Chapter 7

Firm Supply

7.1 Introduction

In Chapter xxxxx we studied the firms profit maximization problem. We obtained factor demands as the solution to the firm’s profit maximization problem. The profit function told us the maximum attainable level of profits for given factor prices and output price. Hotelling’s lemma told us that the derivative of the profit function with respect to the output price was equal to the firms optimal scale decision. The second order condition guaranteed that the scale decision was maximal and required that the technology be decreasing returns to scale. However, that the firm maximizes profits does not guarantee that these profits are non-negative. If profits were negative, then the firm would be better off not entering the industry. The firms entry decision then hinges crucially on whether it obtains positive profits or not.

In this chapter we will study the firm’s entry decision and we will derive the firm’s supply function. We will see that the firm’s supply function is its marginal cost curve above average cost. We will also study properties of the supply function.

7.2 The Firm’s Short Run Supply Function

We have studied the firm’s short run cost minimization problem and we have seen that it decides to set marginal revenue, which under competition is constant and equal to price, equal to marginal cost. The resulting
short run supply function tells us the firm's optimal quantity given cost conditions in the factor market and the firm's technology.

Consider, then, the following short run profit maximization problem for the firm. The firm has already paid \( r \bar{k} \) in fixed capital costs which we assume are sunk. For example, in the short run the firm has already invested \( r \bar{k} \) dollars and built a plant. The firm can not recover these costs regardless of the amount of output it may choose to supply. Therefore, if the firm chooses to supply zero units of output it will incur a loss equal to the sunk cost—in this case, \( r \bar{k} \) dollars. It seems reasonable to believe that \( r \bar{k} \) is the most the firm could lose since it always has the option of producing zero units.

Consider, then, the firm's short run scale decision:

\[
\max_y p y - \omega \omega^r(\bar{k}, y) - r \bar{k}
\]

where we recall that the firm's short run cost function is given by \( C(w, r, \bar{k}, y) = r \bar{k} + \omega \omega^r(\bar{k}, y) \). Recall that the firm is a price taker in goods markets, so its supply decision will be heavily influenced by the unit price of the good. In fact, we know that the firm's optimality condition requires that it price equal to marginal cost:

\[
p = \frac{dC(w, r, \bar{k}, y)}{dy}.
\]

The firm will select a positive level of output as long as price exceeds variable cost:

\[
p > \frac{\omega \omega^r(\bar{k}, y)}{y},
\]

since, under these conditions, producing a positive level of output will not increase the firm's losses beyond its sunk capital costs of \( r \bar{k} \) dollars. In fact, if the price the firm receives for a unit of output exceeds average variable cost, then the firm will reduce its losses or perhaps earn positive profits. since we are studying the short run and we assume that the number of firms is fixed, the firm keeps these profits (under these conditions, in the long run entry of new firms will drive profits to zero). Fixed costs, therefore, are important to the firm when it is deciding whether or not to produce. Once the firm determines that price covers average variable costs, fixed costs do not influence the firm's optimal output choice. In the short run, then, the firm may find it optimal to produce a positive level of output despite the
fact that it may receive a negative profit. The reason is that by doing so, the firm may be able to partially offset its sunk capital costs.

If we look at Figure xxxx, we know that the marginal cost crosses average variable cost at its minimum point. The firm’s supply curve consists of two portions. When price is below the shutdown price, \( p^0 \) in the diagram, the firm produces zero units of output and the supply function is a vertical segment along the y-axis. Once the price reaches \( p^0 \), the firm finds it profitable to produce a positive amount of output and chooses its quantity by equating marginal cost to price. The firm’s supply curve is then the marginal cost curve above minimum average cost.

Insert Figure xxxx.

From Figure xxxx, we can obtain the firm’s total revenue and variable costs.

Insert Figure xxxx.

To be completed.

7.3 Properties of the Supply Function

Let us now look at the properties of the supply function.

**Property I:** Homogeneous of degree 0 in \( w, r, \) and \( p \).

\[ y^* (cw, cr, cp) = y^* (w, r, p) . \]

This essentially follows from the homogeneity of degree 0 of the factor demand functions.

**Property II:** Supply functions must have a non-negative slope.

\[ \frac{dy^* (w, r, p)}{dp} \geq 0 \]

We can decompose the supply function into a scale effect.

To be completed.
7.4 The Firm’s Long Run Supply Function

As we have previously discussed, we know that in the long run the firm is able to vary its inputs. For the profit maximizing competitive firm, this means that capital costs are no longer sunk since in the long run the firm can select the size of its plant. The firm’s long run profit maximization problem is:

$$\max_y py - C^L(w, r, y)$$

and the optimality condition requires that the firm equate marginal revenue—price, in this case—to long run marginal cost:

$$p = \frac{dC^L(w, r, y)}{dy}$$

In the long run, the firm has no sunk costs and, therefore, will choose a non-negative level of output only if it is guaranteed non-negative profits. The firm is guaranteed non-negative profits if price exceed long run average total costs:

$$p > \frac{C^L(w, r, y)}{y}.$$  

The firm’s long run supply curve is, then, its marginal cost curve above long run average cost. The firms long run supply curve is displayed in Figure xxxx.

Insert Figure xxxx.

Consider Figure xxxx. At a price such as $p^*$ the firm is able to cover its average costs and therefore chooses to supply a positive quantity. However, there are two quantities of $y$ that satisfy the price equal marginal cost condition. They are labelled $y^1$ and $y^2$. We have asserted in the preceding that the firm’s supply curve is its marginal cost curve above average costs. That implicitly selects $y^2$ as the optimal quantity and not $y^1$. If we recall the second order condition requires that marginal costs be increases at the optimum. Quantity $y^2$ satisfies this requirement. At quantity $y^1$ the firm is producing where marginal costs are decreasing and therefore violates the second order condition. Quantity $y^1$ minimizes the firm’s profit and so the firm would never find it optimal to produce $y^1$ units of output.

To be completed.
7.5 Applications

To be completed.

7.6 Exercises