^2 - 2c + 2c\sqrt{(1 - c)^2 - 4c/q} \\
< (1 - 4c/q) + 2c^2 - 2c + 2c[(1 - c)^2 - 4c/q] \\
= (1 - 4c/q) - 2c^2 + 2c^3 - 8c^2/q \\
< 1 - 4c/q
\]
where the inequalities are due to facts that \(0 \geq (1 - c)^2 - 4c/q < 1\) and \(c^3 < c^2\). Since \(a^+\) and \(x^+\) are both larger than \(1/2\), we conclude that \(a^+ < x^+\). The expression of the optimal price follows immediately.

Thus, the expected profit in the current period that the seller can generate by optimally choosing prices, given \(\mu_t\), is
\[
\pi(\mu_t) = \begin{cases} 
\mu_t - c, & \text{if } \mu_t \in [a^-, a^+] \\
\frac{c}{(1 - \mu_t)q}, & \text{if } \mu_t \in [x^-, a^-] \cup [a^+, x^+] \\
\mu_t, & \text{if } \mu_t \in [0, x^-] \cup [x^+, 1]
\end{cases}
\]

The relationship between posterior means and seller’s expected profit under the optimal price is shown in Figure 2. The reason that the function is convex over the interval
\([x^-, x^+]\) is that if the posterior mean of the consumer falls in this interval, and if the seller posts a price equal to the posterior mean hoping to fully extract the consumer’s surplus, the consumer will then acquire a signal to learn about the quality of the product instead of purchasing the product directly, thus rendering the seller’s hope for full surplus extraction void. Given the function is convex, it is clear that if the prior mean is not too extreme, it would be beneficial for the seller to garble the information regarding past purchases or to lower price to induce exploration. It is unclear for know which of the two tools (garble of information and pricing) would be more effective. In the next section, we will characterize the optimal strategy for the seller. We are interested in the question: would the ability of dynamic pricing crowd out the use of information garbling, or vice versa?

![Figure 2: Seller Myopic Optimal Profit given Different Posteriors](image)

The seller’s optimal strategy, consisting of pricing as well as disclosure, depends on the prior belief of the consumers \(\mu_0\). It is obvious that if \(\mu_0 < x^-\) or \(\mu_0 > x^+\), not disclosing any information about past purchases to each consumer and setting prices \(p^t = \mu_0\) is optimal for the seller, in which case the seller fully extracts the surplus. The reason is that the per-period value function of the seller \(\pi(\cdot)\) coincides with its concavification when \(\mu < x^-\) or \(\mu > x^-\). To see this, note that even if the seller knows the exact value of \(v\) from the beginning (at \(t = 0\)), he would not want to disclose any information to the buyers. The same strategy is feasible for the seller when he does not know any information about \(v\) at \(t = 0\), and is therefore optimal.

We also note that in the hypothetical case where the seller has full knowledge of \(v\)
at $t = 0$, the optimal strategy is to reveal all the information to the consumer, and then charge $p_t = 1$ if $v = 1$ and $p_t = 0$ when $v = 0$. In this case, the seller also fully extracts the surplus.

The more interesting case occurs when $\mu_0 \in [x^-, x^+]$.

**Lemma 7.5.** It is without loss of generality to consider strategies in which whenever the seller induces a posterior $\mu \in [x^-, a^-]$, he sets a price $p_t = \mu - c$ to induce the buyer to acquire a signal. The same argument in the is also true for $[a^+, x^+]$ if the model primitives is such that the support of $\nu$, i.e. $\cup_t V_t$ does not have an element in the interval $[a^+, x^+]$.

