Credit market conditions for mass enrichment or impoverishment
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Introductory overview:
Many societies in history have developed **increasing concentration of wealth**.
Why? Could it happen for reasons other than mass exploitation by a powerful elite?
Yes, it can be derived, without elite exploitation, from models of **efficient risk sharing** subject to incentive constraints [*Thomas & Worrall 1990, Atkeson & Lucas 1992*].

Efficient risk-sharing subject to incentive constraints can yield a general tendency toward **mass impoverishment** of people **without competitive credit markets**.
But we find a tendency toward **mass enrichment** of people **with competitive credit**.

It might seem unsurprising that a monopolistic credit supply would be associated with a long-run tendency toward mass impoverishment of borrowers.
But in these models, ex ante, **people actually prefer to be without competitive credit**, because competitive credit would reduce their ability to insure against current risks.

Model:

We consider an agent who, in each period $t \in \{1,2,3,...\}$, gets a **risky income** $\tilde{\theta}_t \in \Theta = \{0,G\}$ drawn independently each period with probabilities $p(G)$ and $p(0)=1-p(G)$.

In each period t, the agent holds some **resources** r_t that yield a **reliable income** $(1-\delta)r_t$ where δ is a given per-period discount factor $0<\delta<1$.

The agent's given **initial resource holding** in period 1 is r_1.

In any period t, as a function of the risky income $\tilde{\theta}_t$, the agent may increase current consumption by borrowing $b(\tilde{\theta}_t)$, financed by selling resources $s(\tilde{\theta}_t)$. [May be <0.]

Credit transactions must satisfy the **budget constraint** $\sum_{\theta \in \Theta} p(\theta)b(\theta) \le \delta \sum_{\theta \in \Theta} p(\theta)s(\theta)$.

Then period t+1 the agent will have **updated resource holding** $r_{t+1} = r_t - s(\tilde{\theta}_t)$.

The agent has **constant risk tolerance** $\tau>0$ and gets utility from period-t consumption:

$$u((1-\delta)r_t+\tilde{\theta}_t+b(\tilde{\theta}_t)) = -exp(-((1-\delta)r_t+\tilde{\theta}_t+b(\tilde{\theta}_t))/\tau).$$

The expected discounted value of utilities that the agent wants to maximize is then

$$V(r_1) = E[\sum_{t \in \{1,2,3...\}} \delta^{t-1} u((1-\delta)r_t+\tilde{\theta}_t+b(\tilde{\theta}_t))].$$

We allow resource holdings r_t to go negative (debt).

In all cases here we can get a **simplifying formula**: $V(r_1+r) = V(r_1)\, exp(-(1-\delta)\hat{r}/\tau)$, because holding $\hat{r}$ for fixed supplemental income $(1-\delta)\hat{r}$ and choosing b&s as for r_1 yields utilities $u((1-\delta)(r_t+\hat{r})+\tilde{\theta}_t+b(\tilde{\theta}_t)) = u((1-\delta)r_t+\tilde{\theta}_t+b(\tilde{\theta}_t))exp(-(1-\delta)\hat{r}/\tau)$, which are decision-theoretically equivalent to the utilities $u((1-\delta)r_t+\tilde{\theta}_t+b(\tilde{\theta}_t))$.

With $V(0)<0$, $V(r) = V(0)exp(-(1-\delta)r/\tau)$. Then $V'(r) = -V(0)((1-\delta)/\tau)exp(-(1-\delta)r/\tau)$.

$V'(r)$ is strictly convex & decreasing in r. $1/V'(r)$ is strictly convex & increasing in r.

Adverse selection and monopolistic credit

Suppose: (1) the agent's **risky income is observable only by the agent**, not verifiable; and (2) the agent can borrow and invest only with **one monopolistic credit source** (which could be *the agent's landlord* or *the local community's credit union*).

We look for optimal risk-sharing plans for the agent subject to incentive constraints.

(Optimal for the agent means that the agent is optimally exploiting the monopolist!)

Given any wealth r, the agent's optimal value satisfies the recursive **Bellman equation**:

$$V(r) = \max_{b:\Theta\to\mathbf{R},\, s:\Theta\to\mathbf{R}} \sum_{\theta\in\Theta} p(\theta)[u((1-\delta)r+\theta+b(\theta)) + \delta V(r-s(\theta))] \text{ subject to:}$$

$$u((1-\delta)r+\theta+b(\theta)) + \delta V(r-s(\theta)) \geq u((1-\delta)r+\theta+b(\varphi)) + \delta V(r-s(\varphi)),\ \forall\varphi,\theta\in\Theta,\ \varphi\neq\theta;$$

$$\sum_{\theta\in\Theta} p(\theta)(\delta s(\theta) - b(\theta)) \geq 0.$$

The **Lagrangean** for this problem can be written:

