Monopoly Pricing with Endogenous Information Manipulation

Wenji Xu *

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1 Introduction

Rapid developments in media technology have given consumers access to new information sources that allow them to get information prior to trading. These media sources are typically controlled by third parties such as the government, activist groups, or for-profit firms, etc. For example, IMDb, a website that aggregates customer reviews of films, provides detailed customer ratings, and ranks films based on that. IMDb has much discretion when it comes to how much and what type of information to convey to the viewers. For example, it can decide which film reviews to put on its front page, which films to put under the “IMDb Picks” section of the website, etc. By the nature of IMDb’s business, these ratings and reviews are available to consumers after the film distributors have negotiated with film theaters about ticket prices and scheduling of the films. This paper analyzes how such endogenous informational responses of these third parties (after the seller has determined its selling strategy) affect consumer and producer surplus in a monopolistic pricing setting.

In this paper, I model explicitly what it means for a seller to design a selling mechanism when there is information manipulation by a third party after the mechanism is chosen. In particular, I will show that even though the third party’s strategy is observed by the buyer, the seller cannot design a mechanism that “directly” punishes the third party for choosing

*wenji@uchicago.edu
information structure that the seller does not want. Instead, the seller can only choose mechanisms that are independent of the third party’s information choices and influence the third party’s decision through providing incentives to the buyer.

I will then characterize the optimal selling mechanisms when the third party seeks to maximize a weighted sum of the buyer’s and seller’s surpluses. In particular, if the third party puts weakly higher weight on the seller’s surplus, the seller will always be able to extract the highest surplus. If the third party cares only about the buyer’s surplus, full information is provided and the seller’s surplus is the lowest. For the cases when the third party puts higher weight on the buyer’s surplus, partial information is provided to the buyer and the seller’s payoff is strictly increasing in the weight.

The rest of the paper proceeds in the usual order. Section 2 reviews the literature. Section 3 provides formal definitions and characterization of the choice sets of the seller and the third party. Section 4 characterizes the equilibrium outcomes under different assumptions of the third parties’ objective. Section 5 concludes.

2 Related Literature

My paper is related to the papers on buyer’s incentive to acquire information prior to bilateral trade and auctions. [Roesler and Szentes, 2017] characterized buyer-optimal learning in environments where the monopolist best-responds to the buyer’s signal structure. In their set-up, the decision of acquiring information is made by individual buyers before the seller chooses the selling mechanism. Other papers have analyzed information acquisition after the selling mechanism is determined. [Persico, 2000] shows that if the buyers’ signals are affiliated then they acquire more information in a first-price auction than in a second-price one. [Shi, 2012] analyzed models where it is costly for the buyers to learn about their valuations and identifies the revenue-maximizing auction in private-value environments. In these setups, the seller commits to a selling mechanism before the buyer decides how much information to acquire. The buyers’ learning strategies depend on the selling mechanism announced by the seller.

My work differs from those listed above by focusing on the information choice by a third
party (rather than individual buyer’s information acquisition decision) who has potentially misaligned preference to both the buyer and the seller. The information choice is made after the seller chooses the selling mechanism, similar to [Persico, 2000] and [Shi, 2012], and opposite to [Roesler and Szentes, 2017].

There is also a set of papers in robust mechanisms design that examines the optimal mechanism for an ambiguity averse seller who faces large uncertainty of the information structure. [Bergemann and Schlag, 2011], [Wolitzky, 2015] and many other papers are in this direction. Closely related to my work are [Kos and Messner, 2015], [Carrasco et al., 2015] and [Du, 2017]. The problem in their papers can be interpreted as a special case of my model where the third party chooses the information structure to minimize the seller’s payoff. As shown in their papers, the optimal selling mechanism is a random pricing strategy over an interval of posted prices. Beyond this special case, the problem analyzed in this paper is different from those in robust mechanism design.

3 Model

There is a seller seeking to sell a single indivisible good to a buyer. The good has no value to the seller. The buyer’s valuation of the object $\theta$ is distributed according to CDF $F$ with strictly positive density on the interval support $[0,1]$. He does not know $\theta$, but observes a signal $s$ about $\theta$.

The seller chooses a trading mechanism first. After the mechanism is picked, some third party (e.g. for-profit firms such as IMDb, activist groups, governments, etc) chooses the information structure (i.e. distribution of $s$) for the buyer. Then the buyer chooses an action in the action space specified by the mechanism, and trade occurs according to the mechanism and buyer’s action. Here, we allow the seller to pick any feasible mechanisms. The choice set of the seller is characterized later in this section.

The seller and buyer have quasi-linear preferences. If they trade the good with probability $q \in [0,1]$ and the buyer pays the seller a monetary transfer of $p$, the utilities for the seller and buyer are $U_s(q,p,\theta) = p$ and $U_b(q,p,\theta) = q\theta - p$ respectively.

We could impose different assumptions on the third party’s preferences. For example, if
the third party is the social planner, he may want to maximize social surplus, i.e. \( U_m(q, p, \theta) = q\theta \); if the third party is a competitor of the seller, he may want to minimize the seller’s revenue \( (U_m(q, p, \theta) = -p) \) or sale \( (U_m(q, p, \theta) = -q) \), etc. In the next section, I will mainly focus on the case when the third party maximizes a weighted sum of the buyer’s and seller’s surpluses, i.e. \( U_m(q, p, \theta) = \alpha p + (1 - \alpha)q\theta \) where \( \alpha \in [0, 1] \). Before that, we need to first characterize the choice sets of the seller and the third party.

**Third Party’s Choice Set**

With a quasilinear preference, the buyer’s trading decision only depends on \( E(\theta|s) \). Thus, we may assume without loss of generality that each signal \( s \) provides the buyer with an unbiased estimate about her valuation, that is \( E(\theta|s) = s \). In what follows we restrict attention to such unbiased signals.

To characterize the third party’s choice set, we should notice first that for each unbiased signal \( s \), \( \theta \) can be expressed as \( s + \epsilon \) for a random variable \( \epsilon \) with \( E(\epsilon|s) = 0 \). This means that \( G \) is the CDF of some unbiased signal \( s \) if and only if \( F \) is a mean-preserving spread of \( G \). Let \( \mathcal{G}_F \) denote the set of CDFs of which \( F \) is a mean-preserving spread. By Proposition 6.D.2 of Mas Colell et al. (1995),

\[
\mathcal{G}_F = \left\{ G \in \mathcal{G} : \int_0^x F(\theta)d\theta \geq \int_0^x G(s)ds, \forall x \in [0, 1], \int_0^1 sdG(s) = \mu \right\}
\]

where \( \mathcal{G} \) is the set of CDFs and \( \mu = \int_0^1 \theta dF(\theta) \). Therefore, the third party’s choice set is all signals with CDF in \( \mathcal{G}_F \).