**Proof.** If $\mu \in [x^-, a^-]$ is induced and $b$ is induced, a mean preserving spread to replace the probability mass on $\mu$ with a distribution over $x^-$ and $a^-$ will generate higher profit for the seller, induce more exploration, and is feasible (because the support of the distribution over posteriors does not have any mass point in $[x^-, a^-]$). Similarly, if $\mu \in [a^+, x^+]$ is induced, a mean preserving spread to replace the probability mass on $\mu$ with a distribution over $a^+$ and $x^+$ will generate higher profit for the seller, induce more exploration, and is feasible. \qed

**A Special Case**

To get some sense of what the optimal strategy looks like, we start by considering the special case in which

$$\begin{align*}
\mu_1 &= \frac{\mu_0}{\mu_0 + (1 - \mu_0)(1 - q)} < a^+ \\
\mu_2 &= \frac{\mu_0}{\mu_0 + (1 - \mu_0)(1 - q)^2} > x^+
\end{align*}$$

The two conditions are satisfied when $q$ is large and $c$ is relatively small, for example $p = \mu_0 = 0.5$, $q = 0.8$ and $c = 0.05$. The special case provides the simplest setting in which there is interesting trade-offs of disclosing more information or not.

It is obvious that the first consumer arriving will acquire an outside signal. Thus, at $t = 2$, the support of $\nu_2$ is $V_2 = \{0, \mu_1\}$. If $\epsilon_2(\mu_1) \in (0, 1)$, then at $t = 3$, the support of $\nu_3$ is $V_3 = \{0, \mu_1, \nu_2\}$. If $\epsilon_2(\mu_1) = 0$, then $V_3 = \{0, \mu_1\}$; while if $\epsilon_2(\mu_1) = 1$, then $V_3 = \{0, \mu_2\}$.

**Claim 7.6.** Conjecture: under some assumptions on the parameter, it is without loss of generality to assume that each period the designer only recommends $e$ at $\mu_1$.  

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Suppose the above conjecture is correct. Then the designer’s problem can be transformed into a dynamic programming problem with a one-dimensional choice variable, because all he needs to take care is how much measure at \( \mu_1 \) should be recommended for experimentation, and how much should be recommended for direct purchase.

8 Evolving Product Quality

In the previous sections, consumers’ valuation of a product is constant over time. However, a lot of factors may cause the quality of a product to fluctuate. For example, facilities of the hotels on Bookings.com are constantly updated, and the employees come and go. In addition, consumers’ taste may change over time. This is another reason that we may expect their valuation of a product to change. In this section, we study the optimal disclosure of purchase histories under evolving product quality/valuation. 4

Suppose \( s_t \) follows the following signal structure:

\[
Pr(s_t = 1|v = 1) = q_1, Pr(s_t = 0|v = 0) = q_2
\]

and that the product quality \( v_t \in \{0, 1\} \) is evolving according to the following Markov process:

\[
v_t = \begin{cases} 
  v_{t-1}, & \text{with probability } p_v \\
  1 - v_{t-1}, & \text{with probability } 1 - p_v 
\end{cases}
\]

Suppose at the end of period \( t \), the seller has a distribution over posteriors \( \tau_t \). Let \( \tau_{t+1} \) the distribution over posteriors at the beginning of period \( t + 1 \). For each \( \nu \) on the support of \( \tau_t \), the measure \( \tau_t(\nu) \) will be shifted to

\[
\nu p_v + (1 - \nu)(1 - p_v) = (2p_v - 1)\nu + (1 - p_v)
\]

under \( \tau_{t+1} \), i.e. \( \tau_{t+1}((2p_v - 1)\nu + (1 - p_v)) = \tau_t(\nu) \) for all \( \nu \). Equivalently,

\[
\tau_{t+1}(\nu) = \tau_t \left( \frac{1}{2p_v - 1} \nu - \frac{1 - p_v}{2p_v - 1} \right).
\]

Therefore, for posteriors \( \nu \) at the beginning of period \( t \) upon which the consumer is recommended action \( b \) or \( n \), the measure \( \tau_t(\nu) \) will be shifted to \( \tau_{t+1}((2p_v - 1)\nu + (1 - p_v)) \). For posteriors \( \nu \) upon which the consumer is recommended \( e \), the measure \( \tau_t(\nu) \) will be

