Credit Market Conditions for Mass Enrichment or Impoverishment

by Roger Myerson, September 2025

https://home.uchicago.edu/~rmyerson/research/masscredit.pdf

Abstract: We revisit a simplified version of a model of Thomas and Worrall (1990), on optimal incentive-compatible financial plans for risk-averse agents who face regular income risks but are subject to adverse-selection incentive constraints. Thomas and Worrall found that, when the agents can commit to borrow and invest only with one monopolistic source of credit, efficient signaling requires excess dissaving in bad periods, which leads to the agent's impoverishment almost surely in the long run. We find that, in contrast, when the agent has inalienable access to competitive opportunities for borrowing and investing, incentives to save more in good periods lead instead to long-run enrichment almost surely. Ex ante, however, the agent would prefer the optimal plan with one monopolistic credit source, in spite of its poor long-run prospects, because it offers better insurance against current risks. We show that these results hold also with moral-hazard incentive constraints.

Introduction

In a 1990 paper on "Income fluctuation and asymmetric information," Jonathan Thomas and Tim Worrall offered fundamental insights into the dynamics of wealth and inequality, but some key implications of their analysis have not been widely understood. The purpose of this note is to revisit the Thomas-Worrall results, with some simplifying technical assumptions but with a broader range of economic assumptions, to highlight some of these vital points.

Thomas and Worrall (1990) consider infinite-horizon dynamic models of risk-averse agents who can negotiate efficient contracts with risk-neutral suppliers of credit and insurance. Their model assumes that everyone shares a common discount factor so that, in the absence of any incentive constraints, perfect insurance contracts would lead to a stationary constant distribution of wealth in which risk-averse agents transfer all their personal risk to risk-neutral insurers. But the results change dramatically when we admit imperfections generated by incentive constraints. The main result that Thomas and Worrall (1990) emphasize is a long-term tendency toward mass impoverishment of risk-averse agents who repeatedly enter into economically efficient insurance contracts with a monopolistic supplier of borrowing and investment opportunities. Their proof of this result uses an elegant application of the martingale convergence theorem that applies to a remarkably general class of models of risk-averse agents whose relationships with their source of credit is subject to informational incentive constraints of adverse selection.

In this paper, we compare Thomas and Worrall's results to starkly different results can be derived under an alternative assumption of competitive credit markets. When agents have access to competitive opportunities for borrowing and investing, we find an opposite tendency toward mass enrichment of the risk-averse incentive-constrained agents, and the proof involves a neatly reciprocal application of the martingale convergence theorem. Furthermore, we show that these results about the contrasting long-term tendencies toward mass impoverishment or enrichment with monopolistic or competitive credit markets can also be derived for cases where the imperfection of insurance markets is due to moral hazard, not just the adverse selection case that was analyzed by Thomas and Worrall (1990).

Given the pervasive risks of farming, we could interpret the agents in these models as traditional peasant farmers, and then our results could offer a basic perspective on economic conditions that could have caused impoverishment of peasants in societies throughout history. In this regard, our conclusion is that free peasants much more are likely to become impoverished in communities where local credit is provided monopolistically than in societies where everyone has access to a competitive market for loans and investments.

This conclusion might seem intuitive and even unsurprising, as a local lord who is the sole source of credit for farmers in his community might use his monopolistic power to impose exploitative terms for credit that drive peasant farmers to the brink of ruin after crop failures. Such exploitation may indeed have occurred in many societies throughout history. But the results that we consider here are about a tendency toward impoverishment of risk-averse borrowers even when they get the best terms that they could ask for from a risk-neutral monopolistic supplier of credit. The optimal contracts that we find under monopolistic credit would actually be preferred ex ante by the agents over the best alternative that they could get from a competitive credit market, even though the long-term consequences of this monopolistic credit are almost surely worse.

Thus, our essential condition for avoiding long-term impoverishment is that agents should have an inalienable right to borrow and lend in competitive credit markets. Agents who could sign away their right to participate in competitive credit markets would agree to do so, to get the optimal monopolistic credit terms that we find in the models here, even though these terms lead almost surely to long-term impoverishment. So we find that restrictions on credit contracts may be appropriate when the preservation of a mass of free and prosperous citizens is valued as a public good. Indeed, as Aristotle noted, the first of Solon's reforms to establish democracy in Athens was a law to prohibit Athenian citizens from offering themselves as collateral in loan contracts (Everson, 1996).

For simplicity, our formal analysis here is focused on models where the agent has constant absolute risk aversion or *constant risk tolerance*, but we try to indicate how the derivations may be extended to the wider classes of models that Thomas and Worrall (1990) considered. The assumption of constant risk tolerance greatly simplifies the derivations, because it yields optimal contracts that do not depend on the agent's wealth, but this independence means that our model assumes no lower bound on the agent's wealth. More realistically, we should recognize (as Thomas and Worrall do in their concluding section) that increasing impoverishment would eventually lead to some dismal lower bound where increasing desperation could induce agents to disrupt the social contract or escape from it. Thus, when our models imply a long-run drift toward agents' infinite impoverishment, we may interpret this result as meaning that the agents' free participation in the given social order will eventually become questionable, leading ultimately to fundamental social transformation as people rebel or flee or lose their freedom to do either. On the other hand, when alternative assumptions about the supply of credit imply a long-run drift toward agents' enrichment, these contrasting results may be taken as support for hopes of long-term social stability, as people get steadily greater stakes in the given social order (although such growth may ultimately confront evironmental resource bounds). That is, our analysis may offer some perspective on ideas such as Solon's about the potential importance of credit reforms for a stable democratic society.

Basic elements of our model

We consider a discrete-time infinite-horizon dynamic model of a risk-averse agent who gets some risky income $\tilde{\theta}_t$ each period $t \in \{1,2,3,...\}$ and may also hold some resources r that pay reliable rents $(1-\delta)r$ per period. For simplicity, we assume that the agent's utility for consumption in each period is defined by a utility function with constant risk tolerance $\tau>0$, and the agent's goal is to maximize the expected present discounted value of consumption utilities, with the discount factor δ, where $0<\delta<1$. So the agent's utility of consumption x would be

$$u(x) = -exp(-x/\tau).$$

The agent's risky income is drawn independently in each period t from some probability distribution p over some set of possible values Θ. For simplicity, we may assume here that $\tilde{\theta}_t$ has just two possible values, a good outcome G>0 and a bad outcome 0. That is, the set of possible values of $\tilde{\theta}_t$ is $\Theta = \{0,G\}$, and in each period t the agent may have a high type with $\tilde{\theta}_t=G$ or a low type with $\tilde{\theta}_t=0$. In the first several sections, we take the probabilities p(G)>0 and p(0)=1-p(G)>0 as exogenously given parameters. (Later, in the sections on moral hazard, we will assume take these probabilities to depend on some unobservable effort c, which the agent chooses privately each period, according to some function $p(\theta)=P(\theta|c)$.)

In each period, the agent may enter into credit transactions with one or more risk-neutral bankers who have the same discount factor δ. We will assume one monopolistic banker in some sections here, but we will assume multiple competitive bankers in other sections. By these credit transactions, the agent can increase current consumption by some net borrowing b, which can then be financed by selling some share s of the agent's future resources. These amounts can depend on the current amount of the agent's risky income. So in any period when the agent holds safe resources r and has current risky income θ, the agent's utility of current consumption would be $u((1-\delta)r+\theta+b(\theta))$, and the agent's resource holdings in next period would become $r-s(\theta)$. We assume that a risk-neutral banker would be willing to accept a contract stipulating these transfers $b(\theta)$ and $s(\theta)$ if they satisfy the budget constraint

$$\sum_{\theta\in\Theta} p(\theta)b(\theta) \leq \delta\sum_{\theta\in\Theta} p(\theta)s(\theta).$$

One advantage of assuming constant risk tolerance is that, for all of the various cases that we analyze here, the agent's optimal plan (b,s) will be independent of the agent's given resource holdings. Then, given any initial resource holding $\tilde{r}_1=r$ in the first period, the agent's resource holdings will evolve as a stochastic process $\{\tilde{r}_t\}$ with

$$\tilde{r}_{t+1} = \tilde{r}_t-s(\tilde{\theta}_t), \ \ \forall t\in\{1,2,3...\},$$

where $\tilde{\theta}_t$ denotes the agent's risky income in period t. The expected discounted value of utilities that the agent wants to maximize is then

$$V(r) = E[\sum_{t\in\{1,2,3...\}} \delta^{t-1}u((1-\delta)\tilde{r}_t+\tilde{\theta}_t+b(\tilde{\theta}_t))].$$

The simple recursive structure of this model gives us, for all r, the general Bellman equation

$$V(r) = \sum_{\theta\in\Theta} p(\theta)[u((1-\delta)r+\theta+b(\theta)) + \delta V(r-s(\theta))].$$

Here b and s must be chosen optimally subject to the constraints apply in the case that we are considering (which will be different in each section below). Since the risky income is never worse than 0 for sure or better than G for sure, in all cases here we will find solutions to Bellman equations that satisfy the general bounds:

$$u((1\text{-}\delta)r+0)/(1\text{-}\delta) \leq V(r) \leq u((1\text{-}\delta)r+G)/(1\text{-}\delta).$$

For a simple numerical example, let us consider the parametric values p(G)=0.5, G=2, τ=2, δ=0.9, and $\tilde{r}_1$=0. With these parameters, the expected value of the risky income is p(G)G+(1-p(G))0 = 1. But in autarky, without any access to credit, the agent would be willing to exchange this risky income here for a certainty-equivalent (constant risk-free) income x such that

$$-exp(-x/\tau) = p(G)(-exp(-G/\tau))+(1-p(G))(-exp(-0/\tau)).$$

With these parameters this certainty-equivalent income is about x=0.760 per period. That is, in this example with no access to credit, the risk-averse agent would be willing to give up about 24% of the expected risky income to insure against these risks.

