Fluid level in tandem queues with an On/Off source

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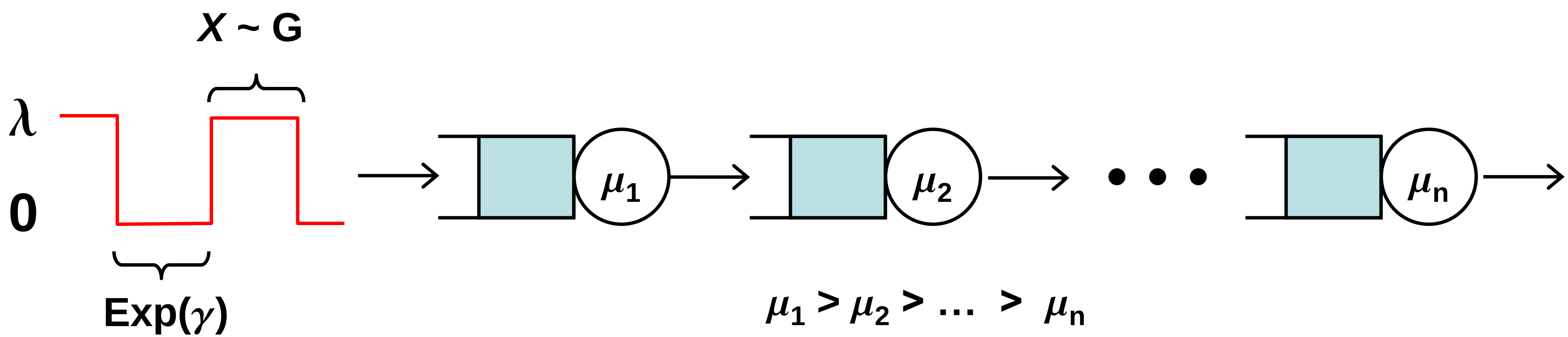
Why fluid queues?

- A simple model for shared resources with high arrival/service rates – e.g. telecommunication networks

- Markov modulated fluid queues can describe correlated input processes – e.g. self-similar traffic

- Often, the only tractable approximation

Tandem fluid queues with On/Off source



Q: First *k* moments of fluid level at queue *n*?

PRIOR WORK: Mostly numerical and iterative

- **Markov modulated queues – Martingale methods [Kella Whitt 92], Sample path SDEs [Brocket Gong Guo 99]**
- **General On periods – approximate *X* by *PH* distribution and solve for moments of fluid level iteratively [Field Harrison 07]**

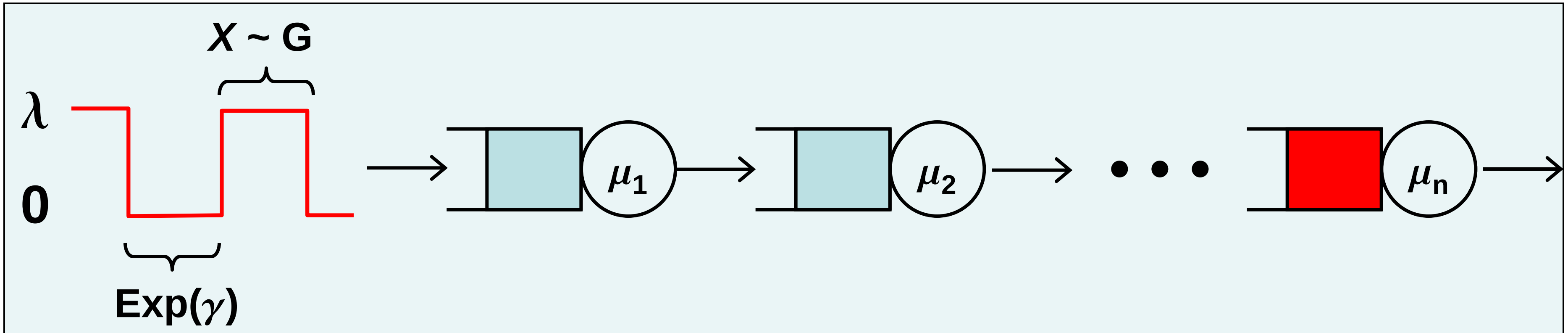
Q': *How many* moments of *X* do you need? (All? *kn*? *k*+*n*?)

OUR CONTRIBUTIONS

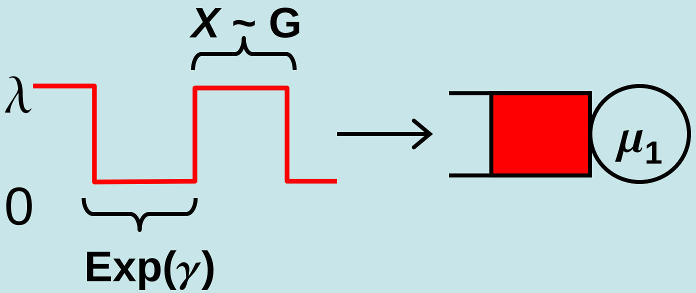
1. **Non-iterative method for Laplace transform of fluid level at each queue**
2. **Closed-form exact expressions for moments of fluid level**