4Moscarini et al. (1998) analyzed a similar model where the underlying state of the world is evolving.
split to $\tau_{t+1}( (2p_v - 1) \frac{\nu q_1}{\nu q_1 + (1 - \nu)(1 - q_2)} + (1 - p_v) ) = (\nu q_1 + (1 - \nu)(1 - q_2)) \tau_t(\nu)$ and $\tau_{t+1}( (2p_v - 1) \frac{\nu (1 - q_1)}{\nu (1 - q_1) + (1 - \nu) q_2} + (1 - p_v) ) = (\nu (1 - q_1) + (1 - \nu) q_2) \tau_t(\nu)$.

Figure 3: Evolution of Posteriors when quality evolves

In Figure 3, I show the simulated distribution of posteriors if the seller fully disclose past purchases, under the parameters $p_v = 0.95, \mu_0 = 0.4, c = 0.002, p = 0.5, q_1 = 0.7, q_2 = 0.9$. The red line is the the mapping of posteriors if the consumer followed his own signal and the revealed action is to purchase the product. The yellow line is the the mapping of posteriors if the consumer followed his own signal and the revealed action is to not purchase the product. The purple line is the mapping if the consumer herds and didn’t follow his private signal. The green line is the 40 degree line. Let $\nu^+, \nu^-$ be the x-coordinates of the intersections of the red line and the yellow line with the green line. It is obvious that the distribution of posteriors will always have support in the interval $[\nu^-, \nu^+]$. It’s also easy to see that $\nu^+$ is increasing in $p_v$, and it is equal to $\frac{1}{2}$ when $p_v = \frac{1}{2}$. Thus, we have the following results.

**Theorem 8.1.** If $\mu_0 > \frac{1}{2}$, there is a threshold $p_v < 1$ such that if $p_v < p_v$, “no disclosure in all periods” is optimal for the seller.

**Proof.** If $\mu_0 > \frac{1}{2}$, and $\mu_0 \in [\mu_n, \mu_0]$. Then we know that $\mu_0 > \frac{1}{2}$. Therefore, for $p_v > \frac{1}{2}$ and sufficiently close to $\frac{1}{2}$, we have $\nu^+ < \mu_0$. In this case, the distribution of posterior means that the seller faces at the beginning of each period has a support on $[0, \mu_0]$, so the optimal disclosure is always to disclose no information as the concavification of the profit function coincides with the profit function itself. □
9 General Signal Structure

In the baseline model, the outside costly signal that consumers can collect on their own is of the “perfect bad news” feature. In this section, we extend the model by allowing the signal structure of \( s_t \) to be such that

\[
Pr(s_t = 1|v = 1) = q_1 \quad \text{and} \quad Pr(s_t = 0|v = 0) = q_2
\]

for any \( q_1, q_2 \in [0, 1] \). Without loss of generality, we assume \( q_1 > 1 - q_2 \),\(^5\) so after observing \( s_t = 1 \), one updates his belief towards \( v = 1 \). It will be shown that the optimality of “delayed garbling” is preserved for a general class of parameter values.

Given this signal structure, upon observing a history \( h \) which reveals that \( n_1 \) private signals observed to be 1, and \( n_2 \) private signals to be 0, the seller’s posterior will be

\[
\nu(n_1, n_2) = \frac{\mu_0 q_1^{n_1}(1 - q_1)^{n_2}}{\mu_0 q_1^{n_1}(1 - q_1)^{n_2} + (1 - \mu_0)(1 - q_2)^{n_1} q_2^{n_2}}
\]

The ex ante probability that a history with \( n = n_1 + n_2 \) revealed signals,\(^6\) leads to a posterior \( \nu(n_1, n_2) \) is

\[
\tau(n_1, n_2) = \mu_0 \left( \frac{n}{n_1} \right) q_1^{n_1}(1 - q_1)^{n_2} + (1 - \mu_0) \left( \frac{n}{n_1} \right) (1 - q_2)^{n_1} q_2^{n_2}
= \left( \frac{n}{n_1} \right) [\mu_0 q_1^{n_1}(1 - q_1)^{n-n_1} + (1 - \mu_0)(1 - q_2)^{n_1} q_2^{n_2-n_1}]
\]