**Seller’s Choice Set**

In the following section, I will characterize the choice set of the seller, i.e. the set of feasible selling mechanisms. I assume that the third party’s choice of information structure is observed by the buyer, but unobservable to the seller. Or equivalently, we can assume that the seller observes the third party’s strategy, but it is not contractible, e.g. not verifiable at court. Therefore, the seller cannot just commit to punish the third party when an information structure that the seller dislike is chosen.
However, since the information choice of the third party is observed by the buyer, the seller can potentially design a mechanism that asks the buyer to report the observed information structure and punish the third party accordingly. For such mechanisms to work, it must be that the buyer is incentivized to report the third party’s strategy truthfully. As we will see, this incentive compatibility constraint substantially restricts the seller’s choice set to mechanisms that are independent of the reported strategy of the third party. \footnote{Strictly speaking, the seller can design any mechanism, including those that are dependent of the reported strategy of the third party. But it is with out loss of generality to only consider mechanisms that are independent of the third party’s information choice because an equilibrium outcome induced by any mechanism can be replicated using a mechanism independent of the reported information structure.} Therefore, the only way the seller can induce desirable information structure from the third party is through providing incentives to the buyer. Hopefully, the conclusion will be clear at the end of this section.

In standard mechanism design where the information structure is exogenous and common knowledge among players, we know from revelation principle [Myerson, 1981] that instead of considering the huge set of all possible extensive form games, the mechanism designer can, without loss of generality, consider only the set of incentive compatible direct mechanisms, in which the designer asks the players to report their private “types” and choose the social outcome based on players’ reports. In my model, the information structure is chosen by the third party after the mechanism is chosen by the seller. Fortunately, an analogous version of the revelation principle holds here:

**Lemma 3.1.** Given an sequential equilibrium (consists of both third party’s and the buyer’s strategies and buyer’s beliefs) in the sequential game induced by an arbitrary selling mechanism, there exists an incentive compatible direct mechanism that induce the same equilibrium outcome. In the direct IC mechanism, the buyer reports both the signal structure (third party’s strategy) and the signal realization he observes, then the trading probability and transfer are determined accordingly.

Lemma 3.1 follows from Theorem 5.1 in the Appendix. Theorem 5.1 shows an analogous version of the revelation principle in a more general setting of mechanism design with endogenous information choice by a third party where there are multiple players. I included this
general result because it may be helpful for future research that go beyond the monopolistic pricing model in this paper where there is only one player in the mechanism.

By Lemma 3.1, we can restrict attention to incentive compatible (IC) direct mechanisms, i.e. a tuple \((G_F \times [0,1], q(\cdot), p(\cdot))\) where buyer submits a report in \(G_F \times [0,1]\) (recall that the signals are unbiased thus the set of possible realizations is \([0,1]\)), and the outcome functions \(q : G_F \times [0,1] \rightarrow [0,1]\) specifies the trading probability, and \(p : G_F \times [0,1] \rightarrow \mathbb{R}\) specifies the transfer, based on buyer’s report of the signal realization and the signal structure. In order to find the optimal mechanism, we need to characterize all IC direct mechanisms.

**Theorem 3.2.** A direct mechanism in our setting is incentive compatible if and only if

1. \(q(\cdot, G)\) is increasing in \(s\), for any \(G \in G_F\)
2. \(p(s, G) = p(0, G) + sq(s, G) - \int_0^s q(x, G) dx\), for any \(s \in [0,1], G \in G_F\)
3. \(sq(s, G) - p(s, G) = sq(s, G') - p(s, G')\), for any \(s \in [0,1], G, G' \in G_F\)

**Proof.** To prove the theorem, we need to show that the three conditions (1)-(3) are necessary and sufficient for incentive compatibility, i.e. the buyer has no incentive to deviate from truthful report in any of the following three manners: misreport \(s\) alone, misreport \(G\) alone, or jointly misreport \(s\) and \(G\). I break the proofs into four lemmas shown below.

By Lemma 3.3 and 3.4, no misreport of \(s\) alone implies condition (1) and (2) in Theorem 3.2. Conversely, we can show that they are also sufficient conditions for no misreport of \(s\) alone (the proof is exactly the same as in standard monopolistic pricing setting without third party information choice). By Lemma 3.5, condition (3) is necessary and sufficient condition for no misreport of \(G\) alone. By lemma 3.6, once no misreport of \(s\) alone and \(G\) alone are both guaranteed, the buyer will not want to misreport \(s\) and \(G\) jointly. \(\square\)

**Lemma 3.3.** \(q(\cdot, G)\) is increasing in \(s\).

**Proof.** No misreport of signal realizations \(s, s'\) implies

\[
\begin{align*}
sq(s, G) - p(s, G) & \geq sq(s', G) - p(s', G) \\
\frac{s'q(s, G) - p(s, G)}{s' - s} & \leq \frac{s'q(s', G) - p(s', G)}{s' - s}
\end{align*}
\]

Thus, \((s - s')q(s, G) \geq (s - s')q(s', G)\) which implies \(q(\cdot, G)\) is increasing. \(\square\)
Let \( u(s, G) \) be buyer’s utility when he reports truthfully, i.e. \( u(s, G) = sq(s, G) - p(s, G) \).

**Lemma 3.4.** \( u(s, G) \) is increasing in \( s \). \( \frac{\partial u(s, G)}{\partial s} = q(s, G) \) whenever it is differentiable.

**Proof.** By incentive compatibility, we know \( u(s, G) = \max_{s', G'} (sq(s', G') - p(s', G')) \). Since \( u(s, G) \) is the maximum of linear (convex) increasing functions of \( s \). Thus \( u \) must be increasing and convex itself, which can be not differentiable in at most countably many points.