The ideal case of perfect insurance

Let us consider first the ideal case in which the agent can perfectly insure against the income risks. In this case, the Bellman equation for any given initial resources r is

$$V(r) = \text{maximum}_{b:\Theta\to\mathbf{R},\ s:\Theta\to\mathbf{R}} \sum_{\theta\in\Theta} p(\theta)[u((1\text{-}\delta)r+\theta+b(\theta)) + \delta V(r\text{-}s(\theta))] \text{ subject to:}$$

$$\sum_{\theta\in\Theta} p(\theta)(\delta s(\theta) - b(\theta)) \geq 0.$$

Letting λ denote the Lagrange multiplier for the budget constraint, the Lagrangean for this optimization problem is

$$L_r(b,s;\lambda) = \sum_{\theta\in\Theta} p(\theta)[u((1\text{-}\delta)r+\theta+b(\theta)) + \delta V(r\text{-}s(\theta))] + \lambda\sum_{\theta\in\Theta} p(\theta)[\delta s(\theta) - b(\theta)].$$

For any r, at the optimal solution we get, from $\partial L_r/\partial b(\theta)=0$ and $\partial L_r/\partial s(\theta)=0$:

$$p(\theta)u'((1\text{-}\delta)r+\theta+b(\theta)) = \lambda p(\theta),\ \forall\theta\in\Theta,$$

$$\lambda p(\theta)\delta = p(\theta)\delta V'(r\text{-}s(\theta)),\ \forall\theta\in\Theta.$$

So $u'((1\text{-}\delta)r+\theta+b(\theta)) = \lambda = V'(r\text{-}s(\theta))$, $\forall\theta\in\Theta$. Then we also have

$$V'(r) = \sum_{\theta\in\Theta} p(\theta)[(1-\delta)u'((1\text{-}\delta)r+\theta+b(\theta)) + \delta V'(r\text{-}s(\theta))] = \lambda.$$

These equations have a solution where θ+b(θ) is constant over θ and s(θ)=0 for all θ. Then the binding budget constraint requires $\sum_{\theta\in\Theta} p(\theta)b(\theta)=0$, and so we get optimal solution:

$$s(\theta) = 0,\ \ b(\theta) = [\sum_{\varphi\in\Theta} p(\varphi)\varphi] - \theta,\ \ \forall\theta\in\Theta.$$

Thus $\tilde{r}_t = r_1$ for all t, as the agent's wealth remains constant forever with perfect insurance. Then

$$V(r) = \sum_{t\geq 1} \delta^{t-1}u((1\text{-}\delta)r+\sum_{\varphi\in\Theta} p(\varphi)\varphi) = u((1\text{-}\delta)r+\sum_{\varphi\in\Theta} p(\varphi)\varphi)/(1\text{-}\delta), \text{ and}$$

$$\lambda = V'(r) = u'((1-\delta)r+\sum_{\varphi\in\Theta} p(\varphi)\varphi).$$

Proposition 0. *Under the general assumptions of our model, if the risk-averse agent with private income risks faces no incentive constraints in devising a plan for risk-contingent borrowing and lending each period with bankers who (by assumption) are risk neutral and have the same discount factor as the agent, then the agent's optimal plan provides perfect insurance against income risks and makes the agent's wealth r constant over time.*

In our simple numerical example with $p(G)=0.5$, $G=2$, and $\tau=2$, perfect insurance would enable the agent to convert the risky income into a risk-free constant income 1 by a contract that has $b(0)=1$ and $b(G)=-1$, so that the agent gets 1 from the banker when $\tilde{\theta}_t=0$ but pays 1 to the banker when $\tilde{\theta}_t=G=2$. With $s(0)=s(G)=0$, the agent's wealth $\tilde{r}_t$ remains constant over time. Given initial wealth $\tilde{r}_1=r$, the agent would then consume $(1-\delta)r+1$ every period forever.

Thus, under our assumption that the risk-averse agents discount the future with the same factor δ as their risk-neutral bankers, this economy could have a steady-state equilibrium in which risk-averse agents' wealth remains constant over time, if all agents could perfectly insure against their personal income risks. We will show next how this result changes when adverse-selection problems constrain agent's ability to insure their risks.

Adverse selection and monopolistic credit

Now suppose that the agent's risky income $\tilde{\theta}_t$ is the agent's private information and cannot be observed by anyone else. Then the ideal perfect insurance is not incentive compatible, because the agent could get $b(0)>0$ by claiming that $\tilde{\theta}_t$ is 0 even when it is actually G. The agent's ability to lie about private information means that a credit contract (b,s) can be incentive compatible only if it satisfies incentive constraints saying that, when the agent's risky income is θ, the agent cannot expect to do better by claiming that it is some other amount φ, to get credit transfers $b(\varphi)$ and $s(\varphi)$ instead of $b(\theta)$ and $s(\theta)$. So with these informational incentive constraints along with the budget constraint, the agent's optimal value V(r) must now satisfy the following Bellman equation for any initial wealth r:

$$V(r) = \max_{b:\Theta\to\mathbf{R},s:\Theta\to\mathbf{R}} \sum_{\theta\in\Theta} p(\theta)[u((1-\delta)r+\theta+b(\theta)) + \delta V(r-s(\theta))] \text{ subject to:}$$

$$u((1-\delta)r+\theta+b(\theta)) + \delta V(r-s(\theta)) \geq u((1-\delta)r+\theta+b(\varphi)) + \delta V(r-s(\varphi)),\ \forall\varphi,\theta\in\Theta,\ \varphi\neq\theta;$$

$$\sum_{\theta\in\Theta} p(\theta)(\delta s(\theta) - b(\theta)) \geq 0.$$

To formulate a Langrangean function for this optimization problem, let $\alpha(\varphi|\theta)\geq 0$ be the Lagrange multipliers of these informational incentive constraints, and let λ be the Lagrange multiplier of the budget constraint. Then for any given r, the Lagrangean function is:

$$L_r(b,s;\lambda,\alpha) = \sum_{\theta\in\Theta} p(\theta)[u((1-\delta)r+\theta+b(\theta)) + \delta V(r-s(\theta))] +$$
$$+ \sum_{\theta\in\Theta} \sum_{\varphi\neq\theta} \alpha(\varphi|\theta)[u((1-\delta)r+\theta+b(\theta))+\delta V(r-s(\theta))-u((1-\delta)r+\theta+b(\varphi))-\delta V(r-s(\varphi))] +$$
$$+ \lambda\sum_{\theta\in\Theta} p(\theta)(\delta s(\theta) - b(\theta)).$$

We get an optimal solution when (b,s) maximizes the Lagrangean for (λ,α) that have nonzero Lagrange multipliers only for binding constraints that are satisfied as equalities. The first-order optimality conditions for the decision variables are, for each θ in Θ:

$$0 = \partial L_r/\partial b(\theta) = [p(\theta)+\sum_{\varphi\neq\theta} \alpha(\varphi|\theta)]u'((1-\delta)r+\theta+b(\theta)) - \sum_{\varphi\neq\theta} \alpha(\theta|\varphi)u'((1-\delta)r+\varphi+b(\theta)) - \lambda p(\theta),$$
$$0 = \partial L_r/\partial s(\theta) = \lambda p(\theta)\delta - [p(\theta) + \sum_{\varphi\neq\theta} (\alpha(\varphi|\theta)-\alpha(\theta|\varphi))]\delta V'(r-s(\theta)).$$

The equations, along with the conditions that each constraint either is satisfied as a binding equation or has a Lagrange multiplier that equals 0, give us conditions for determining the solution $(b(\theta),s(\theta))_{\theta\in\Theta}$ along with the Lagrange multipliers of the binding constraints.