$$L_r(b,s;\lambda,\alpha) = \sum_{\theta\in\Theta} p(\theta)[u((1-\delta)r+\theta+b(\theta)) + \delta V(r-s(\theta))] +$$

$$+ \sum_{\theta\in\Theta}\sum_{\varphi\neq\theta} \alpha(\varphi|\theta)[u((1-\delta)r+\theta+b(\theta))+\delta V(r-s(\theta))-u((1-\delta)r+\theta+b(\varphi))-\delta V(r-s(\varphi))] +$$

$$+ \lambda\sum_{\theta\in\Theta} p(\theta)(\delta s(\theta) - b(\theta)).$$

Consider the equations: $\sum_{\theta\in\Theta} p(\theta)\delta\sigma(\theta) = -(1-\delta)$ and $V'(r-s(\theta))\,(1-\sigma(\theta)) = \omega,\ \forall\theta\in\Theta$.

They have solution: $\omega = 1/[\delta\sum_{\varphi\in\Theta} p(\varphi)/V'(r-s(\varphi))]$ and $\sigma(\theta) = \omega/V'(r-s(\theta)),\ \forall\theta\in\Theta$.

Then the **envelope theorem** and the **Lagrangean optimality conditions** give us

$$V'(r) = \partial L_r(b,s;\lambda,\alpha)/\partial r + \sum_{\theta\in\Theta} [\sigma(\theta)\partial L_r/\partial s(\theta) - (1-\delta)\partial L_r/\partial b(\theta)] =$$

$$= \sum_{\theta\in\Theta} p(\theta)\delta V'(r-s(\theta))(1-\sigma(\theta)) = \delta\omega = 1/[\sum_{\varphi\in\Theta} p(\varphi)/V'(r-s(\varphi))].$$

So for any r, the optimal solution has: $1/V'(r) = \sum_{\theta\in\Theta} p(\theta)[1/V'(r-s(\theta))]$.

The agent's optimal plan makes $1/V'(r_t)$ a **martingale**, with $1/V'(r_t) = E[1/V'(r_{t+1})|r_t]\ \forall t$.
But $1/V'(r_t)$ is a strictly convex & increasing function of r_t.
So in every period t: $1/V'(r_t) = E[1/V'(r_{t+1})|r_t] > 1/V'(E[r_{t+1}|r_t])$, and thus $r_t > E[r_{t+1}|r_t]$.

Proposition 1 *[Thomas & Worrall 1990]. Under the general assumptions of our model, if the risk-averse agent with private income risks can borrow and invest only with one monopolistic banker and faces informational (adverse-selection) incentive constraints in devising a plan for risk-contingent credit transactions each period, then the agent's optimal plan makes* $1/V'(r_t)$ *(the reciprocal of the agent's marginal value of wealth) a martingale, and so the agent's wealth* r_t *decreases in expectation every period and goes to* $-\infty$ *with probability 1 as* $t \to +\infty$.

Adverse selection and competitive credit

Still assuming (1) the agent's risky income is observable only by the agent, not verifiable; suppose now (2') the agent can borrow or invest independently with many different risk-neutral bankers who have the same discount factor δ.

Then agent could adjust any given contractual terms $(b(\theta),s(\theta))$ for current borrowing and future asset sales to any other $(\hat{b},\hat{s})$ that satisfies $\hat{b}-\delta\hat{s} = b(\theta)-\delta s(\theta)$ by independently borrowing $\hat{b}-b(\theta)$ from another banker for the subsequent payment $\hat{s}-s(\theta)$.

Thus, **an agent who has access such a competitive credit market would always prefer to report a type θ that maximizes $b(\theta)-\delta s(\theta)$** [Allen 1985, Thomas & Worrall 1990].

Bankers with the same discount factor δ would refuse terms with any $b(\theta)-\delta s(\theta)>0$, and so the agent's budget constraint becomes: $b(\theta)-\delta s(\theta) \leq 0, \forall\theta$.

The best the agent can do is have a **balanced budget for each type**: $s(\theta) = b(\theta)/\delta\ \forall\theta$.

Then for any wealth r, the agent's optimal value satisfies the recursive Bellman equation:

$$V(r) = \max_{b:\Theta\to\mathbf{R}} \sum_{\theta\in\Theta} p(\theta)[u((1-\delta)r+\theta+b(\theta)) + \delta V(r-b(\theta)/\delta)].$$

At the optimal solution, the first-order optimal conditions for each $b(\theta)$ give us

$$0 = u'((1-\delta)r+\theta+b(\theta)) - \delta V'(r-b(\theta)/\delta)/\delta, \text{ so } V'(r-b(\theta)/\delta) = u'((1-\delta)r+\theta+b(\theta)), \forall\theta\in\Theta.$$

Then $V'(r) = \sum_{\theta\in\Theta} p(\theta)[(1-\delta)u'((1-\delta)r+\theta+b(\theta)) + \delta V'(r-b(\theta)/\delta)] = \sum_{\theta\in\Theta} p(\theta)V'(r-b(\theta)/\delta)$.