For any \( s, G \) where \( u \) is differentiable.

\[
\lim_{\delta \to 0} \frac{u(s + \delta, G) - u(s, G)}{\delta} \geq \lim_{\delta \to 0} \frac{[sq(s + \delta, G) - p(s, G)] - [sq(s, G) - p(s, G)]}{\delta} = q(s, G)
\]

and

\[
\lim_{\delta \to 0} \frac{u(s, G) - u(s - \delta, G)}{\delta} \leq \lim_{\delta \to 0} \frac{[sq(s, G) - p(s, G)] - [(s - \delta)q(s, G) - p(s, G)]}{\delta} = q(s, G)
\]

**Lemma 3.5.** The buyer have no incentive to only misreport \( G \) if and only if \( \forall s, \forall G, G' \), \( sq(s, G) - p(s, G) = sq(s, G') - p(s, G') \)

**Proof.** \( \forall G, G' \), no misreports of \( G \) and \( G' \) imply

\[
sq(s, G) - p(s, G) \geq sq(s, G') - p(s, G')
\]

\[
sq(s, G') - p(s, G') \geq sq(s, G) - p(s, G)
\]

**Lemma 3.6.** If the mechanism is such that the buyer has no incentive to misreport \( s \) or \( G \) alone, then there is no incentive to misreport \( s, G \) jointly.
Proof. We want to show $sq(s,G) - p(s,G) \geq sq(s',G') - p(s',G')$ for all $s, G, s', G'$. Note that $(s, G)$ don’t want to deviate to $(s, G')$ implies

$$sq(s, G) - p(s, G) \geq sq(s, G') - p(s, G')$$

and $(s, G')$ don’t want to deviate to $(s', G')$ implies

$$sq(s, G') - p(s, G') \geq sq(s', G') - p(s', G')$$

Thus, $sq(s, G) - p(s, G) \geq sq(s', G') - p(s', G')$.

By condition (2) and (3) in Theorem 5.1, we can conclude that $\int_0^s q(x, G)dx = \int_0^s q(x, G')dx$ for all $G, G'$ and all $s$, which implies that $q(s, G) = q(s, G')$ for all $G, G'$ and all $s$. Therefore, IC direct mechanisms are independent of the report of the third party’s strategy.

We have now completely characterized the set of all IC direct mechanism. We now bring in individual rationality. In particular, I assume that the buyer can decide whether to participate in the selling mechanism after the third party has chosen the information structure and after he observes the signal realization. Buyer’s outside option is assumed to be zero.

**Lemma 3.7.** An incentive-compatible direct mechanism is individually rational if and only if $u(0, G) \geq 0$ for all $G$.

**Proof.** We know $u$ is increasing in $s$ for IC mechanisms. Thus, for the equilibrium choice of $G$, $u(s, G)$ is non-negative for all $s$ if and only if it is nonnegative for the lowest type. Since the interim payoff is constant across $G$ in any IC mechanism, the inequality holds for off-equilibrium $G'$’s too.

Thus, if an IC and IR direct mechanism maximizes the seller’s expected revenue, we must have $p(0, G) = 0$ for all $G$. Therefore, the seller’s choice set becomes all increasing functions $q(\cdot)$ from $[0, 1]$ to $[0, 1]$, i.e. it is without loss of generality to restrict attention to mechanisms that does not depend on $G$, and the transfers $p(\cdot)$ is uniquely pinned down once $q(\cdot)$ is chosen. Therefore, the choice set of the seller is any mixture of posted price mechanisms, exactly the same as in the standard setting without third party.

We have characterized the choice sets for both the third party and the seller, where it is without loss of generality to assume that the third party chooses from the set of signals that
are mean preserving spreads of $\theta$, and the seller chooses from the set of mixtures of posted price mechanisms.

4 Optimal mechanisms

Knowing the choice sets of the seller and the third party, we can now characterize the optimal mechanisms and the equilibrium payoffs given different preferences of the third party.

In terms of notations, since we already argued that the transfers $p(\cdot)$ are uniquely pinned down given $q(\cdot)$, I will identify mechanisms with their corresponding allocation rule $p(\cdot)$ and omit the transfer rule $p(\cdot)$. Furthermore, I refer to a posted price mechanism at $p$ to be the mechanism with allocation rule $q(s) = 1$ if $s > p$, and $q(s) = 0$ if $s < p$, and transfer rule $p(s) = p$ if $s > p$ and $p(s) = 0$ if $s < p$, i.e. the buyer buys the good with certainty at the posted price $p$ if and only if he reports a value higher than the posted price $p$. Since any increasing function $q : [0, 1] \rightarrow [0, 1]$ is a convex combination of the allocation rules of posted price mechanisms (Proposition 2.4 in Borgers 2013), we know that the seller’s choice set is the set of all mixtures of posted price mechanisms.

4.1 Third party wants to maximize seller’s surplus

First, consider the case when the third party’s preference is aligned with the seller’s, i.e. he wants to maximize seller’s profit. I will first argue that posted price mechanism is optimal among all possible mechanisms, and then characterize the optimal mechanism by finding a posted price mechanism that achieves the highest profit for the seller.

For any posted price mechanism at $p$ chosen by the seller, let $G_p$ denote the signal structure chosen by the third party as a best response to the posted price mechanism at $p$. The seller maximizes his payoff by choosing among posted price mechanisms. To see this, let $q(s) = \alpha q_1(s) + (1 - \alpha) q_2(s)$ and $p(s) = \alpha p_1(s) + (1 - \alpha) p_2(s)$ be the mechanism chosen, and suppose $(q_1, p_1), (q_2, p_2)$ are two posted price mechanisms with posted prices $p_1^*$ and $p_2^*$. The seller’s profit given any signal structure $G$ is

$$\alpha \int_{p_1^*}^{1} p_1^* dG(s) + (1 - \alpha) \int_{p_2^*}^{1} p_2^* dG(s) \leq \max\{ \int_{p_1^*}^{1} p_1^* dG_{p_1}(s), \int_{p_2^*}^{1} p_2^* dG_{p_2}(s) \}$$
Thus, the seller’s payoff under any random pricing mechanism (mixture of posted prices) is weakly lower than the highest payoff he can get from a posted price mechanism.

The seller’s profit is weakly lower than $\mu$ for any mechanism and any $G$, i.e. $\mu \geq pD(G_p,p)$, because

$$\mu = \int_0^1 sdG_p(s) \geq 0 \cdot (1 - D(G_p,p) + pD(G_p,p))$$

where $D(G,p) = 1 - \sup_{x<\mu} G(x)$ is the probability of trade, assuming the buyer buys when he is indifferent.