By envelope theorem, the marginal value of additional wealth can be computed by differentiating the Lagrangean:

$$V'(r) = \partial L_r(b,s;\lambda,\alpha)/\partial r = \sum_{\theta\in\Theta} p(\theta)[(1-\delta)u'((1-\delta)r+\theta+b(\theta)) + \delta V'(r-s(\theta))] +$$
$$+ \sum_{\theta\in\Theta} \sum_{\varphi\neq\theta} \alpha(\varphi|\theta)(1-\delta)[u'((1-\delta)r+\theta+b(\theta)) - u'((1-\delta)r+\theta+b(\varphi)))] +$$
$$+ \sum_{\theta\in\Theta} \sum_{\varphi\neq\theta} \alpha(\varphi|\theta)\delta[V'(r-s(\theta)) - V'(r-s(\varphi))].$$

From this envelope formula, we can derive two more useful formulas by adding linear combinations of the first-order optimality equations for the decision variables $b(\theta)$ and $s(\theta)$. That is, in first-order sensitivity analysis of how small changes in wealth r would affect the value function V(r), we may consider any similarly small changes of the decision variables, because the first-order effect of the optimized decision variables is zero.

First, we can cancel out all the terms in $\partial L_r(b,s;\lambda,\alpha)/\partial r$ that involve u′ and V′ by adding $\partial L_r/\partial s(\theta)=0$ and subtracting $(1-\delta)\partial L_r/\partial b(\theta)$ for each θ:

$$V'(r) = \partial L_r(b,s;\lambda,\alpha)/\partial r + \sum_{\theta\in\Theta} [\partial L_r/\partial s(\theta) - (1-\delta)\partial L_r/\partial b(\theta)] =$$
$$= \sum_{\theta\in\Theta} [\lambda p(\theta)\delta - (1-\delta)\lambda p(\theta)] = \lambda.$$

That is, when we consider the effect on the Lagrangean of marginally increasing r by some small amount ε, the effect on all u(.) terms could be cancelled out by decreasing each $b(\theta)$ by $(1-\delta)\varepsilon$, and the effect on the V(.) terms could be cancelled out by increasing each $s(\theta)$ by ε; but these changes in b and s would also affect the budget constraint, which is multiplied by λ in the Lagrangean. Thus we find that the marginal value of wealth to the agent is equal to the Lagrange multiplier of the budget constraint: $V'(r)=\lambda$.

A second formula for $V'(r)$ can be derived by considering changes in the decision variables that adjust b to cancel out the effect of r on current consumption but then adjust s to avoid any first-order effect on the binding constraints. As before, when wealth r is increased by some small amount ε, current consumption $u((1-\delta)r+\theta+b(\theta))$ could be held constant if we changed borrowing $b(\theta)$ by $-(1-\delta)\varepsilon$ for each θ in Θ. Now suppose that we also changed each $s(\theta)$ by some net increase $\sigma(\theta)\varepsilon$. Then the incentive constraints would depend on ε only through the differences between the future values $V(r+\varepsilon-s(\theta)-\sigma(\theta)\varepsilon)$ that different types θ could get. So the first-order effect on the incentive constraints would be zero if the first-order effect of ε on the future values $V(r+\varepsilon-s(\theta)-\sigma(\theta)\varepsilon)$ were the same for all θ. That is, for some ω, we should have

$$V'(r-s(\theta))\,(1-\sigma(\theta)) = \omega,\ \ \forall\theta\in\Theta.$$

Maintaining equality in the budget constraint would require

$$\textstyle\sum_{\theta\in\Theta} p(\theta)\delta\sigma(\theta) = -(1-\delta).$$

These equations can be satisfied with

$$\omega = 1/[\delta\textstyle\sum_{\varphi\in\Theta} p(\varphi)/V'(r-s(\varphi))];\ \text{ and }\ \sigma(\theta) = \omega/V'(r-s(\theta)),\ \forall\theta\in\Theta.$$

Then we get

$$V'(r) = \partial L_r(b,s;\lambda,\alpha)/\partial r + \textstyle\sum_{\theta\in\Theta} [\sigma(\theta)\partial L_r/\partial s(\theta) - (1-\delta)\partial L_r/\partial b(\theta)] =$$

$$= \textstyle\sum_{\theta\in\Theta} p(\theta)\delta V'(r-s(\theta))(1-\sigma(\theta)) = \delta\omega = 1/[\sum_{\varphi\in\Theta} p(\varphi)/V'(r-s(\varphi))],$$

and so

$$1/V'(r) = \textstyle\sum_{\theta\in\Theta} p(\theta)[1/V'(r-s(\theta))].$$

This last equation says that, in the stochastic process with $\tilde{r}_{t+1} = \tilde{r}_t-s(\tilde{\theta}_t)$, the agent's optimal solutions make the quantity $1/V'(\tilde{r}_t)$ a martingale. Also, this quantity is bounded below by 0, and so the martingale convergence theorem tells us that the sequence $1/V'(\tilde{r}_t)$ must converge with probability 1 as $t\rightarrow\infty$. This is the main result of Thomas and Worrall (1990). They observed that $1/V'(\tilde{r}_t)$ could not have any positive probability of converging to a positive limit $1/V'(r)$ for any finite r. The proof of this observation is that the agent's optimal contract for wealths at r would still have $b(0)>0$, to reduce downside consumption risks, which in turn would require $s(0)>0$, to avoid violating the budget constraint and incentive constraints; but then in every period when $\tilde{r}_t$ is sufficiently near r, there would be a positive probability $p(0)$ of $\tilde{r}_{t+1}=\tilde{r}_t-s(0)$ jumping down farther away from r. So Thomas and Worrall showed that, as $t\rightarrow\infty$, the stochastic process must satisfy $1/V'(\tilde{r}_t)\rightarrow 0$ with probability 1, which implies that $V'(\tilde{r}_t)\rightarrow+\infty$ and so $\tilde{r}_t\rightarrow-\infty$ with probability 1. Selling more $s(\theta)$ when θ is low helps to prove the agent's need, but it leads almost surely to poverty in the long run.

We can characterize the value function V more specifically by using our assumption of constant risk tolerance, which gives us $u(x)=-exp(-x/\tau)$ with the risk-tolerance parameter $\tau>0$. Then once the optimal contracts b and s are determined, the agent's value function is defined by the equation

$$V(r) = -E[\textstyle\sum_{t\in\{1,2,3,...\}} \delta^{t-1}\, exp(-((1-\delta)\tilde{r}_t+\tilde{\theta}_t+b(\tilde{\theta}_t))/\tau)],$$
$$\text{with } \tilde{r}_1 = r \text{ and } \tilde{r}_{t+1} = \tilde{r}_t-s(\tilde{\theta}_t)\ \forall t\in\{1,2,3,...\}.$$

If initial wealth were changed from r to $r+\rho$, for any ρ, then keeping b and s always the same as the original solution for r, but adding a constant $(1-\delta)\rho$ to consumption each period, would still satisfy all conditions for optimality, because the agent's utilities in each period would look like the original multiplied by the decision-theoretically irrelevant constant $exp(-(1-\delta)\rho/\tau) > 0$. Thus, we can conclude that $V(r+\rho) = exp(-(1-\delta)\rho/\tau)V(r)$ for any r and ρ. That is, for an agent with constant risk tolerance τ and discount factor δ, the value function is of the form

$$V(r) = -A\, exp(-(1-\delta)r/\tau) \quad \text{where } A = -V(0).$$

This decision-theoretic equivalence of the agent's problems with different levels of wealth r also implies that, with constant risk tolerance, the optimal contract $(b(\theta),s(\theta))_{\theta\in\Theta}$ is independent of r.

Thus, at least in this case of constant risk tolerance, we know that V(.) is strictly increasing, continuously differentiable, and strictly concave, with $\lim_{r\rightarrow-\infty} V'(r) = +\infty$ and $\lim_{r\rightarrow+\infty} V'(r) = 0$. For numerical calculations, the whole value function is known once we compute this value coefficient A, and then the Lagrange multiplier for the budget constraint is

$$\lambda = V'(r) = (A(1-\delta)/\tau)\, exp(-(1-\delta)r/\tau).$$

Notice also that $1/V'(r) = (\tau/(A(1-\delta)))exp((1-\delta)r/\tau)$ is an increasing strictly convex function of r. So with the martingale result we get, in every period t,

$$1/V'(\tilde{r}_t) = E(1/V'(\tilde{r}_{t+1})\,|\tilde{r}_t) > 1/V'(E(\tilde{r}_{t+1}|\tilde{r}_t)), \text{ and thus } \tilde{r}_t > E(\tilde{r}_{t+1}|\tilde{r}_t).$$

That is, under the agent's optimal incentive compatible insurance contract, the agent's wealth is expected to decrease every period.