   A': # moments of *X* needed = **??**

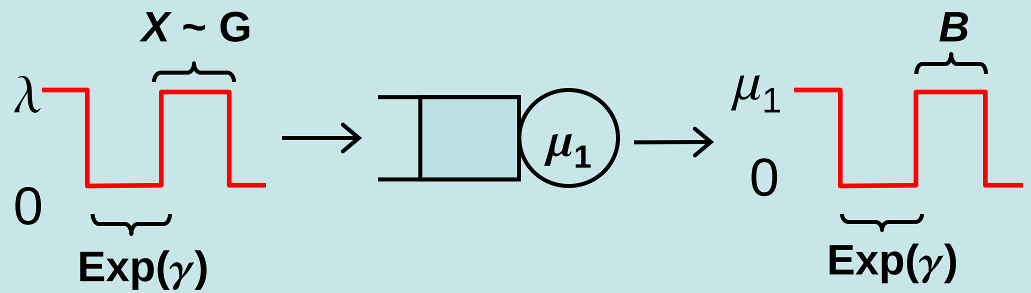
Analysis Roadmap



STEP 1: Fluid level at queue 1

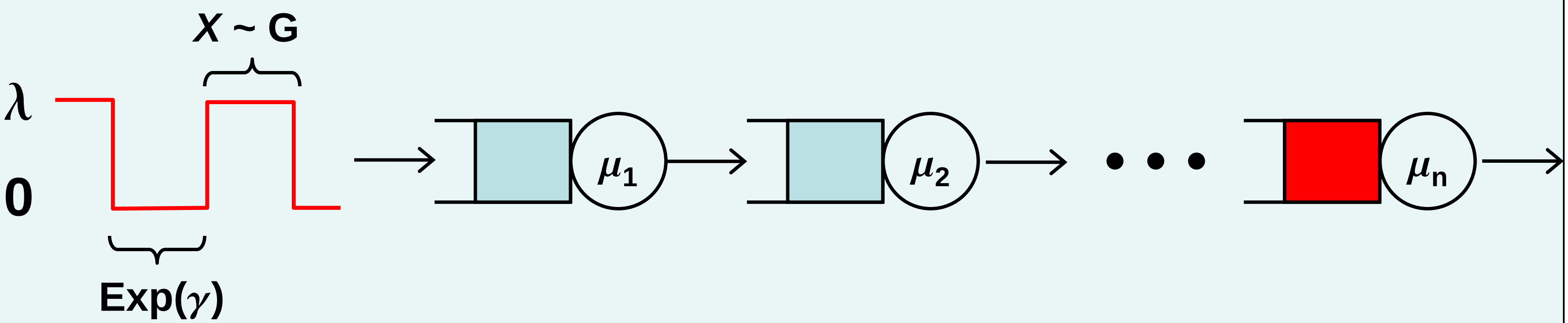


STEP 2: Busy period analysis



STEP 3:
Making the method non-iterative

Analysis Roadmap



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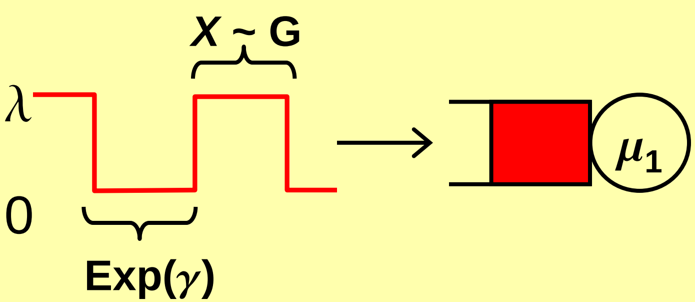

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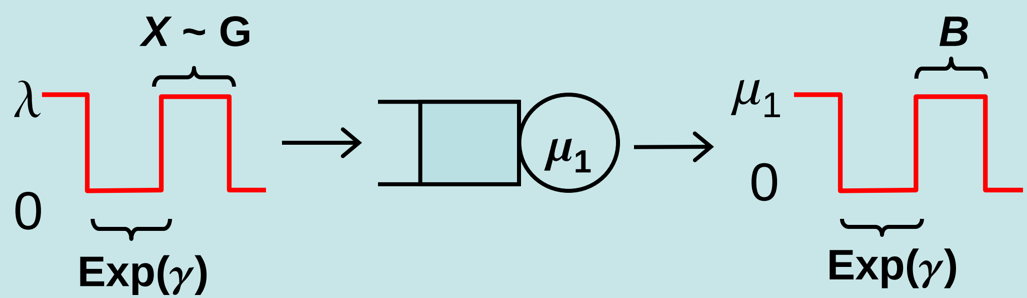
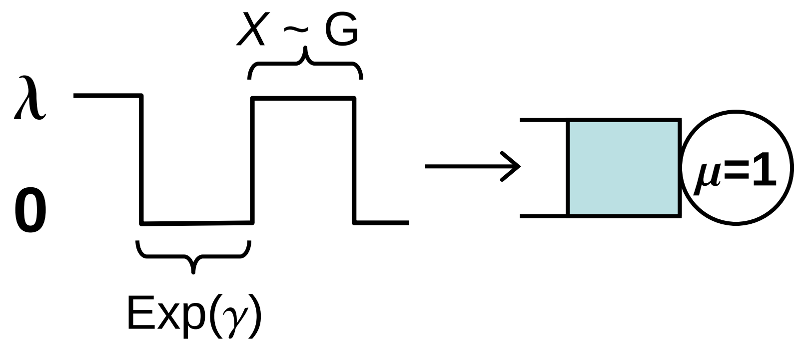
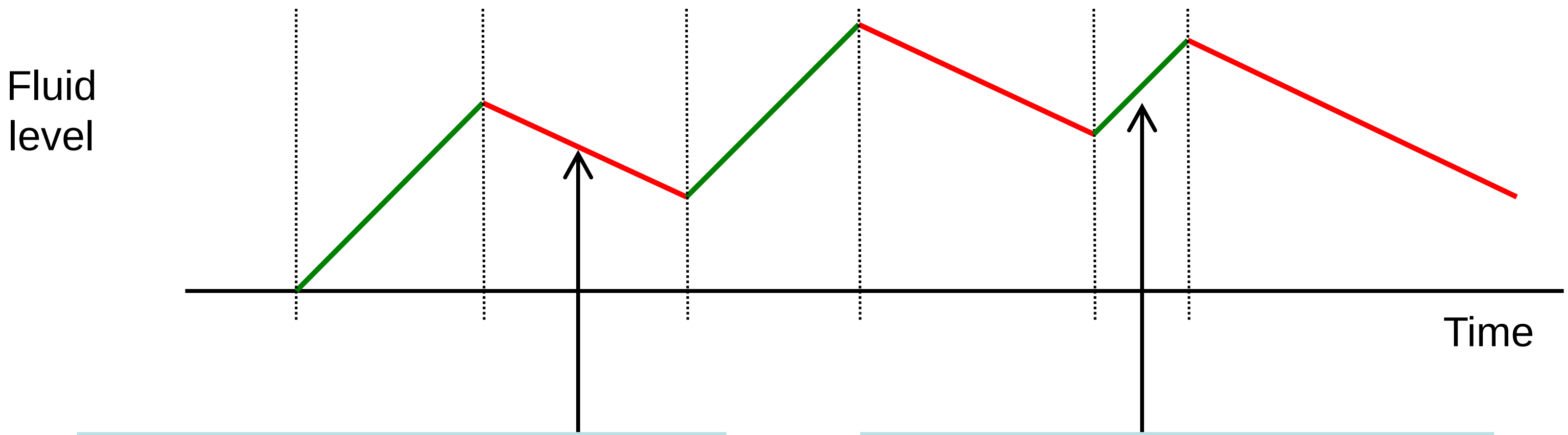


