The University of Chicago  
Department of Economics  
Elements of Economic Analysis I  
Problem Set #5

This problem set is due at the Review Session.

Question 1: Uncertainty

Carlos sells sunglasses. If it is sunny, Carlos makes 30$; if it is rainy, Carlos makes 10$. Suppose that Carlos orders preferences according to:

\[ U(c_s, c_r) = \]

a. A casino in Atlantic City sells dated rain coupons for 1$. If it rains, the casino will give you 2$ for every rain coupon you bought (that has the correct date on it). If it is sunny, then the coupons are worthless. On a graph, draw Carlos’ “endowment” if he buys no rain coupons and label it \( E \).

b. Plot the point consistent with Carlos buying 10 coupons. Label it \( A \).

c. Suppose that Carlos can buy any number of coupons (but not negative amounts). What will his budget set look like? Draw it in blue ink.

d. What is the slope of the budget line above and to the left of \( E \)?

e. Suppose, now, that the casino also sells sunshine coupons. These tickets also cost 1$ and pay 2$ if sunny and 0$ otherwise. In red ink, draw the set of contingent commodity bundles that are feasible if Carlos can buy any amount of sunshine coupons.

f. If the price of a 1$ worth of consumption when it rains is set to 1, what is the price of a dollars worth of consumption if it is sunny?

h. Suppose that \( \pi = 0.5 \). How many units of consumption will Carlos consume contingent on rain? How many rain coupons will Carlos buy?

i. Is there any value of \( \pi \) for which Carlos will buy exactly 0 rain coupons and 0 sun coupons?

Question 2: Equilibrium

Consider a two good, two consumer endowment economy. Suppose that agent 1’s endowment is given by:

\[ \omega^1 = (\omega^1_x, \omega^1_y) \]

and agent 2’s, endowment is given by:

\[ \omega^2 = (\omega^2_x, \omega^2_y) \]
where we assume that \( \omega_x^1 > \omega_x^2 \) and \( \omega_y^2 > \omega_y^1 \). Also assume agent 1’s preferences are given by:

\[
U(x, y) = \alpha \log(x) + (1 - \alpha) \log(y)
\]

and agent 2’s preferences are given by:

\[
U(x, y) = \gamma \log(x) + (1 - \gamma) \log(y)
\]

Finally, let the absolute prices of good \( x \) and good \( y \) be given by \( p_x \) and \( p_y \).

a. Define the competitive equilibrium of the endowment economy.

b. Write down each agent’s maximization problem.

c. Solve each agent’s maximization problem. You should obtain \( x_i^*, y_i^* \), and \( \eta_i^* \) for \( i = 1, 2 \) where \( \eta_i^* \) denotes the Lagrange multiplier on agent \( i \)’s budget constraint.

d. Find \( X^* \) and \( Y^* \), the aggregate demands for the two goods.

e. What are the equilibrium conditions for this economy?

f. Find the market clearing relative price for the \( x - market \).

g. Now use the market clearing condition in the \( y - market \) to find the equilibrium price. How much of good \( y \) does each agent consume?

h. Determine the equilibrium values of \( \eta_i^* \).

i. Graph the solution. Label your graphs clearly.

j. Would the answer be the same if I assumed that \( p_x = 1 \)? Explain your answer clearly.

l. Who trades what with whom?

m. What happens to the equilibrium relative price of good \( x \) if the endowment of \( x \) increases, everything else held constant? Graph your solution, give a mathematical analysis, and explain the economic intuition of your result.