Theoretical Model from Levitt and Syverson (2002)*

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The basic model

We present here a model of a real estate transaction involving an owner selling her house. The primary goal of the model is to understand the distortions that arise in a principal-agent setting when the real estate agent is better informed about the housing market than is the home owner hiring the agent to help sell the home. In order to better isolate the role of information asymmetry, we begin by presenting a highly stylized two-period model that ignores many other aspects of the real estate market that may be important (e.g., the option of selling a house without hiring an agent, risk aversion, shirking by the agent, competition among agents, rising housing prices over time, etc.). In addition, the contractual form is taken as given: the agent is paid a fixed percentage $\alpha$ of the sale price of the home.\footnote{Later, we consider the choice of $\alpha$, as well as allowing more general contractual forms. In practice, however, virtually all real estate transactions in the United States in which the buyer hires an agent conform to the structure of the home seller paying a fixed percentage of the sale price – typically 5-7 percent (Hsieh and Moretti 2003).} After solving this simple model, we then consider how the inclusion of these complicating features affects the conclusions of the basic model.

The structure of the model is presented in Figure 1. Both the seller and agent are risk neutral. In period 0, a home owner, hereafter denoted the seller, decides to sell her house and hires an agent. Once hired, an agent cannot be fired. Beginning in period 1, a fixed cost $c$ is incurred by the agent each period the house is on the market. This reflects either time costs of selling the home (e.g. writing advertisements, showing the house, holding open houses, negotiating with potential buyers) or financial costs such as advertising and promotion expenditures. We expect $c$ to be small relative to the value of the home. For simplicity, we assume that the seller bears no cost of having the home on the market and that both sellers and agents have zero discount rates.\footnote{As discussed later, for any reasonable set of costs borne by the owner, none of the basic conclusions of the model are altered.}
In period 1, exactly one offer (denoted $p_1$) is made on the house. The agent advises the seller to accept or reject the offer (we show below that the agent cannot credibly communicate any finer-grained signal than a simple “accept” or “reject”), and the seller chooses whether or not to accept. If the first-period offer is accepted, the seller receives the sale price minus the agent’s share ($\alpha p_1$) and the agent receives his share minus the costs incurred marketing the property ($\alpha p_1 - c$).

If the period 1 offer is rejected, then that offer disappears and the game proceeds to period 2. Another offer, denoted $p_2$ and drawn from the same distribution of offers, arrives in the second period. The seller is required to accept this offer.\(^3\) The seller receives the payoff $\alpha p_2$. The agent’s payoff when the second-period offer is accepted is $\alpha p_2 - 2c$.

In this model, the benefit to the seller of hiring the agent is that the agent has better information about the value of the home than the seller (although the agent may or may not truthfully reveal this information). In particular, it is assumed that the agent costlessly learns the exact distribution from which take-it-or-leave-it offers to purchase the house are drawn, denoted $f(p)$, where $p$ is the price of the offer.\(^4\) The seller, on the other hand, in period zero observes only a noisy signal of the offer distribution, which we will call $f_s(p)$. More precisely, we assume the following functional forms for the offer distribution and the seller’s signal of it:

\[
\begin{align*}
  f(p) &= \mathcal{U}[p_1, p_3] \\
  f_s(p) &= f(p) + \hat{\epsilon} \quad \text{where } \hat{\epsilon} \sim \mathcal{U}[-\epsilon, \epsilon] \text{ and is independent of } p
\end{align*}
\]

Because $E[\hat{\epsilon}] = 0$, sellers’ initial beliefs about the offer distribution are on average unbiased, although any particular seller may be overly optimistic ($\hat{\epsilon} > 0$) or pessimistic ($\hat{\epsilon} < 0$) in valuing offers.