Our general bounds on the value function imply that the value coefficient A must satisfy

$$-exp(-0/\tau)/(1-\delta) \leq -A \leq -exp(-G/\tau)/(1-\delta).$$

With a computational program that can solve the constrained optimization problem in our Bellman equation, the value coefficient can be found by searching in this interval for a value A such that, when we assume that $V(0-s(\theta))$ is $(-A)exp((1-\delta)s(\theta)/\tau)$ on the right-hand side of the Bellman equation for r=0, we get V(0) equal to -A on the left-hand side.

In our two-state risk model, the binding incentive constraint is the one that would be violated by perfect insurance, the constraint that the high-type agent with $\tilde{\theta}_t=G$ should not expect to gain by pretending to be the low type with $\tilde{\theta}_t=0$. So the incentive constraints will have one positive Lagrange multiplier $\alpha(0|G)>0$, the other Lagrange multiplier being $\alpha(G|0)=0$. With constant risk tolerance, we get

$$u'((1-\delta)r+G+b(0)) = u'((1-\delta)r+0+b(0))\, exp(-(G-0)/\tau), \text{ where } exp(-(G-0)/\tau) < 1,$$

and then the first-order optimality conditions give us:

$$[p(0) - \alpha(0|G)\, exp(-(G-0)/\tau)]u'((1-\delta)r+0+b(0)) = [p(0) - \alpha(0|G)]V'(r-s(0)),$$

$$[p(G)+\alpha(0|G)]u'((1-\delta)r+G+b(G)) = [p(G) + \alpha(0|G)]V'(r-s(G)).$$

(Positivity of u′ and V′ assures us of a solution with $\alpha(0|G)$ between 0 and p(0).) Thus,

$$u'((1-\delta)r+G+b(G)) = V'(r-s(G)), \text{ but } u'((1-\delta)r+0+b(0)) < V'(r-s(0)).$$

That is, for optimal insurance under adverse-selection incentive constraints, the low-type agent should increase the net dissaving amount s(0) until the agent's marginal expected value of future resources in r-s(0) is strictly greater than the agent's marginal value of current consumption, because doing so helps to deter the high-type agent from imitating the low-type and thus enables credible insurance contracts with greater low-type insurance benefits b(0). This increase in the marginal value of future resources mean that the agent is tending to become poorer.

Proposition 1. *Under the general assumptions of our model, if the risk-averse agent with private income risks can borrow and invest only with one monopolistic banker and faces informational (adverse-selection) incentive constraints in devising a plan for risk-contingent credit transactions each period, then the agent's optimal plan makes* $1/V'(\tilde{r}_t)$ *(the reciprocal of the agent's marginal value of wealth) a martingale, and so the agent's wealth* $\tilde{r}_t$ *decreases in expectation every period and goes to* $-\infty$ *with probability 1 as* $t\to+\infty$.

To illustrate, let us consider the solution to our simple numerical example with parameters p(G)=0.5, G=2, τ=2, and δ=0.9. In this case, the numerical solution has

$$b(0)=0.965,\ s(0)=0.672,\ b(G)=-0.945,\ s(G)=-0.650,\ -A=-6.097,\ \alpha(0|G)=0.0165,$$

while $\alpha(G|0)=0$. Under this contract, when the agent's type is low the agent gets a current consumption benefit of 0.965 from the banker but must give the banker future resources worth 0.672, but the banker is compensated for the expected loss in this transaction by expected gains from the high-type agent, who is required to pay the banker 0.945 for increased future resources worth only 0.650. So in any period t when the agent's resources wealth is r, the agent's current consumption $(1-\delta)r+\tilde{\theta}_t+b(\tilde{\theta}_t)$ will be either $(1-\delta)r+1.055$ for the high type with θ=G=2 or $(1-\delta)r+0.955$ for the low type with θ=0; and in the subsequent period t+1 the agent's resource wealth $r-s(\tilde{\theta}_t)$ will be either r+0.650 if $\tilde{\theta}_t$=G or r-0.670 if $\tilde{\theta}_t$=0. The insurance subsidy for the low type here is $b(0)-\delta s(0)=0.360$, financed by an equal budget surplus $\delta s(G)-b(G)$ from the high type, and the expected net decline in the agent's resources each period is

$$E(s(\tilde{\theta}_t)) = 0.5(-0.650) + 0.5(0.672) = 0.011.$$

The incentive constraint that a high type should not gain by claiming to be low is binding in this solution, and this equation is satisfied in this numerical solution with values -5.902 on both sides. The low type would strictly lose by claiming to be high, as

$$-exp(-[0+b(0)]/\tau) + -\delta A\, exp((1-\delta)s(0)/\tau) = -6.292$$
$$> -exp(-[0+b(G)]/\tau) + -\delta A\, exp((1-\delta)s(G)/\tau) = -6.916.$$

If the agent starts with initial wealth $\tilde{r}_1$=r, then the agent's expected present discounted utility value from the whole stochastic consumption stream is $-A\,exp(-(1-\delta)r/\tau)$, where we have computed -A=-6.097. This solution satisfies our recursive value equation

$$-A\, exp(-(1-\delta)r/\tau) = \sum_{\theta\in\{0,G\}} p(\theta)[-exp(-((1-\delta)r+\theta+b(\theta))/\tau) + -\delta A\, exp(-(1-\delta)(r-s(\theta))/\tau)].$$

The agent would get the same expected utility value from a sure constant income $x=(1-\delta)r+0.990$ every period, where this certainty-equivalent income x satisfies

$$-exp(-x/\tau)/(1-\delta) = -A\, exp(-(1-\delta)r/\tau).$$

So in the optimal incentive-compatible contract here, the agent gets more than 95% of the certainty-equivalent gains over autarky that could be achieved with perfect insurance (1-0.760).

The incentive compatibility of this contract would fail, however, if the agent could borrow and invest independently with many different risk-neutral bankers who use the same discount factor δ. With such access to competitive bankers, the high type could do strictly better by pretending to be the low type but then investing $y=1.8$ with another banker in exchange for resources worth $y/\delta=2.0$ next period. Under this plan, the agent's current consumption is

$$(1-\delta)r+G+b(0)-y = (1-\delta)r+2+0.965-1.8 = (1-\delta)r+1.165$$

$$> (1-\delta)r+G+b(G) = (1-\delta)r+2-0.945 = (1-\delta)r+1.055$$

and the agent's wealth next period would be

$$r-s(0)+y/\delta = r-0.672+2.0 = r+1.328 > r-s(G) = r+0.650.$$

So the high type agent could be better off both in the current period and in the future by pretending to be a low type in this contract while making investments with other competitive bankers. But $b(0)>\delta s(0)$, and so the first banker would not be willing to enter into this contract if the agent would always claim to be the low type with $\tilde{\theta}_t=0$. In the next section, we consider how the agent's optimal financial plan chages when the agent has access to many competitive bankers.

Adverse selection and competitive credit

Now consider instead an alternative case where the agent can borrow or invest independently with many different risk-neutral bankers who have the same discount factor δ. Then the agent could adjust any given contractual terms $(b(\theta),s(\theta))$ for current borrowing and future asset sales to any other $(\hat{b},\hat{s})$ that satisfies $\hat{b}-\delta\hat{s} = b(\theta)-\delta s(\theta)$ by independently borrowing $\hat{b}-b(\theta)$ from another banker for the subsequent payment $\hat{s}-s(\theta)$. Thus, as observed by Allen (1985) and Thomas and Worrall (1990), an agent who has access such a competitive credit market would always prefer to report a type θ that maximizes $b(\theta)-\delta s(\theta)$. But competitive bankers would be willing to enter into such transactions only when $b(\theta) \leq \delta s(\theta)$, and so the agent's optimal incentive-constrained credit terms must satisfy $b(\theta)=\delta s(\theta)$ for each type $\theta\in\Theta$.

Thus, for any wealth r, the agent's optimal value V(r) must satisfy the Bellman equation:

$$V(r) = \max_{b:\Theta\to\mathbf{R}} \sum_{\theta\in\Theta} p(\theta)[u((1-\delta)r+\theta+b(\theta)) + \delta V(r-b(\theta)/\delta)].$$

The informational incentive constraints that we analyzed in the preceding section will be automatically satisfied by the optimal solution here because, with no budgetary subsidies from one type to another, each type θ chooses $b(\theta)$ to maximize $u((1-\delta)r+\theta+b(\theta))+\delta V(r-s(\theta))$ subject to the type-specific budget constraint $s(\theta)=b(\theta)/\delta$. So the optimal solution for the agent in a competitive credit market would also be feasible for a monopolistic banker, although it would not be optimal for that case as we analyzed it in the previous section.