STEP 3:
Making the method non-iterative

Fluid level in a queue with On/Off source



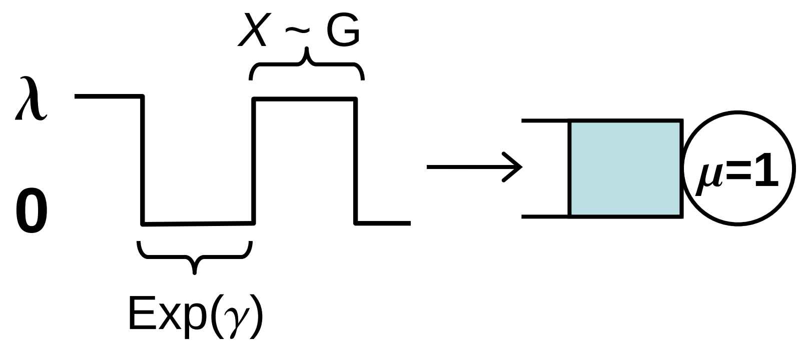


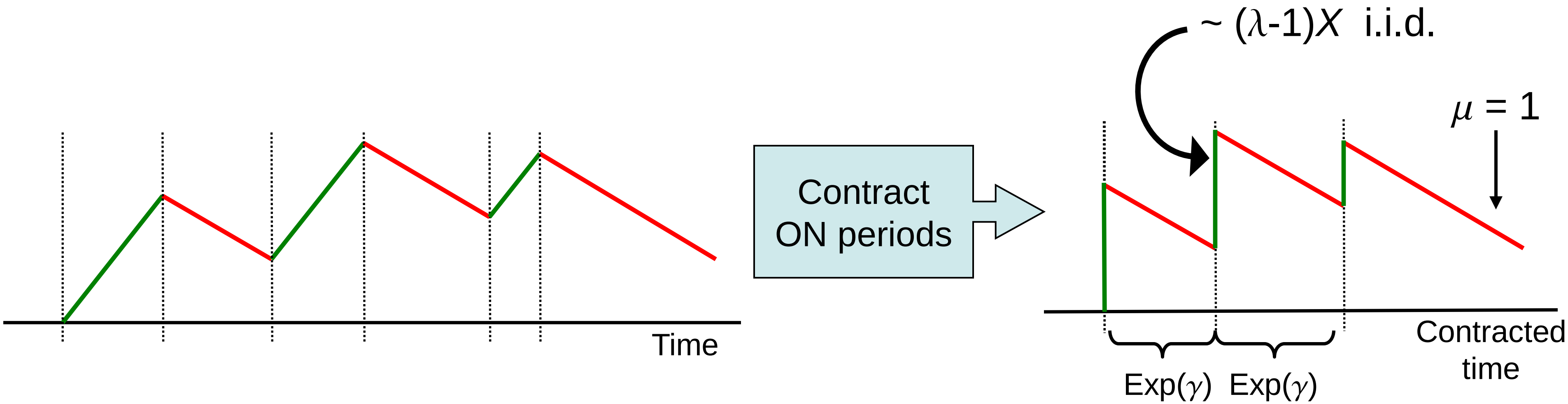
L_{OFF} = Fluid level | source Off

L_{ON} = Fluid level | source On

$$L = L_{OFF}\,\mathbf{1}_{\{\text{source Off}\}} + L_{ON}\,\mathbf{1}_{\{\text{source On}\}}$$

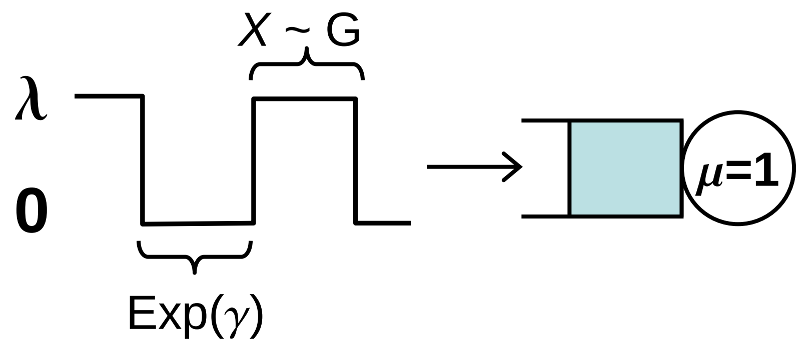
STEP 1a: Analysis of L_{OFF}

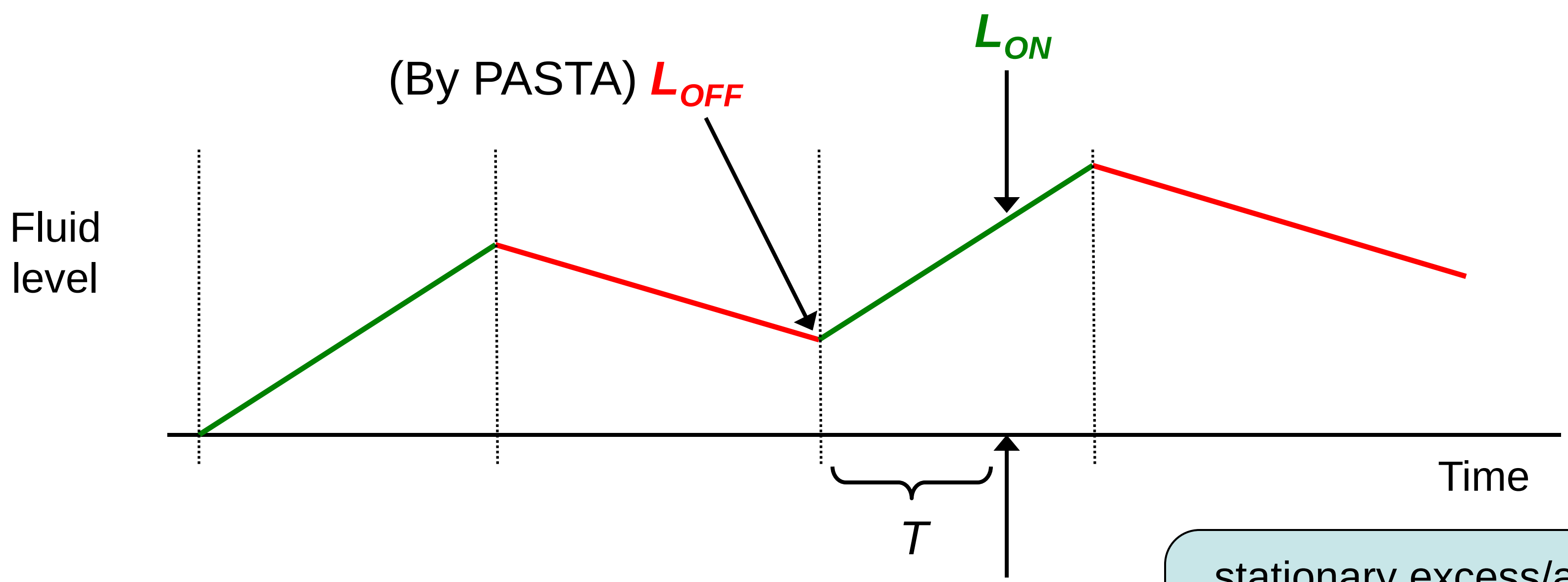




$L_{OFF} \stackrel{d}{=}$ stationary *M/G/1* workload with arrival rate γ and job sizes ~ $(\lambda\text{-}1)X$

