Credit market conditions for mass enrichment or impoverishment

by Roger Myerson, https://home.uchicago.edu/~rmyerson/research/masscredit.pdf

These notes: https://home.uchicago.edu/~rmyerson/research/masscredit_notes.pdf

Introductory overview:

Many societies in history have developed **increasing concentration of wealth**.

Why? Could it happen for reasons other than mass exploitation by a powerful elite?

Yes, it can be derived, without elite exploitation, from models of **efficient risk sharing** subject to incentive constraints [*Thomas & Worrall 1990, Atkeson & Lucas 1992*].

Efficient risk-sharing subject to incentive constraints can yield a general tendency toward **mass impoverishment** of people who borrow & invest only with a **credit monopolist** (which in traditional villages could be a *local landlord*, or a *community credit union*).

But we find an opposite tendency toward **mass enrichment** of people who have inalienable access to a **competitive credit market**.

[*See also Ljungqvist & Sargent (2012) 3rd ed, chapters 17.5, 17.6, 20.5.11, 20.6*]

It might seem unsurprising that a monopolistic credit supply would be associated with a long-run tendency toward mass impoverishment of borrowers.

But in these models, ex ante, **people actually prefer dealing with a credit monopolist**, because competitive credit would reduce their ability to insure against current risks.

Excess borrowing for current consumption is an efficient way to signal bad outcomes in privately observed risks.

Thus, a stable democratic society may need laws to regulate credit contracts [Solon].

Model:

We consider an agent who, in each period $t \in \{1,2,3,...\}$, gets a **risky income** $\tilde{\theta}_t \in \Theta = \{0,G\}$ drawn independently each period with probabilities $p(G)$ and $p(0)=1-p(G)$.

In each period t, the agent holds some **resources** r_t that yield a **reliable income** $(1-\delta)r_t$ where δ is a given per-period discount factor $0<\delta<1$.

The agent's given **initial resource holding** in period 1 is r_1.

In any period t, as a function of the risky income $\tilde{\theta}_t$, the agent may increase current consumption by **borrowing** $b(\tilde{\theta}_t)$, financed by **selling resources** $s(\tilde{\theta}_t)$. [May be <0.]

Credit transactions must satisfy the **budget constraint** $\sum_{\theta\in\Theta} p(\theta)b(\theta) \le \delta\sum_{\theta\in\Theta} p(\theta)s(\theta)$.

Then period t+1 the agent will have **updated resource holding** $r_{t+1} = r_t - s(\tilde{\theta}_t)$.

The agent has **constant risk tolerance** $\tau>0$ and gets utility from period-t consumption:

$$u((1-\delta)r_t+\tilde{\theta}_t+b(\tilde{\theta}_t)) = -exp(-((1-\delta)r_t+\tilde{\theta}_t+b(\tilde{\theta}_t))/\tau).$$

The **expected discounted value** of utilities that the agent wants to maximize is then

$$V(r_1) = E[\sum_{t\in\{1,2,3...\}} \delta^{t-1}u((1-\delta)r_t+\tilde{\theta}_t+b(\tilde{\theta}_t))].$$

We allow resource holdings r_t to be positive (wealth) or negative (debt).

In all cases here we can get a **simplifying formula**: $V(r_1+\hat{r}) = V(r_1)\,exp(-(1-\delta)\hat{r}/\tau)$, because holding $\hat{r}$ for fixed supplemental income $(1-\delta)\hat{r}$ and choosing b&s as for r_1 yields utilities $u((1-\delta)(r_t+\hat{r})+\tilde{\theta}_t+b(\tilde{\theta}_t)) = u((1-\delta)r_t+\tilde{\theta}_t+b(\tilde{\theta}_t))\,exp(-(1-\delta)\hat{r}/\tau)$, which are decision-theoretically equivalent to the utilities $u((1-\delta)r_t+\tilde{\theta}_t+b(\tilde{\theta}_t))$.

With $V(0)<0$, $V(r) = V(0)exp(-(1-\delta)r/\tau)$. Then $V'(r) = -((1-\delta)/\tau)V(0)exp(-(1-\delta)r/\tau)$.

$V'(r)$ is strictly **convex & decreasing** in r. $1/V'(r)$ is strictly **convex & increasing** in r.