The upper bound can be achieved when the seller chooses a posted price mechanism with posted price $p^* = \mu$, and the third party chooses the “no-information” signal as a best response: $G^*(s) = 0$ if $s < \mu$ and $G^*(s) = 1$ if otherwise. This is an equilibrium.

It is the unique equilibrium because (1) if the seller chooses a cutoff $p^* < \mu$, the third party will choose the no-information signal to maximize trade probability (which at the same time maximizes seller’s profit). In this case the seller gets a profit $p^* \cdot 1 < \mu$; (2) if the seller chooses a posted price mechanism at $p^* > \mu$, the third party will choose the signal structure that maximizes trade probability (which at the same time maximizes seller’s profit), i.e.

$$G_{p^*}(s) = \begin{cases} 
F(s), & \text{if } s < F^{-1}(c) \\
c, & \text{if } F^{-1}(c) < s < p^* \\
1, & \text{if } s \geq p^*
\end{cases}$$

where $c$ is a constant such that $\int_{F^{-1}(c)}^p F(s) - c)ds = 1 - \int_p^1 G(s)ds$. In this case the seller gets a profit $p^* \cdot (1 - c) < \mu$. To see this,

$$p^* \cdot (1 - c) = \int_0^{F^{-1}(c)} (1 - c)ds + \int_{F^{-1}(c)}^{p^*} 1 - F(s)ds$$

$$= \mu \cdot 1 - \int_0^{F^{-1}(c)} c - F(s)ds$$

$$< \mu$$

Therefore there is a unique equilibrium with the seller choosing the posted price mechanism at $p^* = \mu$, and the third party choosing the “no-information” signal $G^*(s) = 0$ if $s < \mu$ and $G^*(s) = 1$ if otherwise.
Intuitively, when the third party wants to help the monopolist, he will just choose to shut down the signal so that the buyer does not get any informational rent. For example, suppose IMDb is owned by a single film distributor, and if we ignore the advertising effect, then the film distributor should simply shut down IMDb’s service.

4.2 Third party wants to maximize buyer’s surplus

When the third party wants to maximize buyer’s payoff, that regardless of the mechanism chosen by the seller, the third party will choose the “full-information” signal $G = F$. To see this, consider any posted price mechanism at $p$, the buyer’s surplus is

$$
\int_p^1 (s - p) dG(s) = \int_p^1 sg(s) ds - p[1 - G(p)]
$$

$$
= (s - p) G(s)|_p^1 - \int_p^1 G(s) ds
$$

$$
= 1 - p - \int_p^1 G(s) ds
$$

$$
\leq (1 - p) - \int_p^1 F(s) ds
$$

$$
= \int_p^1 (s - p) dF(s)
$$

The inequality follows from second order stochastic dominance. Thus, choosing $G = F$ maximizes buyer’s surplus regardless of the mechanism chosen by the seller. Since any random pricing mechanism is a convex combination of posted mechanisms, $F$ is also optimal for such mechanisms. To see this, let $q(s) = \alpha q_1(s) + (1 - \alpha) q_2(s)$ and $p(s) = \alpha p_1(s) + (1 - \alpha) p_2(s)$ be the mechanism chosen, and suppose $(q_1, p_1), (q_2, p_2)$ are two posted price mechanisms with posted prices $p_1^*$ and $p_2^*$. The buyer’s surplus under any signal structure $G$ is

$$
\alpha \int_{p_1^*}^1 (s - p_1^*) dG(s) + (1 - \alpha) \int_{p_2^*}^1 (s - p_2^*) dG(s)
$$

$$
\leq \max\{ \int_{p_1^*}^1 (s - p_1^*) dG(s), \int_{p_2^*}^1 (s - p_2^*) dG(s) \}
$$

$$
\leq \max\{ \int_{p_1^*}^1 (s - p_1^*) dF(s), \int_{p_2^*}^1 (s - p_2^*) dF(s) \}
$$

Therefore, the full-information signal $F$ a best-response to any random pricing mechanisms.
Given that the third party will provide full information regardless of the selling mechanism, the seller will choose the posted price mechanism at $p^* = \arg\max_p pD(F,p)$ to maximize his profit. This is, however, not the unique equilibrium because given the posted price mechanism chosen by the seller, the third party have infinitely many best-responses. But it easy to show that for any such best-response choice of signal, we must have $G(p^*) = F(p^*)$ and $\int_{p^*}^{\infty} sdG(s) = \int_{p^*}^{\infty} sdF(s)$, so these equilibriums are payoff equivalent to the one where the third party chooses full information.

Intuitively, when the third party information provider is the consumer himself, he will always get the most accurate information so that he can make informed decisions.

4.3 Third party wants to maximize weighted sum of buyer’s and seller’s surpluses

Now, consider the situation where the third party cares about both the buyer and the seller, and maximizes a weighted sum of the two players’ payoffs (e.g. government seeks to maximize some weighted social surplus through providing information to consumers using media). In this case, I am yet to characterize the optimal random pricing mechanism (i.e. mixture of posted prices), thus, the argument is incomplete. In what follows, I will characterize the optimal posted price mechanism, and analyze how equilibrium surplus evolve as the third party’s preference changes. Then I will provide an example to show that the posted price mechanism is not optimal among all possible mechanisms (therefore, the next step of my research agenda is to characterize the optimal random pricing mechanism).

Suppose third party puts $\alpha$ weight on seller’s payoff and $(1-\alpha)$ on buyer’s. Thus, given a posted price mechanism at $p$, the third party solves

$$
\max_{G \in G_F} \alpha p(1 - \sup_{x < p} G(x)) + (1 - \alpha) \int_{p}^{1} (s - p) dG(s)
\iff
\max_{G \in G_F} (1 - \alpha) \int_{p}^{1} sdG(s) - (1 - 2\alpha)p(1 - \sup_{x < p} G(x))
$$

When $\alpha \geq \frac{1}{2}$, the third party wants to maximize both the social surplus $\int_{p}^{1} sdG(s)$ and the seller’s surplus $p(1 - \sup_{x < p} G(x))$. Therefore, posting a price $p^* = \mu$ is the optimal strategy for the seller which induces the manipulator to choose the no-information signal.
Hence, when the third party cares weakly more about the seller, the seller is always able to extract the highest surplus by committing to a posted price mechanism at the prior mean valuation of the buyer, and in equilibrium social surplus is $\mu$ which is entirely accrued to the seller.