STEP 1b: Analysis of L_{ON}



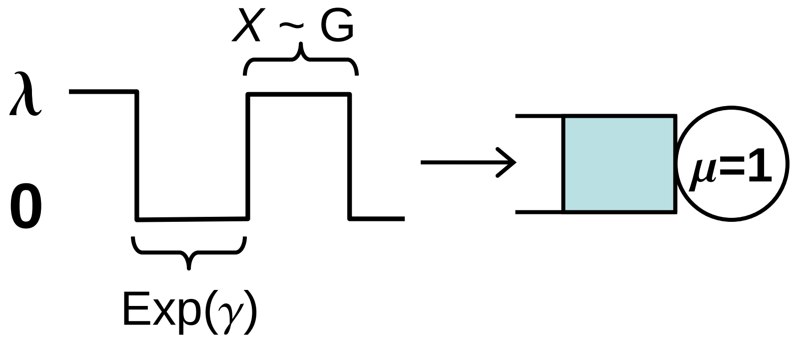


$$L_{ON} = L_{OFF} + (\lambda-1)T$$

$$\stackrel{d}{=} L_{OFF} + (\lambda-1)X_e$$

stationary excess/age in a renewal process with i.i.d. renewals according to X

Finally…



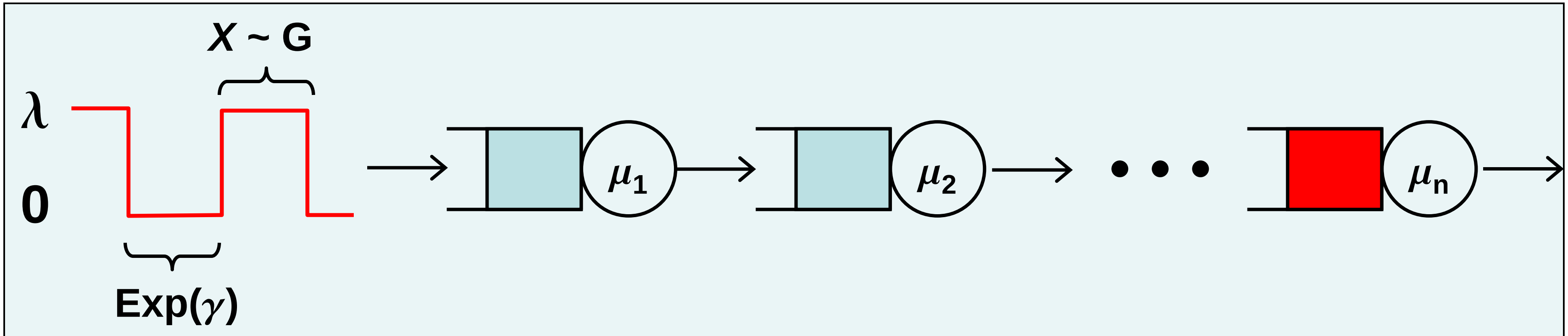
$$L = L_{OFF}\,\mathbf{1}_{\{\text{source Off}\}} + L_{ON}\,\mathbf{1}_{\{\text{source On}\}}$$

$$\stackrel{d}{=} L_{OFF} + (\lambda-1)X_e \,.\, \mathbf{1}_{\{\text{source On}\}}$$

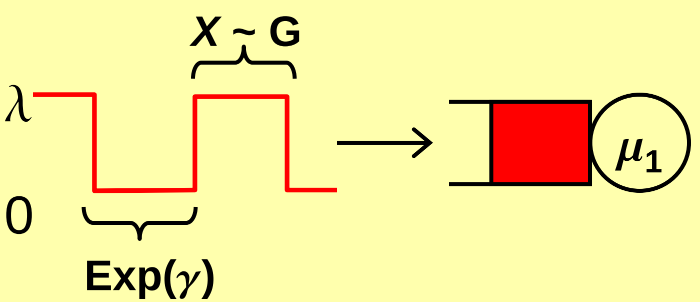
$$\stackrel{d}{=} M(\gamma)/G((\lambda-1)X)/1 \text{ workload} + (\lambda-1)X_e \,.\mathbf{1}_{\{\text{source On}\}}$$

First *k* moments of *L completely determined* by first (k+1) moments of *X*

Analysis Roadmap

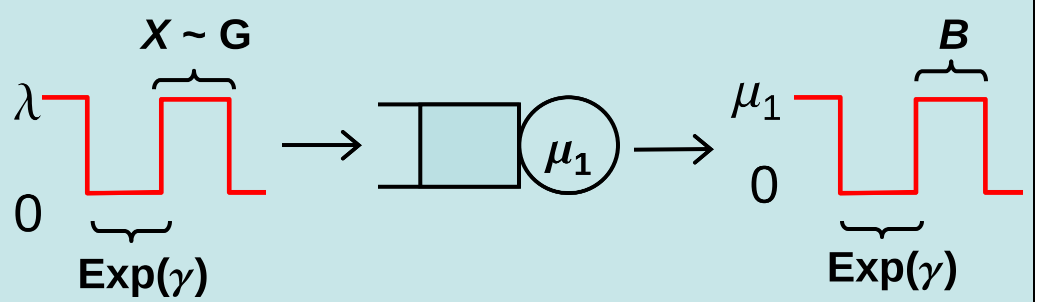


STEP 1: Fluid level at queue 1



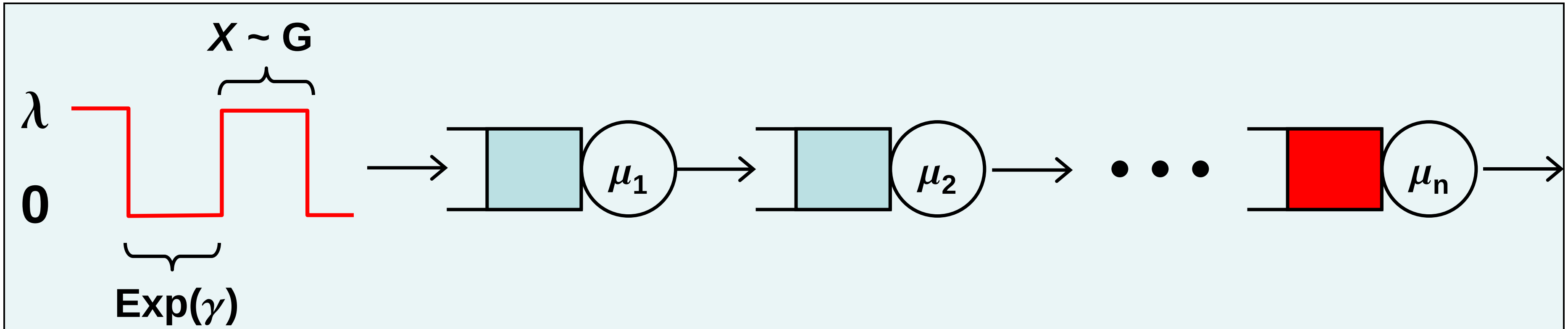
(k+1) moments of *X* ⇒ k moments of fluid level

STEP 2: Busy period analysis



STEP 3:
Making the method non-iterative

Analysis Roadmap



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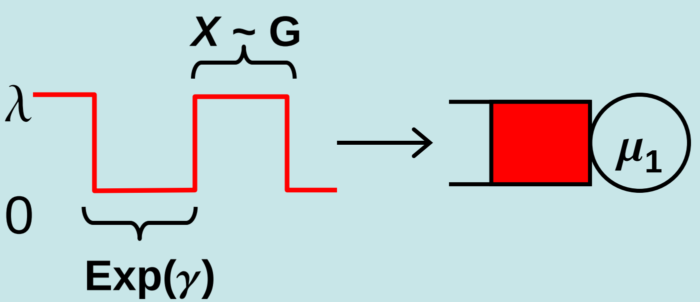
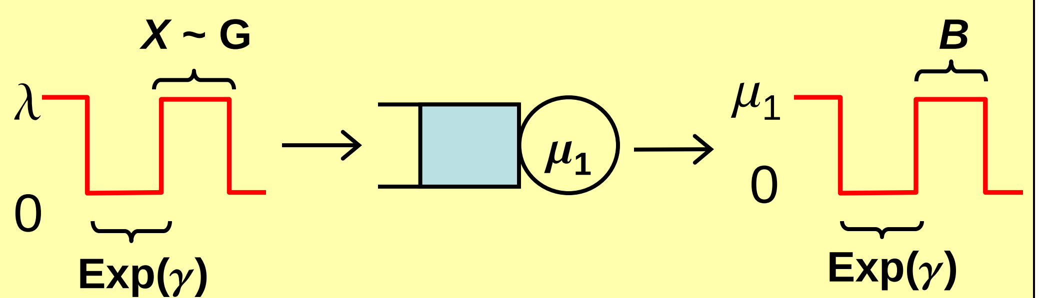