At the optimal solution, the first-order optimal conditions for each $b(\theta)$ give us

$$0 = u'((1-\delta)r+\theta+b(\theta)) - \delta V'(r-b(\theta)/\delta)/\delta, \text{ and so}$$

$$V'(r-b(\theta)/\delta) = u'((1-\delta)r+\theta+b(\theta)), \forall\theta\in\Theta.$$

Then we get

$$V'(r) = \sum_{\theta\in\Theta} p(\theta)[(1-\delta)u'((1-\delta)r+\theta+b(\theta)) + \delta V'(r-b(\theta)/\delta)] = \sum_{\theta\in\Theta} p(\theta)V'(r-b(\theta)/\delta).$$

Thus, when the agent can borrow or invest independently with many different risk-neutral bankers, the agent's stochastic wealth $\tilde{r}_t$ makes $V'(\tilde{r}_t)$ a nonnegative martingale. Then by the martingale convergence theorem, $V'(\tilde{r}_t)$ must be convergent as $t\to\infty$ with probability 1.

As in the previous section, changing the initial wealth by some additive constant ρ would not change the optimal solution, because simply adding the interest income $(1-\delta)\rho$ to consumption in every period would leave the agent's utilities in each period looking like the original utilities multiplied by the decision-theoretically irrelevant constant $exp(-(1-\delta)\rho/\tau) > 0$. Thus, an agent with constant risk tolerance τ and discount factor δ still has a value function of the form

$$V(r) = -A\, exp(-(1-\delta)r/\tau) \text{ where } -A = V(0).$$

Fixing the model's given parameters, the coefficient (-A) here must be lower here than in the previous section, however, because the optimal solution to the Bellman equation here is a feasible alternative (but not optimal) for the Bellman equation in the previous section. That is, for any wealth r, the optimal value V(r) that the agent can achieve in a competitive credit market is less than the optimal value that the agent could get from a monopolistic banker.

Notice that this value function is continously differentiable, and $V'(r) = (A(1-\delta)/\tau)exp(-(1-\delta)r/\tau)$ is a strictly convex and decreasing function of r. Then with the martingale property of $V'(\tilde{r}_t)$, we also get

$$V'(\tilde{r}_t) = E(V'(\tilde{r}_{t+1})|\tilde{r}_t) > V'(E(\tilde{r}_{t+1}|\tilde{r}_t)), \text{ and thus } \tilde{r}_t < E(\tilde{r}_{t+1}|\tilde{r}_t)$$

Now, by an argument similar to the preceding section, we can show that the convergent martingale $V'(\tilde{r}_t)$ cannot have any positive probability of converging to a positive limit $V'(r)$ for any finite r, because the agent's optimal plan at r would have $b(G)<0$, to set aside some of the high current income for consumption in future periods with low income; and so, in every period when $\tilde{r}_t$ is sufficiently near r, there would be a positive probability p(G) of $\tilde{r}_{t+1} = \tilde{r}_t-b(G)/\delta$

jumping up farther away from r. Thus, the stochastic process must satisfy $V'(\tilde{r}_t)\rightarrow 0$ with probability 1 as $t\rightarrow\infty$, which implies that $\tilde{r}_t\rightarrow+\infty$ with probability 1.

Comparing the results in this section to the previous section, we find that changing from monopolistic credit to competitive credit flips the martingale argument from $1/V'$ to V'. But this technical mathematical result has the vital economic consequence that, with probability 1, the agent with access to a competitive credit market ultimately rises to great wealth, instead of falling into deep poverty with monopolistic credit. That is, where we found a long-run tendency to impoverishment with monopolistic credit, we find here a tendency to enrichment with competitive credit.

It might seem unsurprising that monopolistic credit would tend to impoverish people, as a monopolistic banker might exploit his market power to lend at high interest rates while offering little or no interest on deposits. But we derived the impoverishment result in the preceding section under an assumption that the monopolistic banker would accept the agent's optimal incentive-compatible contract, as if the agent had all the bargaining power in negotiating their credit transactions. Competitive credit markets cannot be better for the agent than this optimal contract, because the agent's plan for borrowing and investing with a competitive credit market would also be feasible with a monopolistic banker. If a banker offered the optimal contract conditional on the agent waiving access to all other opportunities for borrowing and investment, the agent would be willing to accept this offer. Thus, the enrichment result in this section actually depends on an assumption that the agent has an inalienable right to borrow and lend in competitive credit markets.

Proposition 2. *Under the geneal assumptions of our model, if the risk-averse agent with private income risks has an inalienable right to borrow and invest with bankers in a competitive credit market with discount factor δ, but the agent faces informational (adverse-selection) incentive constraints in devising a plan for risk-contingent credit transactions each period, then the agent's optimal plan makes the agent's marginal value of wealth $V'(\tilde{r}_t)$ a martingale, and so the agent's wealth $\tilde{r}_t$ increases in expectation every period and goes to $+\infty$ with probability 1 as $t\rightarrow+\infty$. But for any given wealth r, the optimal expected discounted utility value V(r) that the agent can achieve in a competitive credit market is less than the optimal value that the agent could get from a monopolistic banker under Proposition 1.*

It might be helpful to revisit our simple numerical example with the parameters p(G)=0.5, G=2, τ=2, δ=0.9, and $\tilde{r}_1$=0. Then the agent's optimal plan has b(0)=0.878 and b(G)=-0.922. That is, the low-type agent borrows b(0)=0.878 for current consumption and so decreases future wealth (or increases future debt) by s(0)=b(0)/δ=0.975; but the high-type agent consumes G+b(G)=2-0.922=1.078 from current risky income and then invests the savings to increase future wealth by -s(G)=-b(G)/δ=1.025. The expected net increase in the agent's wealth each period is

$$E(-s(\tilde{\theta}_t)) = 0.5(-0.975) + 0.5(1.025) = 0.025.$$

With this optimal plan, the coefficient in the agent's value function is V(0) = -A= -6.1416. So the agent would get the same expected utility value from by replacing the risky income by a certainty-equivalent income of x=0.9750 every period (which satisfies -*exp*(-x/τ)/(1-δ) = -A). For comparison, recall that the agent's certainty equivalent income would be 0.760 under autarky, and it would be 0.990 with monopolistic credit. So in this example, access to competitive credit markets still allows the agent to get more than 93% of the certainty-equivalent gains over autarky that could be offered by a monopolistic banker. Having a monopolistic banker enables the agent to incentive-compatibly reduce short-term income risks, with credit contracts that subsidize a low-income type at the expense of a high-income type, using excess borrowing as a costly signal of the unverifiable low-income type. But when agents are very patient (high δ), most of the possible insurance gains may be achieved just by transfers between periods in a competitive credit market, by saving and investing in periods when income is high, and by borrowing and selling assets when income is low, without contracting for one possible type to subsidize another within the current period.

Moral hazard and competitive credit

Adverse-selection incentive constraints for unverifiable risks are just one reason why perfect full insurance may not be feasible. Moral hazard is another fundamental cause of imperfection in insurance markets. In this section and the next one, we now show that the main qualitative results for adverse-selection models from the preceding two sections can be derived also for models with moral-hazard incentive constraints.

For simplicity, we again consider constant risk tolerance u(x) = -*exp*(-x/τ) and binary risks Θ={0,G} with G>0. Now we let the probability of the good outcome G depend each period on the agent's costly effort c≥0 according to some function P(G|c), with P(0|c) = 1-P(G|c). Let us assume here that the $P(G|.)$ function satisfies $P(G|0)=0$ and $P(G|G)\leq 1$, $P(G|c)$ is concave and increasing in c for all $0\leq c\leq G$, $P(G|c)$ is continuously differentiable in c for $0<c\leq G$, and $\lim_{c\to 0} P'(G|c) = +\infty$.

Here and in the next section, we suppose that the risky income $\tilde{\theta}_t$ is publicly observable in each period t, but the agent chooses c unobservably. The agent's effort cost c is subtracted from the agent's consumption x in the current period.

Under autarky, with no access to any form of credit or insurance, the agent's problem in each period would be to choose effort c to maximize the expected utility

$$\sum_{\theta\in\Theta} P(\theta|c)u((1-\delta)r+\theta-c).$$

With our given assumptions, the optimal effort c must be between 0 and G (because it is never worth spending more than G to increase the probability of getting G), and it satisfies the first-order condition

$$\sum_{\theta\in\Theta} P'(\theta|c)u((1-\delta)r+\theta-c) = \sum_{\theta\in\Theta} P(\theta|c)u'((1-\delta)r+\theta-c).$$

Here $P'(\theta|c) = \partial P(\theta|c)/\partial c$.

For a benchmark numerical example, let us consider the parameters $G=2$, $\tau=2$, $\delta=0.9$, and $P(G|c)=(c/2)^{0.5}$ with $\tilde{r}_1=0$. In this case, the autarky solution has $c=0.381$, yielding $P(G|c)=0.436$, and the agent's certainty-equivalent income each period is $x=0.2646$, which satisfies the equation $u(x)=\sum_{\theta\in\Theta} P(\theta|c)u(\theta-c)$.