When $\alpha < \frac{1}{2}$, given a posted price mechanism at $p$, the third party solves (assume buyers who are indifferent always buy)

$$\max_{G \in \mathcal{G}_F} \alpha p (1 - \sup_{x < p} G(x)) + (1 - \alpha) \lim_{x \to p^-} \int_x^1 (s - p) dG(s)$$

$$\Leftrightarrow \max_{G \in \mathcal{G}_F} \lim_{x \to p^-} \int_x^1 [(1 - \alpha)s + (2\alpha - 1)p] dG(s)$$

$$\Leftrightarrow \max_{G \in \mathcal{G}_F} \lim_{x \to p^-} \int_x^1 [(s - Ap] dG(s)$$

$$\Leftrightarrow \max_{G \in \mathcal{G}_F} \lim_{x \to p^-} \int_x^1 s g(s) ds - Ap [1 - G(x)]$$

$$\Leftrightarrow \max_{G \in \mathcal{G}_F} s G(s)[1] - \int_x^1 G(s) ds - Ap [1 - G(x)]$$

$$\Leftrightarrow \max_{G \in \mathcal{G}_F} 1 - x G(x) - Ap + Ap G(x) - \int_x^1 G(s) ds$$

$$\Leftrightarrow \max_{G \in \mathcal{G}_F} (A - 1) p G(x) - \int_x^1 G(s) ds$$

where $A = \frac{1 - 2\alpha}{1 - \alpha} \in [0, 1]$.

There is a trade-off (due to the two terms in the objective) for the third party between providing more information to the buyer or not. This maximization problem can be solved in two steps: first fix $\int_0^p G(s) ds = M$ and find the optimal $G_M$ in this subset of cdfs, then find the global optimal $G$ among these $G_M$’s.

Let $G_M \in \mathcal{G}_F$ be the cdf that maximizes third party’s objective subject to an additional condition that $\int_0^p G(s) ds = M$. Then $G_M$ is of the form:

$$G_M(s) = \begin{cases} 
F(s), & \text{if } s < c \\
F(c), & \text{if } s \in [c, p] \\
\text{many feasible paths such that } F \text{ is a mean-preserving spread,} & \text{if } s > p
\end{cases}$$

where $c$ is such that $\int_{F^{-1}(c)}^c (c) F(s) ds + \int_{F^{-1}(c)}^p c ds = M$.

Proof. if not of the above form, then can construct a small perturbation that lowers the $G_M$ curve at $s = p$ a little bit, which is an improvement. \qed
Then, given a posted price \( p \), the third party solves

\[
\max_M (A - 1) p G_M(p) + M
\]

by choosing \( M \) in its feasible set given \( p \) (for \( p < \mu \), \( M \in [0, \int_0^p F(s)ds] \), and for \( p \geq \mu \), \( M \in [p - \mu, \int_0^p F(s)ds] \); these bounds follow from \( F \) being a mean-preserving spread of \( G \)).

**Example:** uniform prior \( F(s) = s \)

\[
G_M(s) = \begin{cases} 
  s, & \text{if } s < c \\
  c, & \text{if } s \in [c, p] \\
  \text{irrelevant}, & \text{if } s > p
\end{cases}
\]

where \( c \) satisfies \( \int_c^c sds + \int_c^p cds = M \), which implies \( c = p - \sqrt{p^2 - 2M} \) (ruling out the other root). Thus, the third party solves

\[
\max_{M \text{ feasible}} (A - 1) p(p - \sqrt{p^2 - 2M}) + M.
\]

The FOC gives us the global optimum (without considering whether \( M \) is feasible or not)

\[
M^* = \frac{1}{2} p^2 (1 - (1 - A)^2) < \frac{p^2}{2} = \int_0^p F(s)ds. \quad \text{(the objective is concave, one can check the second order condition.)}
\]

If \( p < \mu \), then \( M^* \) is feasible, thus the third party will choose \( M(p) = M^* \).

If \( p > \mu \), and let \( p^* \) be the cutoff of whether \( M^* \) is feasible, i.e. \( \frac{1}{2} p^2 (1 - (1 - A)^2) = p - \mu \) (\( \mu = \frac{1}{2} \) in the uniform case), which implies \( p^* = \frac{1}{2 - A} \). Thus, when \( p < p^* \), third party’s optimal choice is \( M(p) = p - \mu = p - \frac{1}{2} \); when \( p > p^* \), third party will choose \( M(p) = \frac{1}{2} p^2 (1 - (1 - A)^2) \).

Therefore, when \( p < \max\{\mu, \frac{1}{2 - A}\} \), \( M(p) = M^* = \frac{1}{2} p^2 (1 - (1 - A)^2) \); else, \( M(p) = p - \mu \).

Now, given third party’s choice of \( M(p) \) for each \( p \) (corresponding to a distribution \( G_M(p) \) for each \( p \)), the seller will pick the \( p \) which maximizes his payoff

\[
p(1 - G_M(p)) = p\{1 - [p - \sqrt{p^2 - 2M(p)}]\}.
\]

Substituting \( M(p) \) into the expression, when \( p > \max\{\mu, \frac{1}{2 - A}\} \) the payoff is \( 2p(1 - p) \); when \( p < \max\{\mu, \frac{1}{2 - A}\} \), the payoff is \( p(1 - Ap) \).
As shown in Figure 1, seller’s equilibrium surplus is increasing in the weight that the third party puts on the seller, and it approaches the highest surplus under no-information when the weight approaches $\frac{1}{2}$ and it approaches the surplus under full-information when the weight goes to zero.

![Figure 1: Seller’s equilibrium surplus under different weight $\alpha$](image)

What we have shown above is the equilibrium outcome when we only allow the seller to choose among posted price mechanisms. However as the following example shows, posted price mechanism is not optimal in general. Therefore, we will need to characterize the optimal random pricing mechanism for the argument to be complete.

**Example:** Assume the weight is $\alpha = 0.0476$, and the prior distribution is uniform $F(s) = s$. We can use the procedure above to find the optimal posted price to be $p^* = 0.475$ which generates a surplus of 0.263 for the seller. However, if the seller choose a mixture between two prices $p_1 = 0.2, p_2 = 0.4$ where he uses $p_1$ with probability 0.1 and uses $p_2$ with probability 0.9, the seller will get a surplus of 0.268 which is higher than his payoff in the best posted price mechanism.