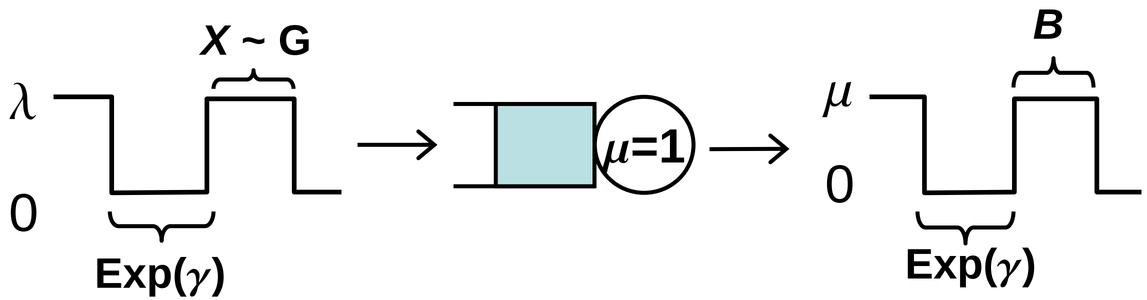
(k+1) moments of *X* $\Rightarrow$ **k moments of fluid level**

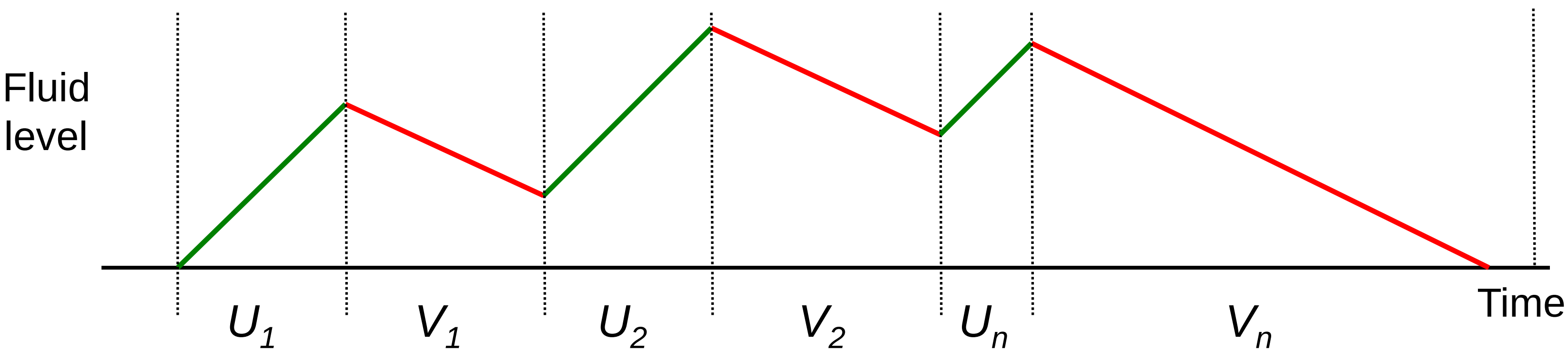
STEP 2: Busy period analysis



STEP 3:
Making the method non-iterative

Busy period analysis [Boxma,Dumas 98]

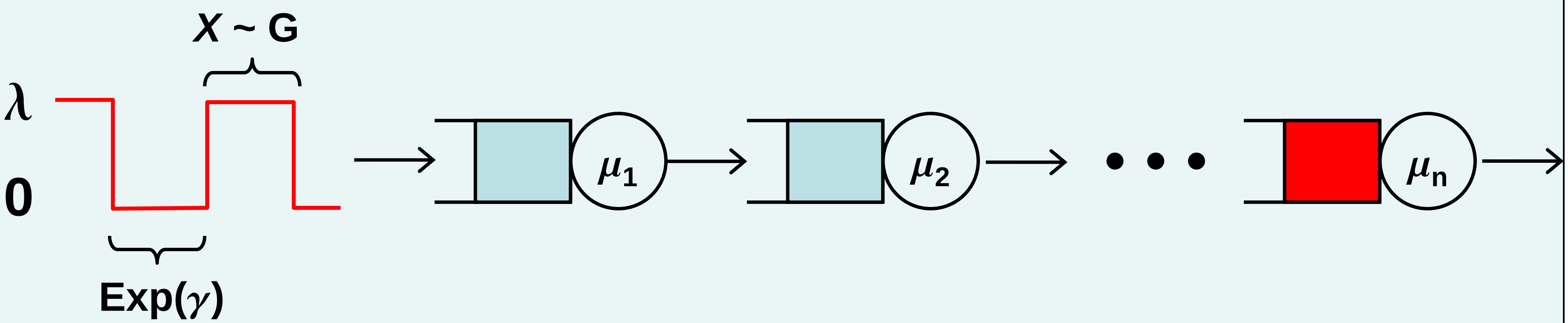




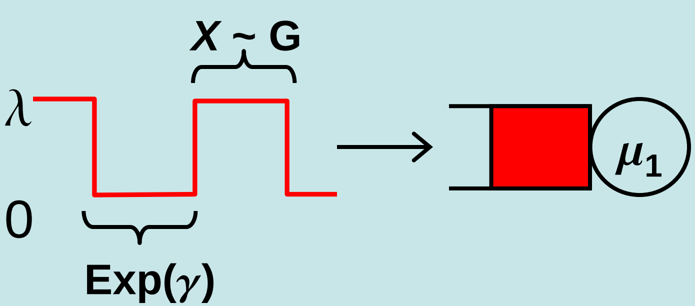
$$\begin{aligned} \boldsymbol{B} &= (U_1+U_2+\ldots+U_n)+(V_1+V_2+\ldots+V_n) \\ &= [1/(\lambda-1)+1](V_1+V_2+\ldots+V_n) \\ &\overset{d}{=} [1/(\lambda-1)+1]\; M(\gamma)/G((\lambda-1)X)/1 \text{ busy period} \end{aligned}$$