The consumer decision is also different under this information structure. At a posterior \( \mu \), if the consumer does not acquire \( s_t \) and purchase the product, his payoff is \( \mu - p \). If the consumer acquire the signal, and observe \( s_t = 1 \), he will update his posterior belief to

\[
\nu_1(\mu) = \frac{\mu q_1}{\mu q_1 + (1 - \mu)(1 - q_2)} = \frac{\mu}{\mu + (1 - \mu) \frac{1-q_2}{q_1}} > \mu,
\]

where the inequality is due to the assumption that \( q_1 > 1 - q_2 \). If he observes \( s_t = 0 \), he will update his belief to

\[
\nu_0(\mu) = \frac{\mu(1 - q_1)}{\mu(1 - q_1) + (1 - \mu)q_2} = \frac{\mu}{\mu + (1 - \mu) \frac{q_2}{1-q_1}} < \mu.
\]

Similar to the baseline model, it is only rational to acquire the signal \( s_t \), if the consumer anticipates that he will follow the signal realizations, i.e. to purchase the product if and

\(^5\)We rule out the case \( q_1 = 1 - q_2 \) because it corresponds to the non-informative signal, which will never be acquired if it is costly.

\(^6\)Recall that the private signal \( s_t \) only got revealed if the consumer’s posterior is in \([\mu_u, \mu_b]\).
only if \( s_t = 1 \). Thus, the expected payoff for the consumer by choosing to acquire the signal is

\[
\begin{align*}
[\mu_1 + (1 - \mu)(1 - q_2)](\nu_1(\mu) - c) & = \mu q_1 (1 - p) + (1 - \mu)(1 - q_2)(-p) - c
\end{align*}
\]

**Lemma 9.1.** Let \( \mu_b = \frac{a_{2p-c}}{1-q_1 (1-p)-(1-q_2)p} \) and \( \mu_n = \frac{(1-q_2)p+c}{q_1(1-p)+(1-q_2)p} \), and assume \( \mu_n < \mu_b \), the optimal action of consumer \( t \) with belief \( \mu \) is

\[
a_t^*(\mu) = \begin{cases} 
  b, & \text{if } \mu \geq \mu_b \\
  n, & \text{if } \mu \leq \mu_n \\
  e, & \text{if } \mu \in [\mu_n, \mu_b].
\end{cases}
\]

We assume \( \mu_0 \in [\mu_n, \mu_b] \) otherwise no consumer will ever acquire private information about \( s_t \). Let \( t_b \) and \( t_n \) be the earliest periods in which the posterior belief upon observing a history in the period may be larger than \( \mu_b \) or smaller than \( \mu_n \), respectively. Thus,

\[
\begin{align*}
t_b & = \max \left\{ t : \frac{\mu q_1 t}{\mu q_1 + (1 - \mu)(1 - q_2)t} < \mu_b \right\} \\
t_n & = \max \left\{ t : \frac{\mu (1 - q_1)t}{\mu (1 - q_1)t + (1 - \mu)q_2 t} > \mu_n \right\}
\end{align*}
\]

**Proposition 9.2.** For any given \( q_2 \), there exists a threshold \( q_1^*(q_2) < 1 \) such that for all \( q_1 > q_1^*(q_2) \), the “delayed-garbling” disclosure policy, which discloses no information about purchase history in periods 1 to \( t_b \), and discloses garbled information to induce posteriors at \( \mu_n \) and \( \mu_b \), is optimal for the seller.