To solve for the third party’s strategy and the payoff for the seller when the seller chooses the random pricing strategy in the above example, we can use a similar method as in the 2-step approach used in the derivation of the optimal posted price mechanism. In particular, we can fix $\int_0^{p_1} G(s)ds = M_1$ and $\int_0^{p_2} G(s)ds = M_2$ and get the best cdf with these additional
restrictions. Then, we vary $M_1, M_2$ to solve for the global optimal CDF using standard Lagrangian method (we can prove that the objective is concave in $M_1, M_2$). I omit the details for sake of space.

4.4 Third party wants to control trade probability

The third party could also seeks to control the trade probability, e.g. when there is positive/negative externalities of trading the good, the government may have a preference to maximize/minimize trade probability. In this section, I characterize the optimal selling mechanism when the third party maximizes trade probability, and shows that posted price mechanism is sub-optimal when the third party minimizes trade probability (again, I am yet to characterize the optimal random pricing mechanism in this case).

**Third party maximizes trade probability**

For any posted price mechanism $p$, the third party solves the following problem:

$$\max_{G \in \mathcal{G}_F}(1 - \sup_{x < p} G(x))$$

If $p \leq \mu$, the solution to the above problem is a cdf $G$:

$$G_p(s) = \begin{cases} 
0, & \text{if } s < \mu \\
1, & \text{if } s \geq \mu
\end{cases}$$

which results in a trading probability of 1. If $p > \mu$, the optimal solution is

$$G_p(s) = \begin{cases} 
F(s), & \text{if } s < F^{-1}(c) \\
c, & \text{if } F^{-1}(c) < s < p \\
1, & \text{if } s \geq p
\end{cases}$$

where $c$ is a constant such that $\int_{F^{-1}(c)}^{p}(F(s) - c)ds = 1 - \int_{p}^{1} G(s)ds$.

For the seller, his optimal choice of mechanism is to set $p = \mu$ which will induce the third party to choose the no-information signal structure. This choice generates $\mu$ to the seller, which is the upper bound that can be achieved from any mechanism.

**Third party minimizes trade probability**
For any posted price mechanism mechanism p, the third party solves the following problem:

\[
\min_{G \in \mathcal{G}_f} (1 - \sup_{x < p} G(x))
\]

If \(p > \mu\), the solution to the above problem is the cdf \(G\):

\[
G_p(s) = \begin{cases} 
0, & \text{if } s < \mu \\
1, & \text{if } s \geq \mu 
\end{cases}
\]

which results in zero trading probability.

If \(p \leq \mu\), assuming the buyer does not buy the product when indifferent (for an equilibrium to exist), the third party’s optimal strategy is

\[
G_p(s) = \begin{cases} 
0, & \text{if } s < p \\
c, & \text{if } p \leq s < F^{-1}(c) \\
F(s), & \text{if } s \geq F^{-1}(c)
\end{cases}
\]

where \(c\) is a constant such that \(\int_0^p F(s)ds = c - \int_p^{F^{-1}(c)} G(s)ds\). The third party achieves a trading probability of \(1 - c\).

For the seller, his optimal choice of posted price mechanism is price \(p^* \in (0, \mu)\) such that

\[
p^* = \arg\max_p pD(G_p, p)
\]

. Therefore, the optimal choice depends on the shape of \(F\).

**Example: uniform \(F(s)=s\)**

Then for \(p \leq \mu\),

\[
G_p(s) = \begin{cases} 
0, & \text{if } s < p \\
2p, & \text{if } p \leq s < 2p \\
s, & \text{if } s \geq 2p
\end{cases}
\]

The seller’s problem is

\[
\max_{p \in [0,0.5]} p(1 - 2p)
\]

. Thus, the optimal cutoff price mechanism is \(p^* = \frac{1}{4}\) which results in a surplus of \(\frac{1}{8}\) for the seller.
However, in this case, the seller can do better by choosing a random pricing mechanism. Consider the following mechanism which is a mixture of two posted prices: 

\[ q(s) = \frac{1}{2}q_1(s) + \frac{1}{2}q_2(s) \] 

and 

\[ p(s) = \frac{1}{2}p_1(s) + \frac{1}{2}p_2(s) \]

be the mechanism chosen, and suppose \((q_1, p_1), (q_2, p_2)\) are two posted price mechanisms with prices \(\frac{1}{6}\) and \(\frac{1}{3}\).

Given this mechanism, the third party’s best response is

\[
G_p(s) = \begin{cases} 
0, & \text{if } 0 < \frac{1}{6} \\
\frac{1}{4}, & \text{if } \frac{1}{6} \leq p < \frac{1}{3} \\
\frac{1}{2}, & \text{if } \frac{1}{3} \leq s < \frac{1}{2} \\
s, & \text{if } s \geq \frac{1}{2}
\end{cases}
\]

The seller gets a profit of

\[
\frac{1}{2}\left(\frac{1}{6}\left(1 - \frac{1}{4}\right)\right) + \frac{1}{2}\left(\frac{1}{3}\left(1 - \frac{1}{2}\right)\right) = \frac{7}{48} > \frac{1}{8}.
\]

Thus, we need to consider what is the optimal random pricing mechanism.

5 Conclusion

In this paper, I studied a monopolistic pricing problem when the information structure is endogenously chosen/manipulated by a third party who has misaligned interest to the buyer and the seller. I characterized the revelation principle in this setting: it is without loss of generality for the seller to restrict attention to incentive compatible direct mechanisms in which buyer report their private signals as well as the signal structure chosen by the third party (observed by the buyer). Then I characterized the set of incentive compatible direct mechanisms to be mechanisms that are independent of the reported strategy of the third part. I used this characterization to solve for the equilibrium outcomes of the monopolistic pricing problem.

Appendix: Revelation principle in the general setting

Traditional mechanism design takes information structure as the model’s primitive and seeks to characterize the optimal mechanism (i.e. rules of the game) from the designer’s perspective. However, in some economic problems, it is reasonable to assume that economic agents
may influence or manipulate other agents’ information structure. In addition, their strategy to manipulate information is often unobservable to the mechanism designer.

In this section, I model explicitly what it means for a mechanism designer to design a game when there is unobservable information manipulation after the mechanism is chosen. In particular, I will characterize a revelation principle in this setting. I will then specialize to the case of monopolistic pricing and use the revelation principle to characterize the set of incentive compatible direct mechanisms as well as the optimal mechanism. This approach invokes no new primitives. Thus, it is a direct check of the robustness of traditional mechanism design results with respect to endogenous information manipulation.