First *k* moments of *B completely determined* by first *k* moments of *X*

Analysis Roadmap

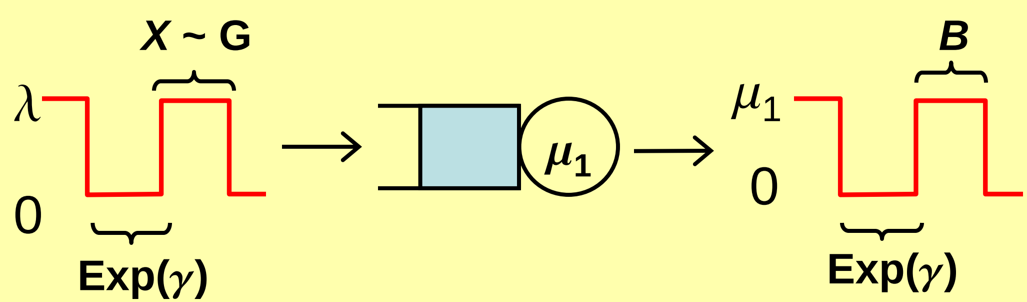


STEP 1: Fluid level at queue 1



(k+1) moments of X $\Rightarrow$ **k moments of fluid level**

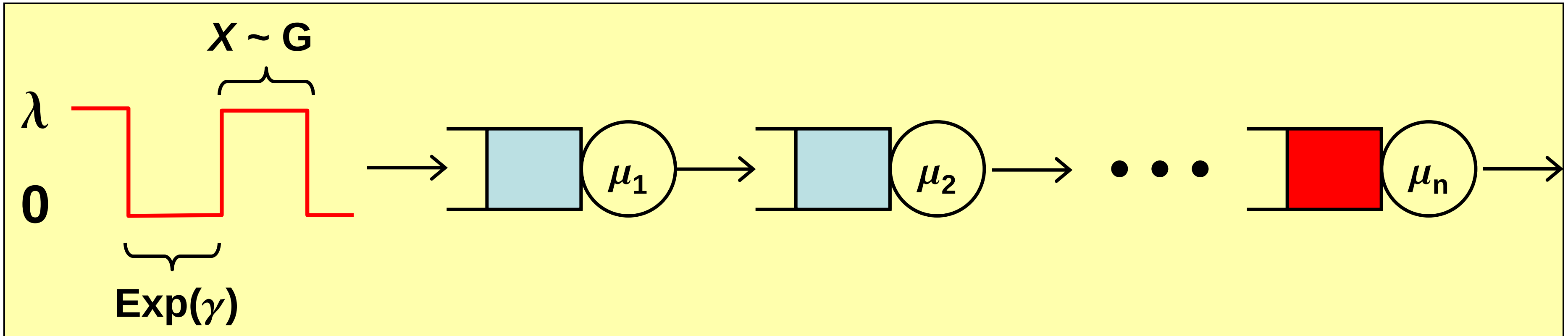
STEP 2: Busy period analysis



(k+1) moments of X $\Rightarrow$ **(k+1) moments of busy period**

STEP 3:
Making the method non-iterative

Analysis Roadmap



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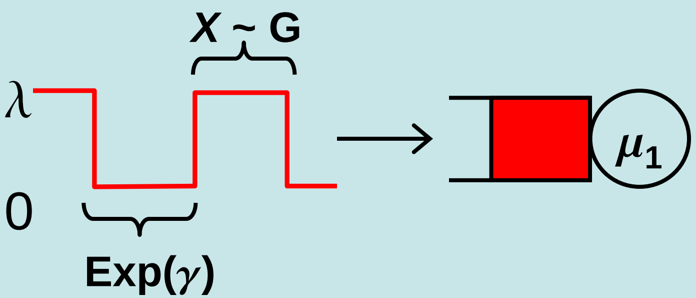
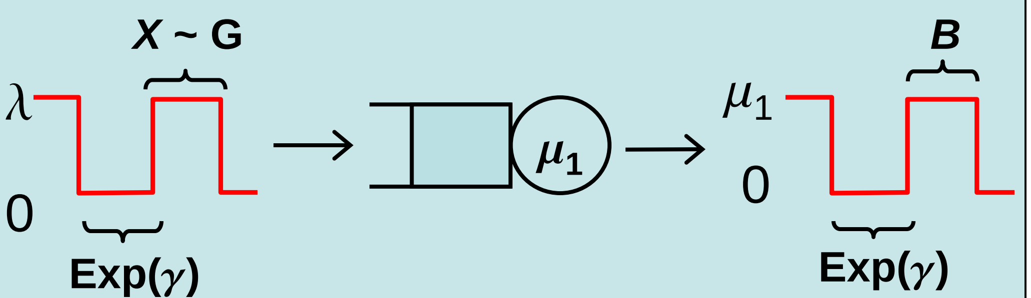