When \( q_1 \) increases, the distribution of posterior means after period \( t_b \) “spread out” on \([0, 1]\). In order for the statement to be true, we need that the distribution of posterior means in period \( t_b \) to be a mean-preserving spread of the distribution that mixes between \( \mu_n \) and \( \mu_b \) with average at \( \mu_0 \). The criterion is satisfied for sufficiently large \( q_1 \), as there exists a threshold \( q_1^*(q_2) < 1 \) above which the distribution has measure zero in \([\mu_n, \mu_b]\). However, we should note that, \( q_1^*(q_2) \) is typically much smaller than \( q_1^*(q_2) \) since to be a mean-preserving spread, there can be a moderate measure over \([\mu_n, \mu_b]\).

**A numerical example:** given parameters \( \mu_0 = 0.6, c = 0.002, p = 0.5, q_2 = 0.8 \), the threshold for the “delayed-garbling” to be optimal is \( q_1^*(q_2) = 0.487 \), while the threshold \( \mu_n < \mu_b \), which can be verified easily.

\[7\text{It is satisfied if and only if } c < p[q_1(1-p)+(1-\mu_0)p] + p(q_2 - p). \text{ We should note that } \mu_n < \mu_b \text{ implies } \mu_n < p < \mu_b, \text{ which can be verified easily.}\]
is optimal, using the same parameters $\mu_0 = 0.4, c = 0.002, p = 0.5$: The yellow region is the subset of values of $q_1$ and $q_2$ under which the “delayed garbling” strategy is optimal. The light blue region is omitted values because it is without loss of generality to assume $q_1 > 1 - q_2$. The dark blue region are values under which $\mu_0$ is outside the interval $[\mu_n, \mu_b]$, and thus the only feasible disclosure is non-informative. The green region are the values of $q_1$ and $q_2$ under which informative disclosure is feasible and “delayed garbling” is not optimal. We can see that “delayed garbling” is optimal unless the signal structure is such that good news is much more informative than bad news. For most of values of $q_1$ and $q_2$, “delayed garbling” is optimal.

Figure 4: (Non-)Optimality of delayed garbling under different signal structure

Theorem 9.3. If $\delta \to 1$, the optimal garbling under full information must be “learned” by the seller in finite time, and implemented afterwards.  

10 Conclusion

In this paper, I study the optimal strategy for a monopolist to disclose purchase histories to sequentially arriving consumers. I show that with exogenous prices, if information

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$^8$The exact form of the optimal disclosure policy during the transitional period, depends on the parameter values, including the discount factor.
environment features bad news, the monopolist will withhold information to induce explo-
ration early on, and reveal a garbled information once and for all at a later time. On the
other hand, if the information environment features good news, the monopolist will reveal
the full histories right from the beginning up until a fixed later time. The model is also
extended to incorporate dynamic pricing, evolving quality, and general signal structures.


References


A Appendix

A.1 Proof of Theorem 5.1

Proof. The proof is broken into two parts: in part one, we solve a myopic maximization problem where the seller designs the disclosure of information in period $t$ to maximize the current period profit; in part two, we provide an argument that the myopic solution also solves the dynamic problem.

Part I: Myopic Problem

Let us first consider the myopic maximization problem where the seller designs the disclosure policy in period $t$ to maximize period $t$ profit. The solution to this static problem will directly help us solve the dynamic programming when seller takes the continuation value into account.

First note that in any period $t$, either $\nu_t = 0$ or $\nu_t > \mu_0$ due to the signal structure of perfect bad news. Note that according to the signal structure of $s_t$ and Bayes rule, $\nu_t$ takes value in a finite set of possible posterior beliefs:

$$\nu_t \in \left\{ \frac{\mu_0}{\mu_0 + (1 - \mu_0)(1 - q)^{i-1}}, i < t \right\}$$  \hspace{1cm} (7)

Denote $\nu_t^*$ as the largest non-zero date-$t$ posterior such that occurs with positive probability given the mechanism that the seller committed to from period 1 to period $t - 1$.

We consider two cases: $\nu_t^* \geq \mu_b$ or $\nu_t^* < \mu_b$.