Now (reversing our order from the previous two sections), in this section we consider the case where the agent has an inalienable right to borrow or invest independently with many competitive bankers who are willing to trade on any terms such that $b(\theta) = \delta s(\theta)$ for each θ.

For any current wealth r, the agent's optimal value $V(r)$ is characterized now by the Bellman equation:

$$V(r) = \max_{c\in[0,G],b:\Theta\to\mathbf{R}} \sum_{\theta\in\Theta} P(\theta|c)[u((1-\delta)r+\theta+b(\theta)-c) + \delta V(r-b(\theta)/\delta)].$$

In this model with hidden effort c and access to competitive credit markets, the agent's optimal solution can be computed from the first-order optimality condition for c

$$\sum_{\theta\in\Theta} P'(\theta|c)[u((1-\delta)r+\theta+b(\theta)-c) + \delta V(r-b(\theta)/\delta)] = \sum_{\theta\in\Theta} P(\theta|c)u'((1-\delta)r+\theta+b(\theta)-c),$$

and the first-order optimality conditions for each $b(\theta)$

$$u'((1-\delta)r+\theta+b(\theta)-c) = \delta V'(r-b(\theta)/\delta)/\delta = V'(r-b(\theta)/\delta),\ \forall\theta\in\Theta.$$

Then we get

$$V'(r) = \sum_{\theta\in\Theta} P(\theta|c)[(1-\delta)u'((1-\delta)r+\theta+b(\theta)-c) + \delta V'(r-b(\theta)/\delta)] =$$

$$= \sum_{\theta\in\Theta} P(\theta|c)V'(r-b(\theta)/\delta).$$

So we again find in this moral-hazard case that, when the agent can borrow or invest independently with many different risk-neutral bankers, the agent's stochastic wealth $\tilde{r}_t$ makes $V'(\tilde{r}_t)$ a nonnegative martingale.

As before, the constant risk tolerance assumption assures us here that the value function will be of the form $V(r) = -A\,exp(-(1-\delta)r/\tau)$, where $-A = V(0)$; and so

$$V'(r) = (A(1-\delta)/\tau)exp(-(1-\delta)r/\tau)$$

is a strictly convex and decreasing function of r (but with a different coefficient A from previous sections). Then with the martingale property for $V'(\tilde{r}_t)$, at each period t we get

$$V'(\tilde{r}_t) = E(V'(\tilde{r}_{t+1})|\tilde{r}_t) > V'(E(\tilde{r}_{t+1}|\tilde{r}_t)), \text{ and so } \tilde{r}_t < E(\tilde{r}_{t+1}|\tilde{r}_t).$$

That is, the expected value of the agent's wealth increases from each period to the next. Furthermore, the martingale convergence theorem can also be used, as in the preceding section, to show that $V'(\tilde{r}_t)$ converges to 0 with probability 1, and so $\tilde{r}_t \to +\infty$ with probability 1 as $t\to\infty$.

Proposition 3. *Under the geneal assumptions of our model, if the risk-averse agent with private income risks has an inalienable right to borrow and invest with bankers in a competitive credit market but faces a moral-hazard incentive constraint in devising a plan for risk-contingent credit transactions each period, then (as in Proposition 2) the agent's optimal plan makes $V'(\tilde{r}_t)$ a martingale, and so the agent's wealth $\tilde{r}_t$ inceases in expectation every period and goes to $+\infty$ with probability 1 as $t\to+\infty$.*

For our benchmark numerical example with G=2, τ=2, δ=0.9, and $P(G|c)=(c/2)^{0.5}$, the first-order optimality conditions give us b(2)=-0.923, s(2)=b(2)/δ =-1.026, b(0)=0.877, s(0)=b(0)/δ=0.974, c=0.4991, P(G|c)=0.4996, -A=-7.8859. Starting with wealth $\tilde{r}_1=0$, the agent's consumption net of effort cost $\theta+b(\theta)-c$ would be 0.578 for the high type and 0.378 for the low type. Then the agent's expected wealth increases each period by $E(-s(\tilde{\theta}_t))=0.025$, but the agent's certainty-equivalent constant income would be x=0.4750 (which satisfies $u(x)/(1-\delta)=-A$).

Moral hazard and monopolistic credit

Finally, let us return to consider the case where the agent with hidden effort can contract with one monopolistic banker who is risk neutral and discounts the future with the same factor δ per period. So in this section we assume that, in each period t, the agent can contract with only one risk-neutral banker to get $b(\tilde{\theta}_t)$ for current consumption by selling some share $s(\tilde{\theta}_t)$ of the agent's future resources. But although the agent's risky income $\tilde{\theta}_t$ now is assumed to be fully observable, the banker understands that, for any given contractual terms (b,s), the agent will choose the hidden effort c to maximize the agent's long-run expected utility value

$$\sum_{\theta\in\Theta} P(\theta|c)[u((1-\delta)r+\theta+b(\theta)-c) + \delta V(r-s(\theta))].$$

Thus, with $P'(\theta|c) = \partial P(\theta|c)/\partial c$, the promised effort c must satisfy the moral-hazard constraint:

$$\sum_{\theta\in\Theta} P'(\theta|c)[u((1-\delta)r+\theta+b(\theta)-c) + \delta V(r-s(\theta))] = \sum_{\theta\in\Theta} P(\theta|c)u'((1-\delta)r+\theta+b(\theta)-c).$$

This moral-hazard constraint is the agent's first-order optimality condition for c when b and s are taken as fixed. The moral-hazard constraint becomes problematic when we consider contracts where banker offers an insurance subsidy for the agent's low type that is financed by a budget surplus from the high type. The advantage of having a monopolistic banker is that, given the banker's risk neutrality, the budget constraint can be relaxed to apply just in expectation

$$\sum_{\theta\in\Theta} P(\theta|c)(b(\theta) - \delta s(\theta)) \leq 0,$$

instead of requiring budget balance for each θ separately. So part of the agent's motivation for promising higher effort c would come from the hope of inducing the banker to accept a contract with a greater insurance subsidy $b(0)-\delta s(0)$ for the low type, which has higher marginal utility of income. But the agent's actual effort cannot be observed by the banker, and so there is nothing to prevent the agent from promising one effort and choosing another. Thus, for the banker to trust the agent, the promised effort must satisfy the first-order optimality conditions for c when the contractual terms (b,s) are fixed.

Then for each initial r, the optimal value V(r) for an agent with hidden effort and one monopolistic banker satisfies the Bellman equation:

$$V(r) = \max_{c\in[0,G],b:\Theta\to\mathbf{R},s:\Theta\to\mathbf{R}} \sum_{\theta\in\Theta} P(\theta|c)[u((1-\delta)r+\theta+b(\theta)) + \delta V(r-s(\theta))] \text{ subject to:}$$

$$\sum_{\theta\in\Theta} P(\theta|c)(\delta s(\theta) - b(\theta)) \geq 0, \text{ and}$$

$$\sum_{\theta\in\Theta} P'(\theta|c)[u((1-\delta)r+\theta+b(\theta)-c) + \delta V(r-s(\theta))] = \sum_{\theta\in\Theta} P(\theta|c)u'((1-\delta)r+\theta+b(\theta)-c).$$

Let μ be the Lagrange multiplier for the moral-hazard equation, and let $\lambda\geq 0$ be the

Lagrange multiplier for the budget constraint. So for any given r, the optimal solution (c,b,s) maximize the Lagrangean:

$$L_r(c,b,s;\lambda,\alpha) = \sum_{\theta\in\Theta} P(\theta|c)[u((1\text{-}\delta)r+\theta+b(\theta)\text{-}c)+\delta V(r\text{-}s(\theta))] + \lambda\sum_{\theta\in\Theta} P(\theta|c)(\delta s(\theta) - b(\theta)) +$$
$$+ \mu[\sum_{\theta\in\Theta} P'(\theta|c)(u((1\text{-}\delta)r+\theta+b(\theta)\text{-}c)+\delta V(r\text{-}s(\theta))) - \sum_{\theta\in\Theta} P(\theta|c)u'((1\text{-}\delta)r+\theta+b(\theta)\text{-}c)].$$

For any r, the optimal solution satisfies the moral-hazard constraint equation, the binding budget constraint equation, and the first-order Lagrangean optimality conditions: $\partial L_r/\partial b(\theta)=0$ and $\partial L_r/\partial s(\theta)=0$, $\forall\theta\in\Theta$, as well as $\partial L_r/\partial c = 0$. These optimality conditions give us the equations:

$$[P(\theta|c)+\mu P'(\theta|c)]u'((1\text{-}\delta)r+\theta+b(\theta)\text{-}c) - \mu P(\theta|c)u''((1\text{-}\delta)r+\theta+b(\theta)\text{-}c) = \lambda P(\theta|c) \quad \forall\theta\in\Theta,$$

$$\lambda P(\theta|c)\delta = [P(\theta|c) +\mu P'(\theta|c)]\delta V'(r\text{-}s_r(\theta)) \quad \forall\theta\in\Theta, \text{ and}$$

$$\mu\sum_{\theta\in\Theta} [P''(\theta|c)(u((1\text{-}\delta)r+\theta+b(\theta)\text{-}c)+\delta V(r\text{-}s(\theta))) + P(\theta|c)u''((1\text{-}\delta)r+\theta+b(\theta)\text{-}c) +$$
$$- 2P'(\theta|c)u'((1\text{-}\delta)r+\theta+b(\theta)\text{-}c)] = \lambda\sum_{\theta\in\Theta} P'(\theta|c)(b(\theta) - \delta s(\theta)).$$

(This last equation is derived from $\partial L_r/\partial c=0$ by netting out terms that must sum to 0 by the moral-hazard constraint.)