Adverse selection and monopolistic credit

Suppose: (1) the agent's **risky income is observable only by the agent**, not verifiable;
and (2) the agent can borrow and invest only with **one monopolistic credit source**
(which could be *the agent's landlord* or *the local community's credit union*).

We look for **optimal risk-sharing plans** for the agent subject to incentive constraints.
(Optimal for the agent means that the agent is optimally exploiting the monopolist!)

Given any wealth r, the agent's optimal value satisfies the recursive **Bellman equation**:

$$V(r) = \max_{b:\Theta\to\mathbf{R},\, s:\Theta\to\mathbf{R}} \sum_{\theta\in\Theta} p(\theta)[u((1-\delta)r+\theta+b(\theta)) + \delta V(r-s(\theta))] \text{ subject to:}$$

$$u((1-\delta)r+\theta+b(\theta)) + \delta V(r-s(\theta)) \geq u((1-\delta)r+\theta+b(\varphi)) + \delta V(r-s(\varphi)), \quad \forall \varphi,\theta\in\Theta,\ \varphi\neq\theta;$$

$$\sum_{\theta\in\Theta} p(\theta)(\delta s(\theta) - b(\theta)) \geq 0.$$

The **Lagrangean** for this problem can be written:

$$L_r(b,s;\lambda,\alpha) = \sum_{\theta\in\Theta} p(\theta)[u((1-\delta)r+\theta+b(\theta)) + \delta V(r-s(\theta))] +$$

$$+ \sum_{\theta\in\Theta}\sum_{\varphi\neq\theta} \alpha(\varphi|\theta)[u((1-\delta)r+\theta+b(\theta))+\delta V(r-s(\theta))-u((1-\delta)r+\theta+b(\varphi))-\delta V(r-s(\varphi))] +$$

$$+ \lambda\sum_{\theta\in\Theta} p(\theta)(\delta s(\theta) - b(\theta)).$$

Consider the equations: $\sum_{\theta\in\Theta} p(\theta)\delta\sigma(\theta) = -(1-\delta)$ and $V'(r-s(\theta))(1-\sigma(\theta)) = \omega$, $\forall\theta\in\Theta$.
They have solution: $\omega = 1/[\delta\sum_{\varphi\in\Theta} p(\varphi)/V'(r-s(\varphi))]$ and $\sigma(\theta) = 1-\omega/V'(r-s(\theta))$, $\forall\theta\in\Theta$.
Then the **envelope theorem** and the **Lagrangean optimality conditions** give us

$$V'(r) = \partial L_r(b,s;\lambda,\alpha)/\partial r + \sum_{\theta\in\Theta} [\sigma(\theta)\partial L_r/\partial s(\theta) - (1-\delta)\partial L_r/\partial b(\theta)] =$$

$$= \sum_{\theta\in\Theta} p(\theta)\delta V'(r-s(\theta))(1-\sigma(\theta)) = \delta\omega = 1/[\sum_{\varphi\in\Theta} p(\varphi)/V'(r-s(\varphi))].$$

So for any r, the optimal solution has: $1/V'(r) = \sum_{\theta\in\Theta} p(\theta)[1/V'(r-s(\theta))]$.
The agent's optimal plan makes $1/V'(r_t)$ a **martingale**, with $1/V'(r_t) = E[1/V'(r_{t+1})|r_t]$ $\forall t$.

But $1/V'(r_t)$ is a strictly convex & increasing function of r_t.
So in every period t: $1/V'(r_t) = E[1/V'(r_{t+1})|r_t] > 1/V'(E[r_{t+1}|r_t])$, and thus $r_t > E[r_{t+1}|r_t]$.
...long-run empoverishment!

Proposition 1 *[Thomas & Worrall 1990]. Under the general assumptions of our model, if the risk-averse agent with private income risks can borrow and invest only with one monopolistic banker and faces informational (adverse-selection) incentive constraints in devising a plan for risk-contingent credit transactions each period, then the agent's optimal plan makes $1/V'(r_t)$ (the reciprocal of the agent's marginal value of wealth) a martingale, and so the agent's wealth r_t decreases in expectation every period and goes to $-\infty$ with probability 1 as $t \to +\infty$.*

Excess borrowing for current consumption is an efficient way to signal bad outcomes in privately observed risks.