(k+1) moments of *X* $\Rightarrow$ **k moments of fluid level**

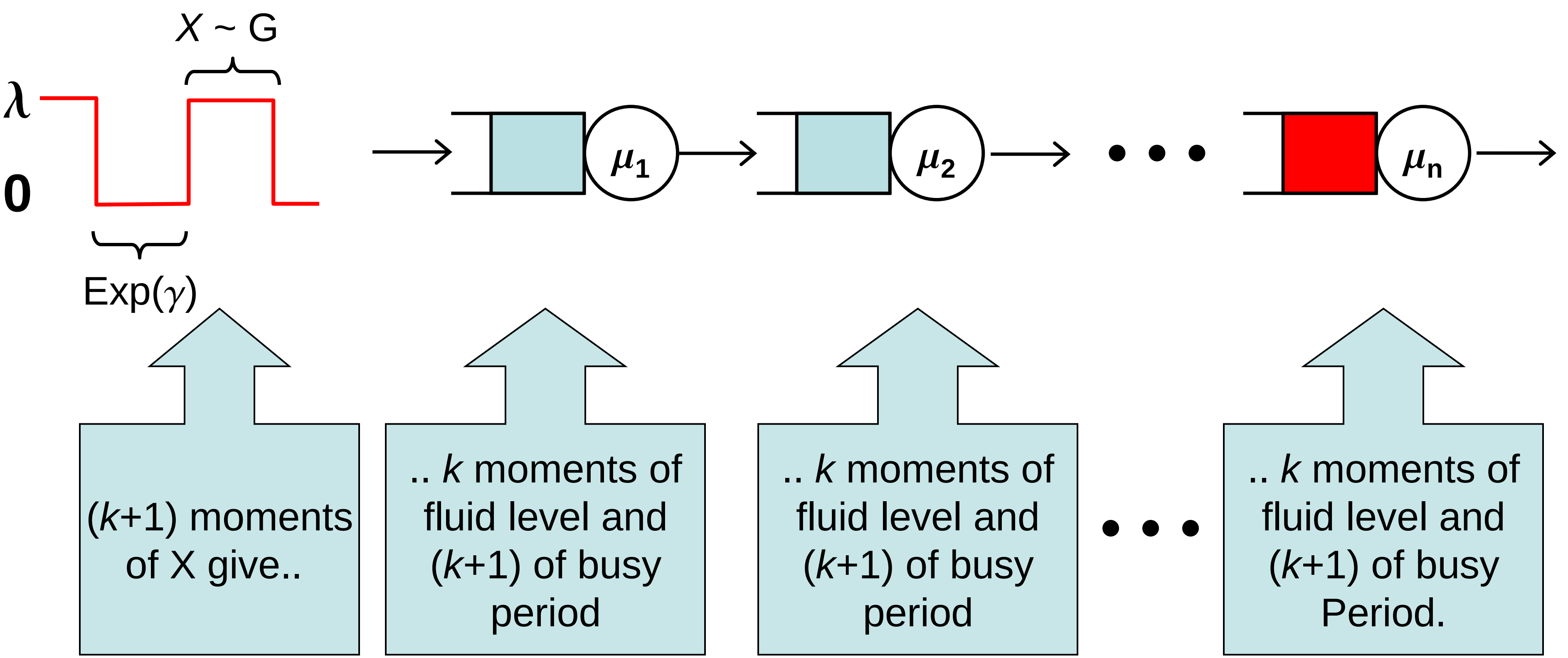
STEP 2: Busy period analysis



(k+1) moments of *X* $\Rightarrow$ **(k+1) moments of busy period**

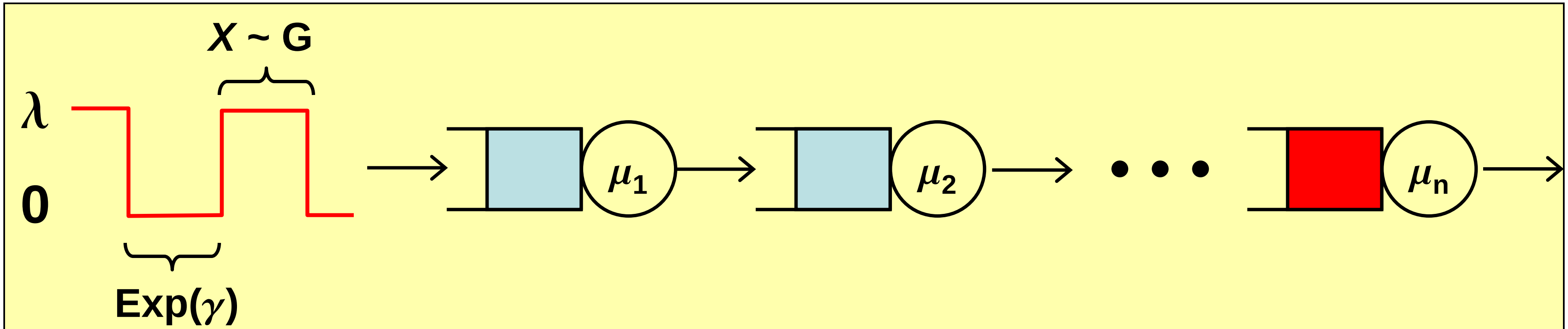
STEP 3:
Making the method non-iterative

Putting it together



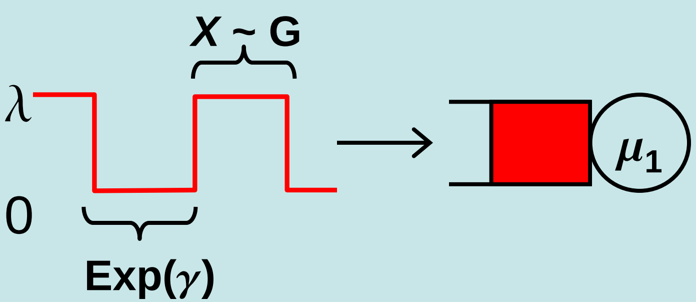
First (k+1) moments of *X completely determine* first k moments of fluid level at each queue

Analysis Roadmap



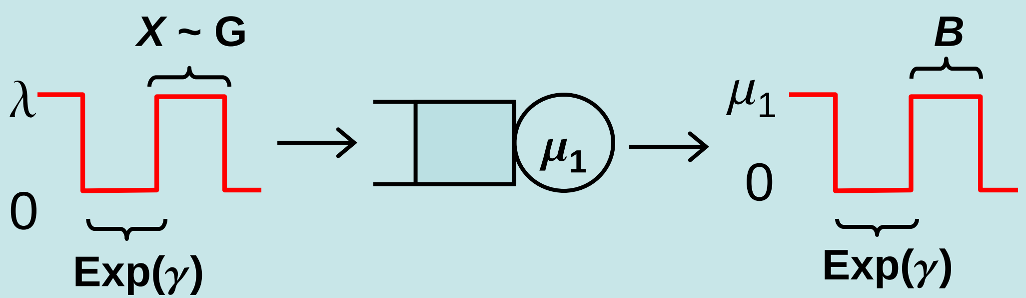
(k+1) moments of *X*   ⇒   k moments of fluid level at all queues

STEP 1: Fluid level at queue 1



(k+1) moments of *X*   ⇒   k moments of fluid level

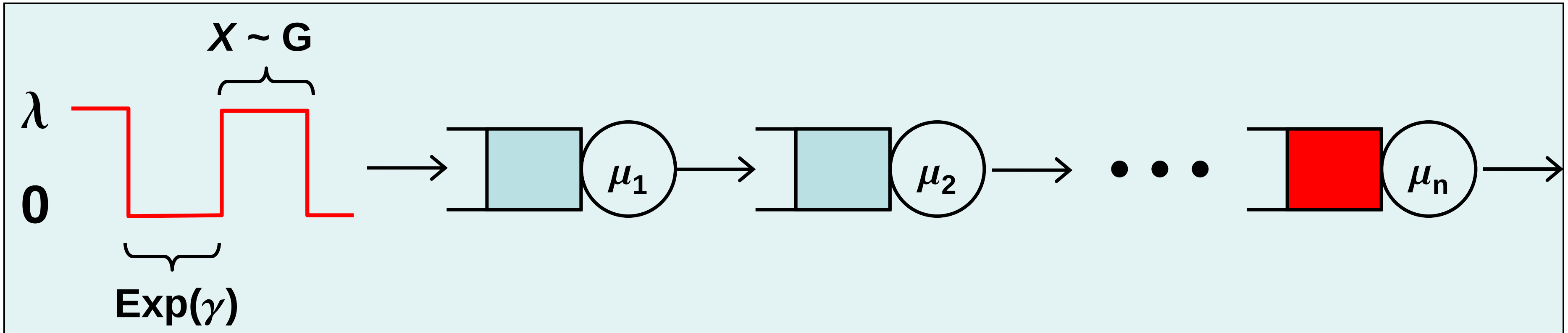
STEP 2: Busy period analysis



(k+1) moments of *X*   ⇒   (k+1) moments of busy period

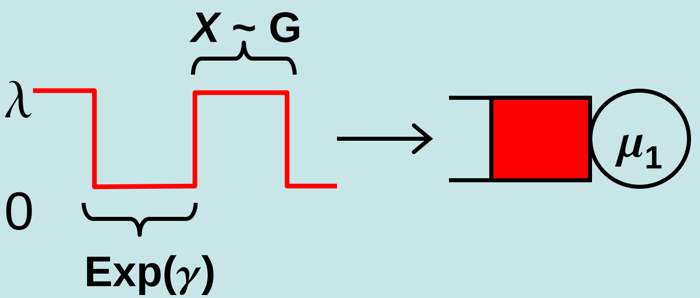
STEP 3:
Making the method non-iterative

Analysis Roadmap



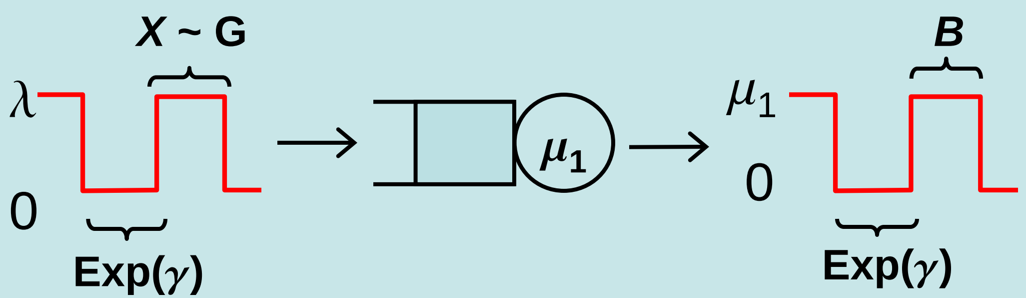
(k+1) moments of *X* $\Rightarrow$ **k moments of fluid level at all queues**