So the agent's optimal plan makes $V'(r_t)$ a **martingale**, as $V'(r_t) = E[V'(r_{t+1})|r_t]\ \forall t$.

But $V'(r_t)$ is a strictly convex & decreasing function of r_t. So in every period t:

$$V'(r_t) = E[V'(r_{t+1})|r_t] > V'(E[r_{t+1}|r_t]), \text{ and thus } r_t < E[r_{t+1}|r_t].$$

For each type θ, the optimal solution b here satisfies

$u((1-\delta)r+\theta+b(\theta)) + \delta V(r-b(\theta)/\delta) = \max_{\beta\in\mathbf{R}} u((1-\delta)r+\theta+\beta) + \delta V(r-\beta/\delta).$

So with $s(\theta)=b(\theta)/\delta$, the optimal solution for this case with competitive credit would also be **feasible for the previous case of monopolistic credit**, as it could satisfy the budget constraint and the incentive constraints in the monopolistic-credit problem.

Thus, the optimal solution to the competive credit case has a value V(r) that is less than the value for the monopolistic credit case.

That is, a monopolistic credit supplier could always offer the agent an insurance contract that the agent would strictly prefer over the best he could get with competitive credit.

Proposition 2. *Under the general assumptions of our model, if the risk-averse agent with private income risks has an inalienable right to borrow and invest with bankers in a competitive credit market with discount factor δ, but the agent faces informational (adverse-selection) incentive constraints in devising a plan for risk-contingent credit transactions each period, then the agent's optimal plan makes the agent's marginal value of wealth $V'(r_t)$ a martingale, and so the agent's wealth r_t increases in expectation every period and goes to $+\infty$ with probability 1 as $t\to+\infty$. But for any given wealth r, the optimal expected discounted utility value V(r) that the agent can achieve in a competitive credit market is less than the optimal value that the agent could get from a monopolistic banker under Proposition 1.*

Numerical example:

Suppose $G=2$, $\tau=2$, $\delta=0.9$, $p(G)=0.5$, and so $p(0)=0.5$.

Without incentive constraints, **ideal full insurance** would yield constant consumption $(1-\delta)r+1$ each period, yielding $V(0) = u(1)/(1-\delta) = -6.0653$ for $r=0$.

With **adverse selection and competitive credit**, the agent's optimal plan is: $b(0) = 0.878$, $s(0) = 0.975$, $b(G) = -0.922$, $s(G) = -1.025$, $V(0) = -6.1416$.

The value $V(r)$ then corresponds to a constant certainty-equivalent consumption $(1-\delta)r+0.975$ per period.

Then $E[s(\tilde{\theta}_t)] = -0.025$ (expected net investment 0.025 each period).

We have ex-post budget balance $\delta s(0)-b(0)=0$, $\delta s(G)-b(G)=0$, and value ratios $u'(\theta+b(\theta))/V'(-s(\theta)) = 1$ for both $\theta=0$ and $\theta=G$, but the agent saves more when $\theta=G$ than the agent borrows when $\theta=0$.

With **adverse selection and monopolistic credit**, the agent's optimal plan is: $b(0) = 0.965$, $s(0) = 0.672$, $b(G) = -0.945$, $s(G) = -0.650$, $V(0) = -6.0971$.

The value $V(r)$ then corresponds to a constant certainty-equivalent consumption $(1-\delta)r+0.990$ per period.

Then $E[s(\tilde{\theta}_t)] = 0.011$ (expected net disinvestment 0.011 each period).

We get ex-post budget balance $\delta s(0)-b(0) = -0.360$, $\delta s(G)-b(G) = 0.360$, and value ratios $u'(0+b(0))/V'(-s(0)) = 0.979$ and $u'(G+b(G))/V'(-s(G)) = 1$, that is, excess borrowing is an efficient costly signal for $\theta=0$.

Other results in the paper:

The same results hold when the agent's ability to insure the risky income is limited by moral-hazard incentive constraint instead of adverse-selection incentive constraints, that is, when we allow that the agent's risky income is publicly observable but the probability of high income depends on an unobservable costly effort by the agent.

With **moral hazard & monopolistic credit**, the agent's optimal plan makes $1/V'(r_t)$ a martingale, and $r_t \to -\infty$ as $t \to \infty$ with probability 1. [*Proposition 3*.]

With **moral hazard & competitive credit**, the agent's optimal plan makes $V'(r_t)$ a martingale, and $r_t \to +\infty$ as $t \to \infty$ with probability 1; but the agent's optimal value V(r) is less with this case of moral hazard & competitive credit than in the corresponding case with moral hazard & monopolistic credit. [*Proposition 4*.]