Adverse selection and competitive credit

Still assuming (1) the agent's risky income is observable only by the agent, not verifiable; suppose now (2') the agent can borrow or invest independently with **many bankers** who are risk-neutral and have the same discount factor δ.

Then agent could adjust any given contractual terms $(b(\theta),s(\theta))$ for current borrowing and future asset sales to any other $(\hat{b},\hat{s})$ that satisfies $\hat{b}-\delta\hat{s} = b(\theta)-\delta s(\theta)$ by independently borrowing $\hat{b}-b(\theta)$ from another banker for the subsequent payment $\hat{s}-s(\theta)$.

Thus, **an agent who has access such a competitive credit market would always prefer to report a type θ that maximizes $b(\theta)-\delta s(\theta)$** [*Allen 1985, Thomas & Worrall 1990*].

Bankers with the same discount factor δ would refuse terms with any $b(\theta)-\delta s(\theta)>0$, and so the agent's budget constraint becomes: $b(\theta)-\delta s(\theta) \le 0$, $\forall\theta$.

The best the agent can do is have a **balanced budget for each type**: $s(\theta) = b(\theta)/\delta$ $\forall\theta$.

Then for any wealth r, the agent's optimal value satisfies the **Bellman equation**:
$V(r) = \max_{b:\Theta\to\mathbf{R}} \sum_{\theta\in\Theta} p(\theta)[u((1-\delta)r+\theta+b(\theta)) + \delta V(r-b(\theta)/\delta)]$.

At the optimal solution, the first-order optimal conditions for each $b(\theta)$ give us
$0 = u'((1-\delta)r+\theta+b(\theta)) - \delta V'(r-b(\theta)/\delta)/\delta$, so $V'(r-b(\theta)/\delta) = u'((1-\delta)r+\theta+b(\theta))$, $\forall\theta\in\Theta$.

Then $V'(r) = \sum_{\theta\in\Theta} p(\theta)[(1-\delta)u'((1-\delta)r+\theta+b(\theta)) + \delta V'(r-b(\theta)/\delta)] = \sum_{\theta\in\Theta} p(\theta)V'(r-b(\theta)/\delta)$.
So the agent's optimal plan makes $V'(r_t)$ a **martingale**, as $V'(r_t) = E[V'(r_{t+1})|r_t]$ $\forall t$.

But $V'(r_t)$ is a strictly convex & decreasing function of r_t. So in every period t:
$V'(r_t) = E[V'(r_{t+1})|r_t] > V'(E[r_{t+1}|r_t])$, and thus $r_t < E[r_{t+1}|r_t]$. *...long-run enrichment!*
Optimal saving in good periods tends to be more than optimal borrowing in bad periods.

For each type θ, the optimal solution b here satisfies
$u((1-\delta)r+\theta+b(\theta)) + \delta V(r-b(\theta)/\delta) = \max_{\beta \in \mathbf{R}} u((1-\delta)r+\theta+\beta) + \delta V(r-\beta/\delta)$.
So with $s(\theta)=b(\theta)/\delta$, the optimal solution for this case with competitive credit would also be **feasible for the previous case of monopolistic credit**, as it could satisfy the budget constraint and the incentive constraints in the monopolistic-credit problem.
Thus, the optimal solution to the competive credit case has a value V(r) that is less than the value for the monopolistic credit case.
That is, a monopolistic credit supplier could always offer the agent an insurance contract that the agent would strictly prefer over the best he could get with competitive credit.

Proposition 2. *Under the general assumptions of our model, if the risk-averse agent with private income risks has an inalienable right to borrow and invest with bankers in a competitive credit market with discount factor δ, but the agent faces informational (adverse-selection) incentive constraints in devising a plan for risk-contingent credit transactions each period, then the agent's optimal plan makes the agent's marginal value of wealth $V'(r_t)$ a martingale, and so the agent's wealth r_t increases in expectation every period and goes to $+\infty$ with probability 1 as $t \to +\infty$. But for any given wealth r, the optimal expected discounted utility value V(r) that the agent can achieve in a competitive credit market is less than the optimal value that the agent could get from a monopolistic banker under Proposition 1.*

Numerical example: Suppose $G=2$, $\tau=2$, $\delta=0.9$, $p(G)=0.5$, and so $p(0)=0.5$.
In **autarky** ($b(\theta)=s(\theta)=0\ \forall\theta$): $V(0) = -6.839$, certainty-equivalent consumption **0.760**.
Without incentive constraints, **ideal full insurance** would yield constant consumption
$(1-\delta)r+$**1** each period, yielding $V(0) = u(1)/(1-\delta) = -6.0653$ for $r=0$.