Now the main martingale result here, with moral hazard and monopolistic credit, can be derived similarly to the earlier section on adverse selection and monopolistic credit. As before, when c and each $(1\text{-}\delta)r+b(\theta)$ are held fixed, the moral-hazard equation depends on s only through the difference $V(r\text{-}s(G))-V(r\text{-}s(0))$, because $P'(0|c)=\text{-}P'(G|c)$. So again, if we consider increasing wealth r by some small ε with c fixed, then current consumption can be held constant by changing each $b(\theta)$ by $\text{-}(1\text{-}\delta)\varepsilon$, and the first-order impact on the moral-hazard incentive constraint and the expected budget-balance constraint will be 0 if the net increase for each $s(\theta)$ is $\sigma(\theta)\varepsilon$, where these $\sigma(\theta)$ satisfy

$$V'(r+s(\theta))\,(1\text{-}\sigma(\theta)) = \omega, \quad \forall\theta\in\Theta, \text{ and}$$

$$\sum_{\theta\in\Theta} p(\theta)\delta\sigma(\theta) = \text{-}(1\text{-}\delta).$$

As before, we get $\omega = 1/[\delta\sum_{\varphi\in\Theta} p(\varphi)/V'(r\text{-}s(\varphi))]$. Then by the envelope theorem,

$$V'(r) = \partial L_r(b,s;\lambda,\alpha)/\partial r + \sum_{\theta\in\Theta} [\sigma(\theta)\partial L_r/\partial s(\theta) - (1\text{-}\delta)\partial L_r/\partial b(\theta)] =$$
$$= \sum_{\theta\in\Theta} p(\theta)\delta V'(r\text{-}s(\theta))(1\text{-}\sigma(\theta)) = \delta\omega = 1/[\sum_{\varphi\in\Theta} p(\varphi)/V'(r\text{-}s(\varphi))],$$

and so

$$1/V'(r) = \sum_{\theta\in\Theta} p(\theta)[1/V'(r\text{-}s(\theta))].$$

Thus, we find again that, with moral hazard and monopolistic credit, the agent's optimal solution makes $1/V'(\tilde{r}_t)$ is a martingale. But $1/V'(\tilde{r}_t)$ always takes nonnegative values and cannot have positive probability of converging to any finite positive value; and so we must have $1/V'(\tilde{r}_t)\to 0$, $V'(\tilde{r}_t)\to+\infty$, and $\tilde{r}_t\to-\infty$ with probability 1 as $t\to\infty$.

As before, the constant risk tolerance case gives us $V(r) = -A\, exp(-(1-\delta)r/\tau)$ for $-A = V(0)$. Fixing the model's given parameters, the coefficient (-A) here will be greater here than in the previous section, because the agent's plan with a competitive credit market would be feasible (but not optimal) here with a monopolistic credit source. Then $1/V'(r) = (\tau/(A(1-\delta)))exp((1-\delta)r/\tau)$ is an increasing strictly convex function of r. So with the martingale result we again get, in every period t,

$$1/V'(\tilde{r}_t) = E(1/V'(\tilde{r}_{t+1})\,|\tilde{r}_t) > 1/V'(E(\tilde{r}_{t+1}|\tilde{r}_t)),\ \text{and thus}\ \tilde{r}_t > E(\tilde{r}_{t+1}|\tilde{r}_t).$$

So again in this moral-hazard case, under the agent's optimal incentive-compatible insurance contract with a monopolistic banker, the agent's wealth is expected to decrease every period.

As before, the envelope theorem also implies

$$V'(r) = \partial L_r(b,s;\lambda,\alpha)/\partial r + \sum_{\theta\in\Theta} [\partial L_r/\partial s(\theta) - (1-\delta)\partial L_r/\partial b(\theta)] = \sum_{\theta\in\Theta} [\lambda p(\theta)\delta - (1-\delta)\lambda p(\theta)] = \lambda,$$

and so $\lambda = V'(r) = (A(1-\delta)/\tau)exp(-(1-\delta)r/\tau)$.

Proposition 4. *Under the general assumptions of our model, if the risk-averse agent with private income risks can only borrow and invest with one monopolistic banker and faces a moral-hazard incentive constraint in devising a plan for risk-contingent credit transactions each period, then (as in Proposition 1) the agent's optimal plan makes* $1/V'(\tilde{r}_t)$ *a martingale, and so the agent's wealth* $\tilde{r}_t$ *decreases in expectation every period and goes to* $-\infty$ *with probability 1 as* $t\to+\infty$. *But for any given wealth r, the optimal expected discounted utility value V(r) that the agent can achieve with a monopolistic banker is greater than the optimal value that the agent could get from a competitive credit market under Proposition 3.*

For our benchmark numerical example with G=2, τ=2, δ=0.9, and $P(G|c)=(c/2)^{0.5}$, the optimal solution is b(2)=-0.931, s(2)=-0.961, b(0)=0.886, s(0)=0.917, c=0.4544, P(G|c)=0.4767, with -A=-7.8797 and μ=0.0447. Starting with wealth $\tilde{r}_1=0$, the agent's consumption net of effort cost $\theta+b(\theta)-c$ would be 0.615 if for the high type and 0.431 for the low type. The insurance subsidy for the low type here is $b(0)-\delta s(0)=0.060$, financed by a budget surplus $\delta s(G)-b(G)=0.066$ from the high type. Then the agent's expected wealth decreases each period by $E(s(\tilde{\theta}_t))=0.022$; but the agent's certainty-equivalent constant income would be x=0.4766 (which satisfies $u(x)/(1-\delta)=-A$).

Comparing these numerical results to those in the preceding section, we find that access to competitive credit markets would allow the agent to get more than 99% of the certainty-equivalent gains over autarky that could be offered by this optimal contract with a monopolistic banker. This suggests that, in such examples where the agent is patient, most of the possible insurance gains may be achieved just by transfers between periods in a competitive credit market, without contracting for insurance subsidies from one current type to the other, which would require an exclusive commitment to one banker.

Equilibrium discounting in competitive credit markets without growth

In Propositions 2 and 3, the prediction of unbounded long-run enrichment for almost all agents in an economy with competitive credit sounds good; but it obviously would require unbounded economic growth. A model with such growth could be justified by assuming that the economy has a constant-returns capital-production technology such that any investment $\eta\geq 0$ at any time t could produce a durable asset returning rent income $(1-\delta)\eta/\delta$ in every period after t.

But now if we assume instead that the aggregate supply of real capital resources is fixed, then we cannot hope for unbounded long-run enrichment for everyone. In such a bounded economy, the increasing wealth of our risk-averse agents could only come from impoverishment of the supposed risk-neutral bankers, whose bankruptcy would mean credit-market failure.

For a sustainable equilibrium in competitive credit markets without a real investment technology to determine the price of capital assets, we must admit the possibility of a market discount factor γ that is different from the agents' given personal-utility discount factor of δ per period. So let us reconsider the case of adverse selection and competitive credit markets (as in Proposition 2), but now allowing the market discount factor γ to differ from δ.

Considering alternative discount factors requires some change of notation here, because the price of capital assets and thus the wealths of individual agents will depend on the market discount factor. Instead of taking an agent's initial wealth as given, we should now take the agent's initial holdings of rent-income streams as given.

Consider an agent who owns assets that generate a reliable rent income of g every period. (Here g may be negative if the agent is a debtor owing rents to others.) Let $h(\theta)$ denote the net amount of future rent income per period that the agent would sell when the agent's current risky income is θ, to finance net current borrowing $b(\theta)$. In a competitive credit market with discount factor γ, this agent's wealth would be worth $g/(1-\gamma)$, and if current risky income is θ then the agent's wealth next period would be worth $(g-h(\theta))/(1-\gamma)$. Competitive bankers in this market would accept the future rents $h(\theta)$ as payment for current borrowing $b(\theta)$ as long as $b(\theta) \leq \gamma h(\theta)/(1-\gamma)$. So to borrow $b(\theta)$ now requires selling future income $h(\theta) = b(\theta)(1-\gamma)/\gamma$.