Case 1: $\nu_t^* < \mu_b$

![Figure 5: Feasible information structures when $\nu_t^* \leq \mu_b$](image-url)
According to (Gentzkow and Kamenica., 2016), there is an isomorphism between the set of distributions over posteriors means that can be induced by some signal structure, and the set of convex functions “sandwiched” between the curve \( \int_0^{\mu} F^0(s)ds \) and \( \int_0^{\hat{c}} F(s)ds \), where \( c^0(s) \) and \( \hat{c}(s) \) are the CDF of the null signal and the fully informative signal. Therefore, the maximization problem can be worked in the space of such convex functions as shown in Figure 5.

Given a posterior mean \( \mu_t \), the seller’s profit in the current period is

\[
v_t(\mu_t) = \begin{cases} 
  p, & \text{if } \mu_t > \mu_b \\
  0, & \text{if } \mu_t < \mu_n \\
  (\mu + (1 - \mu)(1 - q))p, & \text{if } \mu_t \in [\mu_n, \mu_b]
\end{cases}
\]

Thus, given a distribution over posterior means with CDF \( F \), the seller’s expected profit in the current period is:

\[
p(1 - F(\mu_b)) + \int_{(\mu_n, \mu_b)} (\mu p + (1 - q)p)dF(\mu)
\]

where \( \mu_b \) indicates that we are taking the left limit of the CDF.

Since we are considering the case such that \( \nu_t^* < \nu_b \), for all \( F \) that can be induced, it has to be that \( F(\mu_b^-) = 1 \), as can be seen in Figure 5 where the only possible slope at \( \mu_b \) for any convex function between the grey and blue curves is 1. Thus we can discard terms involving \( F(\mu_b^-) \) in the maximization problem. We can transform the expression to become fully in terms of \( F \):

\[
\max_F \int_{(\mu_n, \mu_b)} (\mu p + (1 - q)p)dF(\mu)
\]

\[
\Leftrightarrow \max_F (1 - q)p[F(\mu_b^-) - F(\mu_n^-)] + p[qF(\mu)\big|_{\mu_n}^{\mu_b} - \int_{(\mu_n, \mu_b)} F(\mu)\,d\mu]
\]

\[
\Leftrightarrow \max_F pq(\mu_b - 1)F(\mu_b^-) + (pq - 1 - pq\mu_n)F(\mu_n^-) - pq \int_{(\mu_n, \mu_b)} F(\mu)\,d\mu
\]

Since \( F \) must be increasing, the second term in the expression is minimized when \( F(\mu) \) is set to be \( F(\mu_n^-) \) on the interval \([\mu_n, \mu_b)\). Thus, the objective function becomes

\[
(-1 - pq\mu_n)F(\mu_n^-)
\]

Thus, the optimal disclosure policy is such that \( F(\mu_n^-) \) is minimized, which occurs under non-informative signal structure, i.e. the blue curve in Figure 5. In addition to the non-informative signal, there are infinitely many other information structure that also achieves
the same optimal outcome. Since they are payoff-equivalent, and induce the same action from the consumers, they are essentially the same when the seller decides how to disclose information.

Case 2: \( \nu_t^* \geq \nu_b \)

In this case, we will show that the optimal disclosure mechanism induces an integrated CDF over posteriors such that \( \int_0^{\mu_b} F(s)ds = \mu_b - \mu_0 \), namely, the optimal information structure is either the red curve or the green curve in Figure 6. To show this, we recall that the objective function of the seller for the static problem is

\[
pq(\mu_b - 1)F(\mu_b^-) + (pq - 1 - pq\mu_n)F(\mu_n^-) - pq \int_{(\mu_n^-; \mu_b^-)} F(\mu)d\mu
\]

Let \( a = \int_0^{\mu_n} F(s)ds \) and \( b = \int_0^{\mu_b} F(s)ds \) denote the value of the integrated CDF curve at \( \mu_n \) and \( \mu_b \) respectively. The problem of maximizing the expected profit by choosing a feasible integrated CDF function is equivalent to maximizing expected profit by choosing \( a \) and \( b \) as follows:

\[
\max_{a,b} \quad pq(\mu_b - 1) \frac{b - a}{\mu_b - \mu_n} + (pq - 1 - pq\mu_n) \frac{a}{\mu_n} - pq(b - a)
\]

s.t.