With **adverse selection and competitive credit**, the agent's optimal plan is:
$b(0) = 0.878$, $s(0) = 0.975$, $b(G) = -0.922$, $s(G) = -1.025$, $V(0) = -6.1416$.
The value $V(r)$ then corresponds to a constant certainty-equivalent consumption
$(1-\delta)r+$**0.975** per period.
Then $E[s(\tilde{\theta}_t)] = -0.025$ (expected net investment 0.025 each period).
We have ex-post budget balance $\delta s(0)-b(0)=0$, $\delta s(G)-b(G)=0$,
and value ratios $u'(\theta+b(\theta))/V'(-s(\theta)) = 1$ for both $\theta=0$ and $\theta=G$,
but the agent saves more when $\theta=G$ than the agent borrows when $\theta=0$.

With **adverse selection and monopolistic credit**, the agent's optimal plan is:
$b(0) = 0.965$, $s(0) = 0.672$, $b(G) = -0.945$, $s(G) = -0.650$, $V(0) = -6.0971$.
The value $V(r)$ then corresponds to a constant certainty-equivalent consumption
$(1-\delta)r+$**0.990** per period.
Then $E[s(\tilde{\theta}_t)] = 0.011$ (expected net disinvestment 0.011 each period).
We get ex-post budget balance $\delta s(0)-b(0) = -0.360$, $\delta s(G)-b(G) = 0.360$,
and value ratios $u'(0+b(0))/V'(-s(0)) = 0.979$ and $u'(G+b(G))/V'(-s(G)) = 1$,
that is, excess borrowing is an efficient costly signal for $\theta=0$.

Computations at https://home.uchicago.edu/~rmyerson/research/thomasworrall.xlsx

Other results in the paper:

The same results hold when the agent's ability to insure the risky income is limited by moral-hazard incentive constraint instead of adverse-selection incentive constraints, that is, when we allow that the agent's risky income is publicly observable but the probability of high income depends on an unobservable costly effort by the agent.

With **moral hazard & monopolistic credit**, the agent's optimal plan makes $1/V'(r_t)$ a martingale, and $r_t \to -\infty$ with probability 1 as $t \to \infty$. [*Proposition 3*.]

With **moral hazard & competitive credit**, the agent's optimal plan makes $V'(r_t)$ a martingale, and $r_t \to +\infty$ with probability 1 as $t \to \infty$; but the agent's optimal value $V(r)$ is less with this case of moral hazard & competitive credit than in the corresponding case with moral hazard & monopolistic credit. [*Proposition 4*.]

Limits to aggregate enrichment with competitive credit markets?

Consider the case of competitive credit markets with adverse-selection incentive constraints, but now assume a fixed aggregate supply of capital resources.

Now the competitive market discount factor γ may differ from the given personal-utility discount factor δ. Then there exists a no-growth equilibrium in which the agents' expected net borrowing is zero, with a market discount factor γ that is greater than δ, and each agent's rent-income holdings will be an unbounded martingale.

But as in Proposition 2, the optimal expected discounted utility value that an agent could achieve in this competitive credit market would still be less than the optimal value that the agent could get from a monopolistic banker under Proposition 1.

Moral hazard and monopolistic credit

Suppose now the agent's risky income θ **is publicly observable** but outcome probabilities **$P(G|c)$ & $P(0|c)=1-P(G|c)$ depend on the agent's unobservable effort-cost $c\geq 0$**, where $P(G|0)=0$, $P(G|G)\leq 1$, $P(G|c)$ is concave, increasing, and continuously differentiable, with $P'(G|0)=+\infty$. (E.g.: $P(G|c)=(c/G)^{0.5}$.) Assume monopolistic credit.