Consider a setting with $I$ agents, indexed by $i \in \{1, 2, ..., I\} = \mathcal{I}$. These agents must make a collective choice from some set $X$ of possible social outcomes. All agents (including the mechanism designer and the manipulator) have a common prior $\mu_0$ about a (random) state of nature $\theta \in \Theta$. Agents’ VNM utilities $U_i : \Theta \times X \rightarrow \mathbb{R}$ depend on the realized state of nature and the social outcome.

A third party or manipulator $m \notin \mathcal{I}$ endogenously chooses the information structure $\eta = (\{R_i\}_{i=1}^I, \{\phi(\cdot|\theta)\}_{\theta \in \Theta})$, where $R_i$ is the set of possible realizations of agent $i$’s private signal $s_i$, and $\phi(\cdot|\theta)$ is the joint distribution of the signals $(s_1, ..., s_I)$ over $\prod_{i=1}^I R_i$ conditional on the state $\theta$. Let $M$ be the set of feasible information structures that the manipulator can choose from. In this paper, I will assume $R_i$ to be exogenous, to simplify the exposition.\(^2\) The manipulator has VNM utility $U_m : \Theta \times X \rightarrow \mathbb{R}$, which represents his objective to manipulate the information structure.

Because the agents’ preferences depend on the state of nature, the agents may want the collective decision to depend on $\theta$. To capture this dependence formally, we introduce the notion of a social choice function.

**Definition 5.1.** A social choice function is a function $f : \Theta \rightarrow \Delta(X)$, that for each possible state, assigns a collective choice according to the randomization specified by $f(\theta) \in \Delta(X)$.\(^3\)

\(^2\) It is fine to relax the assumption to allow the manipulator to choose any $R_i$ below a certain cardinality. However, if we allows the most general space of information structures with $R_i$ of any cardinality, the revelation principle proposed in this paper will fail because the information designer may always choose a $R_i$ that has a larger cardinality than the action space specified in the mechanism.

\(^3\) This definition differs from traditional mechanism design in which $f$ specifies a deterministic collective
The timing of the events is as follows. At $t=0$, mechanism designer picks a mechanism $((A_i)_{i \in \mathcal{I}}, g(\cdot))$, where $A_i$ is the action space for player $i$, and $g : \prod_{i \in \mathcal{I}} A_i \to X$ is an outcome function which specifies the social outcome given the strategy profile of the players.\(^4\) At $t=1$, manipulator $m$ chooses an information structure $\eta \in M$. The manipulator’s choice is observed by all players $i \in \mathcal{I}$ but not observed by the mechanism designer. At $t=2$, players privately observe their signals according to the information structure $\eta$. At $t=3$, the players proceed to play the game characterized by $((A_i)_{i \in \mathcal{I}}, g(\cdot))$.

In this setup, the mechanism designer have no control over the set of information structures $M$. In addition, since he does not observe the information structure chosen by the manipulator, the mechanism $((A_i)_{i \in \mathcal{I}}, g(\cdot))$ does not depend on the manipulator’s strategy (nor can it depend on the private signals as in standard mechanism design). Also, since manipulator is a third party who does not participate in the subsequent mechanism, the mechanism only specifies action spaces for players in $\mathcal{I}$.

At $t = 0$, any mechanism $((A_i)_{i \in \mathcal{I}}, g(\cdot))$ chosen by the mechanism designer induces an extensive form game with $|\mathcal{I}| + 1$ players (the players in $\mathcal{I}$ and the manipulator), in which the manipulator moves first by choosing the information structure, and then players in $\mathcal{I}$ chooses their actions in $A_i$ based on their realized private signals.

A strategy for agent $i \in \mathcal{I}$ in the game created by a mechanism $((A_i)_{i \in \mathcal{I}}, g(\cdot))$ is a function $\sigma_i : R_i \times M \to A_i$ giving agent $i$’s choice from $A_i$ for each possible observation of information structure and signal realization that he might have. Loosely speaking, we say that a mechanism implements social choice function $f(\cdot)$ if there is an equilibrium of the game induced by the mechanism that yields the same outcomes as $f(\cdot)$ for each possible $\theta$. Formally,

**Definition 5.2.** The mechanism $\Gamma = (\{A_i\}_{i \in \mathcal{I}}, g(\cdot))$ implements social choice function $f(\cdot)$ if there is a sequential equilibrium strategy profile $(\eta^*, \{\sigma^*_i(\cdot)\}_{i=1}^I)$ of the game induced by $\Gamma$ choice, because in the current setup we cannot equivalently specify $\Theta$ to be the product space of private signals, due to the endogenous information structure.

\(^4\)In principle, the designer can pick any extensive form game, but it suffices to consider its normal representations in strategic form.
such that for all $\theta \in \Theta$, and all $(s_1, \ldots, s_I) \in \prod_{i=1}^I R_i$, we have

$$[f(\theta)](x^*) = \int_{\{(s'_1, \ldots, s'_I) | g(\sigma^*_1(s'_1, \eta^*), \ldots, \sigma^*_I(s'_I, \eta^*)) = x^*\}} \phi^*(s'_1, \ldots, s'_I | \theta),$$

where $x^* = g(\sigma^*_1(s_1, \eta^*), \ldots, \sigma^*_I(s_I, \eta^*))$ is the equilibrium social outcome when $(s_1, \ldots, s_I)$ is realized, $\phi^*$ is the distribution of signals chosen in $\eta^*$, and $[f(\theta)](x)$ represents the probability measure on $x \in X$ according to the distribution $f(\theta) \in \Delta(X)$.

We restrict attention to sequential equilibrium because unlike standard mechanism design problems, in the current setup, each mechanism induces an extensive form game in which the manipulator moves first, after which the mechanism is played by $I$. The mechanism designer has no control over the first stage of the game at all. In a sequential equilibrium, we want to impose belief consistency on off-equilibrium information sets of players in $I$ corresponding to information structures not chosen by the manipulator. This is because if the manipulator were to deviate to an off-equilibrium choice of information structure, he should anticipate that the players form beliefs according to the Bayes’ rule.