\[
a \in [0, \pi_t(0)\mu_n]
\]

\[
b \in [\mu_b - \mu_0, \pi_t(0)\mu_b]
\]

\[
b \geq a + (\mu_b - \mu_n) \frac{a}{\mu_n}
\]

which is equivalent to

\[
\max_{a,b} \quad \frac{pq(\mu_b - 1)}{\mu_b - \mu_n} b + \left(\frac{pq(q - \mu_n) - 1}{\mu_n} - \frac{pq(\mu_n - 1)}{\mu_b - \mu_n}\right) a
\]

s.t. the same constraints

Figure 6: Feasible information structures when \( \nu_t^* \geq \nu_b \)
Thus, given any \( a \) the optimal solution minimizes \( b \) subject to the feasibility constraints. Denote \( a^* \) the solution to the optimization problem above. Thus,

\[
b^* = \max \left\{ \frac{\mu_b}{\mu_n} a^*, \mu_b - \mu_0 \right\}.
\]

If \( \frac{\mu_b}{\mu_n} a^* > \mu_b - \mu_0 \), the problem becomes

\[
\max_a \left( \frac{pq(q - \mu_n)}{\mu_n} - \frac{pq(\mu_n - 1)}{\mu_n} \right) a
\]

s.t. \( a \in \left[ \frac{\mu_n (\mu_b - \mu_0)}{\mu_b}, \pi_t(0) \mu_n \right] \)

Hence \( a^* = \frac{\mu_n (\mu_b - \mu_0)}{\mu_b} \) and \( b^* = \mu_b - \mu_0 \). On the other hand, if \( \frac{\mu_b}{\mu_n} a^* < \mu_b - \mu_0 \), then \( b^* = \mu_b - \mu_0 \).

Thus, the feasibility restricts \( a^* \) to be either 0 or \( \frac{\mu_n (\mu_b - \mu_0)}{\mu_b} \) depending on the parameter values. It is easy to check that the mechanism specified in Theorem 5.1, is the solution to this static problem.

### Part II: Dynamic Problem

Now that we have solved the “static” profit maximization problem, we can turn to the dynamic programming problem where future expected profits are considered jointly with current profit.

It should be first noted that every incentive compatible mechanism must recommend consumer 1 to acquire a signal, simply because the seller have no information to disclose yet, and acquiring a signal is what the first consumer is going to do. Also note that seller’s choice in the current period affects future profits only through the fact that if the consumer acquires a signal in the current period, it will change \( \pi_{t+1}(\nu_{t+1}) \) in the next period, and therefore affects the feasible information structures in the future. Also, we should note that if the seller induces the consumer to choose \( b \) or \( n \), then \( \pi_{t+1}(\nu) \) is the same as \( \pi_t(\nu) \) and the feasible information structures next period are the same as those in the current period. However, if the seller induces the consumer to acquire a signal with positive probability in the current period, then the feasible set of information structures will be “enlarged” in the next period, which corresponds to a counter-clock rotation of the grey curve in the interval of \([0, \mu_b] \) in Figure 6 (recall that only the feasible region in the interval \([0, \mu_b] \) is relevant for the static information design problem). However, in the static problem, we have seen that the only benefit for enlarging the feasible set is to make \( \mu_b \) inducible, and once it is inducible there is no additional benefit of enlarging the feasible set of signals.
When $t \leq \bar{t}$, the static optimal information structure characterized earlier, achieves the highest current profit, and at the same time, maximizes the rate at which the feasible set of signals is enlarged. Thus, it is optimal even in the dynamic programming problem. When $t \geq \bar{t}$, there is no additional benefit from enlarging the feasible set of signals anymore, thus, the static solution is also the solution to the dynamic programming problem. □