Given any wealth r, the agent's optimal value satisfies the recursive **Bellman equation**:

$$V(r) = \max_{c\in[0,G],\, b:\Theta\to\mathbf{R},\, s:\Theta\to\mathbf{R}} \sum_{\theta\in\Theta} P(\theta|c)[u((1-\delta)r+\theta+b(\theta)-c) + \delta V(r-s(\theta))] \text{ subject to:}$$
$$\sum_{\theta\in\Theta} P'(\theta|c)[u((1-\delta)r+\theta+b(\theta)-c) + \delta V(r-s(\theta))] = \sum_{\theta\in\Theta} P(\theta|c)u'((1-\delta)r+\theta+b(\theta)-c);$$
$$\sum_{\theta\in\Theta} P(\theta|c)(\delta s(\theta) - b(\theta)) \geq 0.$$

The **Lagrangean** for this problem can be written:

$$L_r(b,s,c;\lambda,\alpha) = \sum_{\theta\in\Theta} P(\theta|c)[u((1-\delta)r+\theta+b(\theta)-c) + \delta V(r-s(\theta))] +$$
$$+ \mu[\sum_{\theta\in\Theta} P'(\theta|c)(u((1-\delta)r+\theta+b(\theta)-c)+\delta V(r-s(\theta))) - \sum_{\theta\in\Theta} P(\theta|c)u'((1-\delta)r+\theta+b(\theta)-c)] +$$
$$+ \lambda\sum_{\theta\in\Theta} P(\theta|c)(\delta s(\theta) - b(\theta)).$$

As before, use $\omega = 1/[\delta\sum_{\varphi\in\Theta} P(\varphi|c)/V'(r-s(\varphi))]$ and $\sigma(\theta) = 1-\omega/V'(r-s(\theta))$, $\forall\theta\in\Theta$, satisfying: $\sum_{\theta\in\Theta} P(\theta|c)\delta\sigma(\theta) = -(1-\delta)$ and $V'(r-s(\theta))(1-\sigma(\theta)) = \omega$, $\forall\theta\in\Theta$.

Then the **envelope theorem** and the **Lagrangean optimality conditions** give us

$$V'(r) = \partial L_r(b,s,c;\lambda,\alpha)/\partial r + \sum_{\theta\in\Theta} [\sigma(\theta)\partial L_r/\partial s(\theta) - (1-\delta)\partial L_r/\partial b(\theta)] =$$
$$= \sum_{\theta\in\Theta} P(\theta|c)\delta V'(r-s(\theta))(1-\sigma(\theta)) = \delta\omega = 1/[\sum_{\varphi\in\Theta} P(\varphi|c)/V'(r-s(\varphi))].$$

The agent's optimal plan makes $1/V'(r_t)$ a **martingale**, with $1/V'(r_t) = E[1/V'(r_{t+1})|r_t]$ $\forall t$.

But $1/V'(r_t)$ is a strictly convex & increasing function of r_t.

So in every period t: $1/V'(r_t) = E[1/V'(r_{t+1})|r_t] > 1/V'(E[r_{t+1}|r_t])$, and thus $r_t > E[r_{t+1}|r_t]$.

Moral hazard and competitive credit

With competitive credit, for any wealth r, the agent's optimal plan satisfies **Bellman** eq'n:

$$V(r) = \max_{c\in[0,G],\, b:\Theta\to\mathbf{R},\, s:\Theta\to\mathbf{R}} \sum_{\theta\in\Theta} P(\theta|c)[u((1-\delta)r+\theta+b(\theta)-c) + \delta V(r-s(\theta))] \text{ subject to:}$$

$$\sum_{\theta\in\Theta} P'(\theta|c)[u((1-\delta)r+\theta+b(\theta)-c) + \delta V(r-s(\theta))] = \sum_{\theta\in\Theta} P(\theta|c)u'((1-\delta)r+\theta+b(\theta)-c);$$

$$\sum_{\theta\in\Theta} P(\theta|c)(\delta s(\theta) - b(\theta)) \geq 0;$$

$$V'(r-s(\theta)) = u'((1-\delta)r+\theta+b(\theta)-c), \ \forall\theta\in\Theta.$$

The last constraints (from ability to borrow independently with $\partial b(\theta)/\partial s(\theta)=\delta$) imply

$$V'(r) = \sum_{\theta\in\Theta} P(\theta|c)[(1-\delta)u'((1-\delta)r+\theta+b(\theta)-c) + \delta V'(r-s(\theta))] = \sum_{\theta\in\Theta} P(\theta|c)V'(r-s(\theta)).$$

So the agent's optimal plan makes $V'(r_t)$ a **martingale**, as $V'(r_t) = E[V'(r_{t+1})|r_t]\ \forall t$.