To illustrate this idea, and to facilitate the proof of the revelation principle introduced later, let’s consider a simple example represented by the (incomplete) game tree in Figure 2. Consider a case in which there is only one agent, i.e. $I = \{1\}$. The manipulator can only choose between two signals $M = \{\eta_1, \eta_2\}$ for player 1, where $\eta_1$ reveals the true state $\theta$ and $\eta_2$ is a noisy signal. After observing the choice of the manipulator as well as the realization of the signal, player 1 chooses his strategy in $A_1$ in the mechanism (not shown in the game tree), and then the social outcome is determined according to $g : A_1 \to X$.

Notice first that if the manipulator chooses $\sigma_1$, then the information sets corresponding to $\eta_2 = H$ and $\eta_2 = L$ not reached in equilibrium. Thus, Bayes’ rule permits arbitrary beliefs of player 1 on these information sets. To avoid incredible threats by player 1 which supports irrational choice by the manipulator, we restrict attention to sequential equilibrium.

Secondly, we should also notice that player 1 may potentially choose different strategies in $A_1$ at each of the four information sets he faces. Thus, in the revelation principle that we are about to introduce, it is not enough to ask the player to only report realizations of their private signals. We must also ask him to report his observation of the manipulator’s choices.
Figure 2: Manipulator chooses one of two signal structures \( M = \{\sigma_1, \sigma_2\} \) for another player.

To characterize the optimal mechanism, we need to consider the huge set of all possible mechanisms. Fortunately, as in standard mechanism design, we have a revelation principle which says that we can restrict attention to direct mechanisms.

**Definition 5.3.** A direct mechanism is a mechanism in which \( A_i = R_i \times M \) for all \( i \in \mathcal{I} \). We say that a direct mechanism is incentive compatible if truth telling by each agent \( i \), i.e. \( \sigma_i^*(\eta, s_i) = (\eta, s_i) \), constitutes an equilibrium.

**Theorem 5.1** (The Revelation Principle with Information Manipulation). Suppose there exists a (potentially indirect) mechanism \( (\{A_i\}_{i \in \mathcal{I}}, g(\cdot)) \) that implements the social choice function \( f(\cdot) \) in sequential equilibrium. Then, there exists an incentive compatible direct mechanism which also implements \( f(\cdot) \) in the truth telling equilibrium. Furthermore, in the extensive form games induced by both (the indirect and direct) mechanisms, the manipulator chooses the same information structure \( \eta^* \in M \) in the equilibriums.

**Proof.** If \( (\{A_i\}_{i \in \mathcal{I}}, g(\cdot)) \) implements the social choice function \( f(\cdot) \), then there exists a strategy profile \( (\eta^*, \{\sigma_i^*(\cdot)\}_{i=1}^I) \) such that

(1) for all \( \theta \in \Theta \), and all \( (s_1, ..., s_I) \in \prod_{i=1}^I R_i \), we have

\[
[f(\theta)](x^*) = \int_{(s'_1, ..., s'_I)|g(\sigma_1^*(s'_1, \eta^*), ..., \sigma_I^*(s'_I, \eta^*))=x^*} \phi^*(s'_1, ..., s'_I | \theta),
\]

where \( x^* = g(\sigma_1^*(s_1, \eta^*), ..., \sigma_I^*(s_I, \eta^*)) \) is the equilibrium social outcome when \( (s_1, ..., s_I) \) is
realized, \( \phi^* \) is the distribution of signals chosen in \( \eta^* \), and \([f(\theta)](x)\) represents the probability measure on \( x \in X \) according to the distribution \( f(\theta) \in \Delta(X) \):

(2) for all \( i \in \mathcal{I} \), all \( s_i \in R_i \), all \( \eta \in \mathcal{M} \), and any deviating strategy \( \hat{a}_i \in A_i \),

\[
\int_{\theta, s_{-i}} U_i(\theta, g(\sigma_i^*(s_i, \eta), \sigma_{-i}^*(s_{-i}, \eta))) d\phi^*(s_{-i}|s_i, \theta) \mu_0(\theta) \geq \int_{\theta, s_{-i}} U_i(g(\hat{a}_i, \sigma_{-i}^*(s_{-i}, \eta))) d\phi^*(s_{-i}|s_i, \theta) \mu_0(\theta),
\]

(3) for any deviating information structure \( \eta \in \mathcal{M} \) with conditional distribution \( \phi(\cdot|\theta) \) (by the manipulator),

\[
\int_{\theta, s_1, \ldots, s_I} U_m(\theta, g(\sigma_1^*(s_1, \eta^*), \ldots, \sigma_I^*(s_I, \eta^*))) d\phi^*(s_1, \ldots, s_I|\theta) \mu_0(\theta) \geq \int_{\theta, s_1, \ldots, s_I} U_m(\theta, g(\sigma_1^*(s_1, \eta), \ldots, \sigma_I^*(s_I, \eta))) d\phi(s_1, \ldots, s_I|\theta) \mu_0(\theta).
\]

where (1) is just the condition of implementation, (2) guarantees the sequential rationality of each player \( i \in \mathcal{I} \) at all information sets (corresponding to all signal realization under all possible information structure), and (3) guarantees the sequential rationality of the manipulator.

Notice that, (2) implies, in particular, that for all \( i \in \mathcal{I} \), all \( s_i \in R_i \), all \( \eta \in \mathcal{M} \),

\[
\int_{\theta, s_{-i}} U_i(\theta, g(\sigma_i^*(s_i, \eta), \sigma_{-i}^*(s_{-i}, \eta))) d\phi^*(s_{-i}|s_i, \theta) \mu_0(\theta) \geq \int_{\theta, s_{-i}} U_i(g(\hat{a}_i, \sigma_{-i}^*(s_{-i}, \eta))) d\phi^*(s_{-i}|s_i, \theta) \mu_0(\theta)
\]

for all \( \hat{s}_i \in R_i \) and all \( \hat{\eta} \in \mathcal{M} \). Consider the direct mechanism with \( A_i = R_i \times \mathcal{M} \) for all \( i \) and define its outcome function as \( \tilde{g}(s_1, \eta, \ldots, s_I, \eta) = g(\sigma_1^*(s_1, \eta), \ldots, \sigma_I^*(s_I, \eta)) \). Then, from (1)-(3), we know that truth-telling of all players in \( \mathcal{I} \) and manipulator choosing \( \eta^* \) is a sequential equilibrium in the direct mechanism, which implements the same social choice function.

Equipped with the revelation principle, we are ready to answer questions about optimal mechanism under specific settings. I will address this question in the next section by specializing into a simple setup of monopolistic pricing.

References