STEP 1: Fluid level at queue 1



(k+1) moments of *X* $\Rightarrow$ **k moments of fluid level**

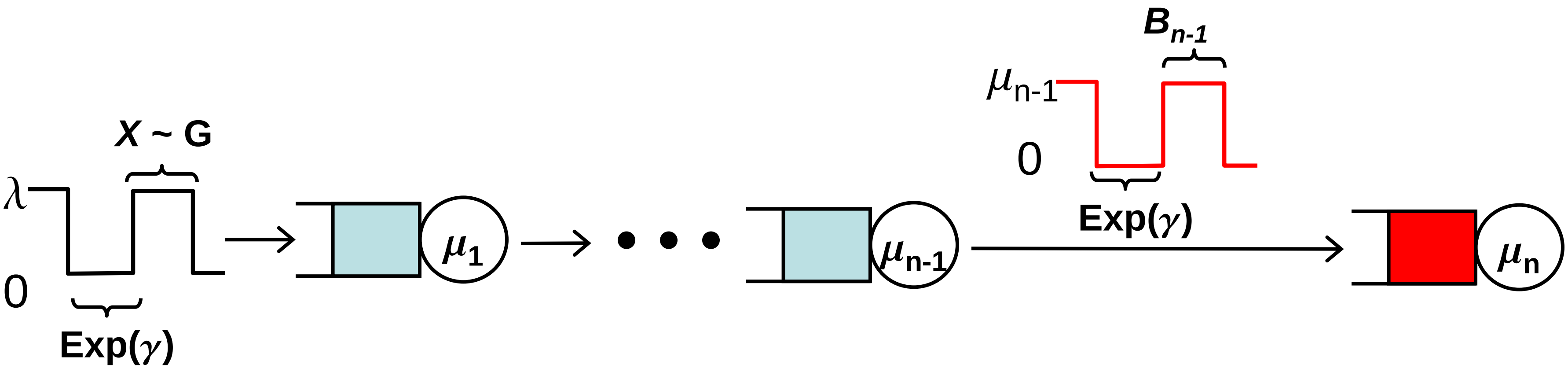
STEP 2: Busy period analysis



(k+1) moments of *X* $\Rightarrow$ **(k+1) moments of busy period**

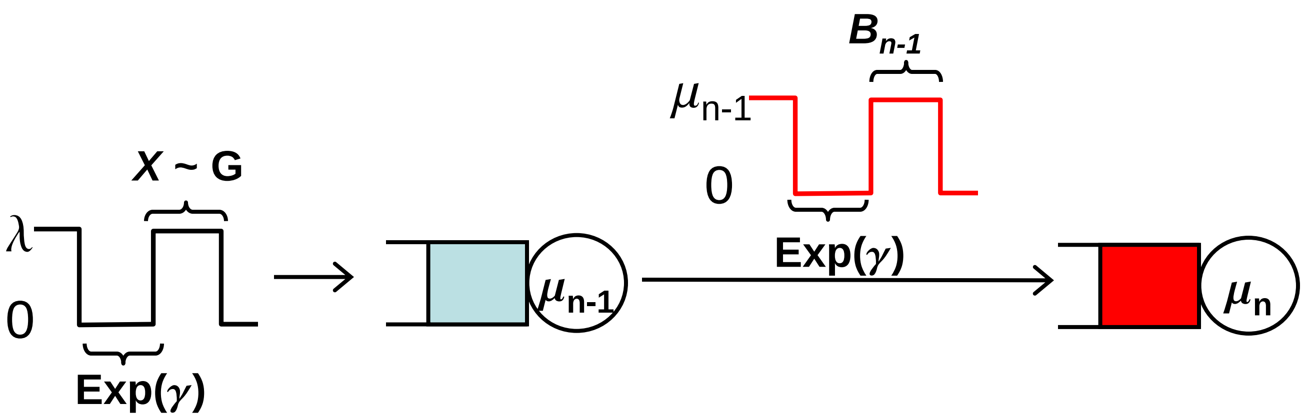
STEP 3:
Making the method non-iterative

Getting rid of busy period iteration



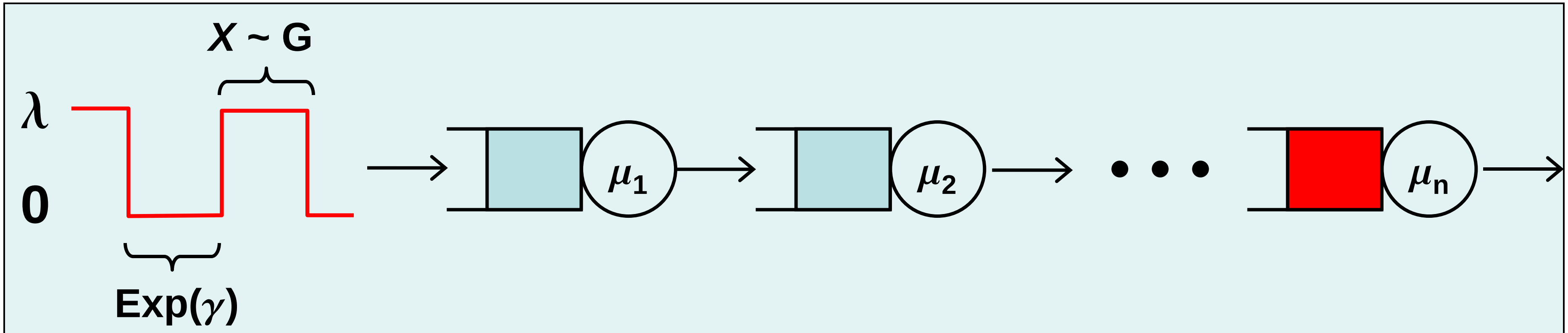
B_{n-1} = function of B_{n-2} = …

$\boldsymbol{\mu_1 > \mu_2 > \ldots > \mu_{n-1}} \Rightarrow B_{n-1}$ is *identical* to:



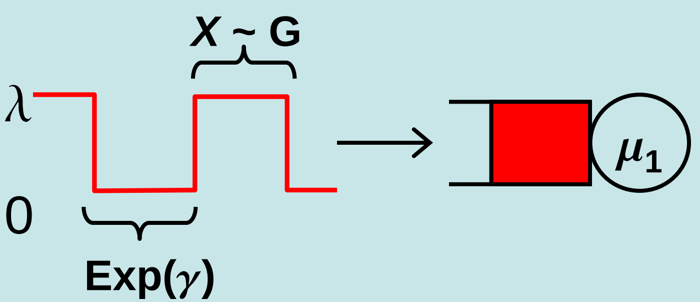
Can obtain B_{n-1} in one step (no need to iterate)

Analysis Roadmap



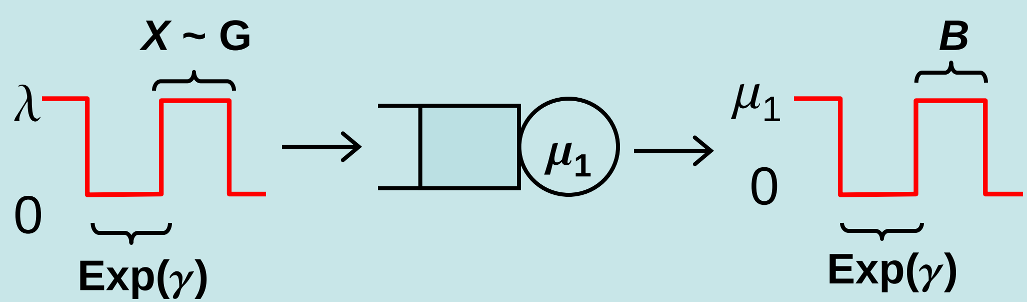
(k+1) moments of *X* $\Rightarrow$ **k moments of fluid level at all queues**

STEP 1: Fluid level at queue 1



(k+1) moments of *X* $\Rightarrow$ **k moments of fluid level**

STEP 2: Busy period analysis



(k+1) moments of *X* $\Rightarrow$ **(k+1) moments of busy period**

STEP 3:
Making the method non-iterative

Contributions/Conclusion

- Non-iterative method to obtain fluid level transform and moments in a tandem network
- Show that first (k+1) moments of On period determine first k moments of fluid level at each queue
- Method generalizes to a wider class of input processes