But $V'(r_t)$ is a strictly convex & decreasing function of r_t. So in every period t:

$V'(r_t) = E[V'(r_{t+1})|r_t] > V'(E[r_{t+1}|r_t])$, and thus $r_t < E[r_{t+1}|r_t]$. *...long-run enrichment!*

Compared to this competitive solution, the credit monopolist would not face the constraints that require ex-post optimality of current borrowing decisions.

Excess borrowing for current consumption could mitigate moral hazard by reducing the marginal disutility of effort c.

Equilibrium discounting in competitive credit markets without growth

Proposition 5. *Consider the case of competitive credit markets with adverse-selection incentive constraints, as in Proposition 2, but now suppose that the aggregate supply of capital resources is fixed, and so the competitive market discount factor γ may differ from the given personal-utility discount factor δ. Then a no-growth equilibrium in which the agents' expected net borrowing is zero can be found with a market discount factor γ that is greater than δ, and then each agent's rent-income holdings form an unbounded martingale. But as in Proposition 2, the optimal expected discounted utility value that an agent could achieve in this competitive credit market would still be less than the optimal value that the agent could get from a monopolistic banker under Proposition 1.*

Consider an agent who owns assets that generate a **reliable rent income g** every period.
With competitive **market discount factor γ**, this agent's wealth would be **worth g/(1-γ)**.
Let **h(θ)** denote the net **future rent income per period that the agent would sell** when the agent's current risky income is θ, to finance net current borrowing b(θ).
Adverse selection on θ & competitive credit factor γ imply: **$b(\theta) = \gamma h(\theta)/(1-\gamma)$**, $\forall \theta \in \Theta$.
The optimal value W(g) for an agent with rent income g satisfies the **Bellman equation**:

$$W(g) = \max_{b:\Theta \to \mathbf{R}} \sum_{\theta \in \Theta} p(\theta)[u(g+\theta+b(\theta)) + \delta W(g - b(\theta)(1-\gamma)/\gamma)].$$

With the no-growth equilibrium γ, the optimal solution has **$\sum_{\theta \in \Theta} p(\theta)b(\theta)=0$**, and so it would also be **feasible for a monopolistic banker** with any discount factor.
We get **$\gamma > \delta$**, to reduce the positive expected net saving from Proposition 2 (for $\gamma=\delta$).

(*For G=2, p(G)=0.5, τ=2, δ=0.9, get γ=0.901, -b(G)=b(0)=0.901, -h(G)=h(0)=0.099 .*)

In our case of ***monopolistic credit***, agents could not buy any durable assets without permission of the "banker."

This may seem strange, because the opposite assumption that *agents can always buy & own any good that they can afford* is pervasive in economic analysis.

But **agents' ownership rights exist only with general acceptance by their community**.

So monopolistic credit can be realistic when the "banker" is a communal credit association in a traditional autonomous village.

(We might define a community to be a group of people who reside near each other and share a general consensus about their individual rights to use or own local resources.)

Our analysis shows why people may favor community policies that tie hardship assistance to restriction of investment rights, although this ultimately tends to increase poverty.

Other explanations for a tendency toward a narrow concentration of wealth have included forceful expropriation by a powerful elite.

(Bowles: since the agricultural revolution, inequality has grown with the aggregate value of resources that can be owned & inherited: land, animals, slaves, capital.)

Demands of defending the whole community against external raiders can be a force for wider distribution of property rights, to give more people a stake in this defense.

Solon's law against Athenian citizens' offering themselves as collateral for debts prevented narrowing of the class of citizens who would serve in the city's armed forces.

Henry Sumner Maine (1871, p164-168): *Even without a lord, autonomous villages tend toward oligarchy when a wider state provides even a minimal quality of regional order.*