Thus, in a competitive credit market where γ is the market discount factor, but δ is (still) the agents' given personal-utility discount factor, the optimal expected discounted utility value $W(g)$ for an agent who owns a rent-income stream g would satisfy the Bellman equation

$$W(g) = \max_{b:\Theta\to\mathbf{R}} \sum_{\theta\in\Theta} p(\theta)[u(g+\theta+b(\theta)) + \delta W(g-b(\theta)(1-\gamma)/\gamma)].$$

The optimal $b(\theta)$ here would satisfy the first-order optimality conditions

$$0 = u'(g+\theta+b(\theta)) - \delta W'(g-b(\theta)(1-\gamma)/\gamma)(1-\gamma)/\gamma, \quad \forall\theta\in\Theta.$$

As before, with constant risk tolerance τ, the value function will have the form

$$W(g) = -A\, exp(-g/\tau), \text{ where } -A = W(0).$$

For a no-growth stationary equilibrium in a competitive credit market with a large population of such agents, each of whom gets a risky income $\tilde{\theta}_t$ independently drawn from the distribution p, the market discount factor γ would adjust to satisfy the market-clearing condition

$$\sum_{\theta\in\Theta} p(\theta)b(\theta) = 0.$$

So in equilibrium, the agents' expected net trades are zero, as $E(b(\tilde{\theta}_t))=0$ and so $E(h(\tilde{\theta}_t))=0$. Then each agent's rent-income holdings $\tilde{g}_t$ in each period t form a martingale with $\tilde{g}_{t+1} = \tilde{g}_t-h(\tilde{\theta}_t)$. (This is not a bounded martingale, however, so it can diverge to $\pm\infty$.) We found expected net borrowing to be negative in the competitive credit market when we assumed γ equal to δ; and so the market-clearing condition should be satisfied by some higher market discount factor γ between δ and 1, corresponding to a lower interest rate which encourages more borrowing.

For welfare comparisons with the previously considered cases, we must recognize that the agent who has rent income g would have wealth $r = g/(1-\delta)$ in our previous cases where the market discount factor was δ, and a transfer of the future rent income stream $h(\theta)=b(\theta)(1-\gamma)/\gamma$ would then be accounted as a sale of assets worth $s(\theta) = h(\theta)/(1-\delta)$. The utility values $W(g)$ here would be comparable to values $V(g/(1-\delta))$ in Propositions 1 and 2. With these formulas to translate (g,h,W) to (r,s,V), the (b,h) plan in equilibrium here corresponds to a (b,s) credit plan that would satisfy the adverse-selection incentive constraints and the expected budget constraint for transactions with the risk-neutral monopolistic banker. The monopolistic banker's budget constraint uses the discount factor δ instead of γ, but expected budget balance is still satisfied because the expected net trades in each period are zero. Thus, the agent's financial plan in this competitive credit market would still be feasible for the agent with a monopolistic banker, and so the agent's expected utility value W(g) in this competitive market would still be less than the optimal value V(r) that the agent could get from a monopolistic banker under Proposition 1.

Proposition 5. *Consider the case of competitive credit markets with adverse-selection incentive constraints, as in Proposition 2, but now suppose that the aggregate supply of capital resources is fixed, and so the competitive market discount factor γ may differ from the given personal-utility discount factor δ. Then a no-growth equilibrium in which the agents' expected net borrowing is zero can be found with a market discount factor γ that is greater than δ. But as in Proposition 2, the optimal expected discounted utility value that an agent could achieve in this competitive credit market would still be less than the optimal value that the agent could get from a monopolistic banker under Proposition 1.*

For our numerical example with p(G)=0.5, G=2, τ=2, and δ=0.9, the no-growth equilibrium market discount factor is γ=0.9011, corresponding to a slightly lower interest rate than the agents' personal discount factor δ. Then the agents' value function has coefficient -A=-6.1403, and the agents' optimal financial plan is b(G)=-0.9011, b(0)=0.9011, h(G)=b(G)(1-γ)/γ=-0.0989, and h(0)=0.0989. For an agent with initial wealth g=0 (and so r=0), the expected discounted utility value from this plan would be the same as what the agent could get by replacing the risky income by a certainty-equivalent income of x=0.9754 every period (which satisfies -*exp*(-x/τ)/(1-δ) = -A). So the agent would slightly prefer this no-growth credit market equilibrium over a competitive credit market where a capital-investment technology could keep the market discount factor equal to δ (for which we previously found the certainty-equivalent income of x=0.9750). But the agent here would strictly prefer to use the optimal financial plan with a monopolistic banker trading at discount factor δ (where we previously found that the certainty equivalent income for an agent with r=0 would be x=0.990).

Conclusions

For simplicity, we have used the term "banker" here to denote the agent's counterparty in credit transactions of borrowing and investing, but our results could be applicable to many cases where people get credit from sources other than formal banks. When we interpret the risk-averse agents as peasant farmers in a traditional agricultural society, they might look to neighbors or kinsmen or a landlord for credit transactions to help smooth consumption against income shocks. (For example, see Dennison 2011, chapter 7, on credit and savings for serfs in early 19th-century Russia.) Then the "banker" in our model might be interpreted as a local landlord who supplies credit to the peasants on his estate, or local villagers might have norms for mutual assistance that make the village-community itself a monopolistic source of credit to its members.

The case of credit from a communal credit union would fit the related model Atkeson and Lucas (1992), who showed that incentive-efficient risk sharing in such a community leads to ever-widening inequality, with an increasing fraction of the community's resources benefitting only a diminishing fraction of the community's members. The key is that dissaving is required for efficiently signaling the need of an agent who has suffered an unverifiable current income shock. Villagers would be understandably skeptical if a neighbor asked for their help against starvation, after reporting an unverifiable current-income shock, but then used some of their assistance to make investments for the future. So efficient insurance against unverifiable income shocks would involve restrictions on any other investments that the agent might want to make, but these restrictions are only possible when the insuring group has power to control all of the agent's financial transactions. One can imagine a monopolistic banker having such power, but a traditional village-community might also have such power over its members, when the ability of any member to claim any local resources depends on their neighbors recognizing and acceptng these claims (Myerson, 2025).

The assumption that anyone can buy anything that they can afford at given market prices is standard in economic theory, and this restriction on the agent's ability to invest at the market interest rate in the monopolistic credit cases here goes against this standard assumption. But property rights depend on social enforcement, and the rights to acquire property can be restricted by those with responsibility for this enforcement, which could be a local lord or a village council in many traditional communities. The results reviewed here in Propositions 1 and 4 show that when risk-averse agents face income risks that are unverifiable or have probabilities dependent on unobservable actions, the agents would voluntarily accept such restrictions on their ability to make other investments, to credibly demonstrate their need for assistance in bad periods. But this enforced dissaving, to prove their needs in bad periods, pushes the agents to mortgage their future for current relief, creating a long-run tendency towards impoverishment.

The assumption of inalienable right to borrow and invest in competitive credit markets is a better fit for modern states with legal systems that reliably enforce property rights for everyone. Contrasting Propositions 2, 3, and 5 with Propositions 1 and 4 here, our results suggest that the introduction of such a reliable rule of law could actually reduce agents' ability to insure themselves against current risks. But while agents might prefer not to lose the safety net that their traditional communities could provide, the competitive credit markets of the new state would provide a stronger incentive to save in good periods, with the likely result being long-run enrichment instead of impoverishment.

References

Franklin Allen (1985). "Repeated principal-agent relationships with lending and borrowing." *Economics Letters* 17(1-2):27-31 (1985). https://doi.org/10.1016/0165-1765(85)90121-1

Andrew Atkeson and Robert E. Lucas Jr. (1992). "On efficient distribution with private information." *The Review of Economic Studies* 59(3):427-453. https://www.jstor.org/stable/2297858

Tracy Dennison (2011). *The institutional framework of Russian serfdom*. Cambridge University Press. [Chapter 7 on "Credit and savings."]

Stephen Everson, ed (1996). *Aristotle: The politics and the constitution of Athens*. Cambridge University Press.

Roger Myerson (2015). "Moral hazard in high office and the dynamics of aristocracy." *Econometrica* 83:2083-2126. https://doi.org/10.3982/ECTA9737

Roger Myerson (2025). "Local politics in nations and empires." Chapter 9 in *Handbook of New Institutional Economics*, 2nd edition, edited by Claude Ménard and Mary M. Shirley, p. 185-208 (Springer Nature). https://doi.org/10.1007/978-3-031-50810-3_9

Jonathan Thomas and Tim Worrall (1990). "Income fluctuation and asymmetric information: An example of a repeated principal-agent problem." *Journal of Economic Theory* 51(2):367-390. https://doi.org/10.1016/0022-0531(90)90023-D