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\(^3\) This period-2 default is one of the least satisfying assumptions of the basic model, but is enormously useful for two reasons. First, it makes it unnecessary to specify the precise process by which the seller updates her beliefs concerning the value of the home as a series of offers arrives. Second, it makes the agent’s motives for falsely or truthfully advising the seller transparent. In a version of the model with three or more periods, the agent will have an additional incentive for distorting his information transmission to the agent, namely to convince the seller that the house has a lower value than is actually the case, which increases the offer acceptance rate in later periods.
her own home. Both the agent’s information and the seller’s information are private, non-verifiable, and non-contractible.\(^5\) We assume, however, that the range of \(f(p)\), which is \(p_h - p_l\), and the distribution of \(\hat{\epsilon}\), are common knowledge to the seller and agent. The offer distribution is assumed to be stationary over the relevant time frame (periods 1 and 2).

We begin our analysis of the model with the special case in which a real estate agent sells his own house.\(^6\) This special case is of particular importance for two reasons. First, because the owner in this case has both full information and bears the costs of selling the house, the first-best outcome will be achieved, making this case an important benchmark. Second, in our empirical work, the key distinction that we focus on in testing the model is the differential market outcomes for agent-owned and non-agent-owned houses.

Defining \(\bar{p} = E[p_2] = E[p]\), the expected price obtained if the period 1 offer is rejected, it is straightforward to demonstrate the following set of results:

**Proposition 1: The agent-as-seller outcome**

When the agent who knows the true distribution of offers sells his own home,

(a) The first period offer is accepted if and only if \(p_1 > \bar{p} - c\)

(b) The expected sale price is \(\bar{p} + \frac{1}{8} \theta - \frac{c^2}{2\theta}\), where \(\theta = p_h - p_l\).

(c) The average time on the market for the house is \(1.5 - \frac{c}{\theta}\) periods.

(d) The first-best outcome is achieved.

\(^4\) Presumably, the agent has previously incurred some fixed cost in order to acquire the human capital necessary to be able to make this assessment.

\(^5\) In the two-period version of the model, it is irrelevant whether the seller’s information is private or not. When the model is extended to three or more periods, it will be to the seller’s advantage to keep this signal private, as the agent could use knowledge of the seller’s information to manipulate her beliefs in the agent’s favor.

\(^6\) We assume that a real estate agent represents himself when selling his own home, and thus bears the selling cost \(c\) and does not have to pay \(\alpha\) to another agent.
Proof: See appendix.

Proposition 1 describes the outcome of the model when the seller has accurate information about the distribution of possible offers and internalizes the costs of selling the home. The seller accepts any offer in period 1 that exceeds the expected offer in period 2 (remember that the period 2 offer must be accepted if the first period offer is rejected), but also accepts offers up to \( c \) units below \( \bar{p} \) since selling in the first period allows the owner to avoid paying the cost \( c \) in the second period. Note that the expected sales price in Proposition 1(b) consists of three terms. The first term \( \bar{p} \) is the mean of the offer distribution. The second term is the option value associated with having the opportunity to reject the first period offer if it is too low. The greater is the range of possible offers, the higher the option value. The third term embodies the effect of selling costs—both the effects on the probability that a first-period offer is accepted and on the average value of that offer. The average time on the market is slightly below 1.5 periods since the seller accepts any first-period offer above \( \bar{p} \) (fifty percent of such offers because the distribution of \( p \) is symmetric), as well as those offers within \( c \) of \( \bar{p} \).

The more general case of the model involves a seller who is less well informed than the agent and thus potentially derives benefit from the agent’s advice. Note that in the setup of this model, the agent cannot credibly convey any information beyond a simple recommendation to either accept or reject the offer (Crawford and Sobel 1982). Any message that attempted to further signal the intensity of the agent’s preferences is cheap talk. For any offer that is high enough that the agent would like the seller to accept, the agent will have the incentive to falsely portray the offer as being extremely high, even if it is just above the cutoff. As consequence, the seller will ignore such information.\(^7\)

\(^7\) If we were allow cash transfers from the agent to the seller, this would provide a means for the agent to convey more information about the attractiveness of the offer. When a really good offer arrives, the agent could credibly signal this by offering to make an additional payment to the seller if the offer is accepted.
In light of the above restriction on the agent’s available strategy space, the following proposition characterizes the agent’s recommendation and the seller’s optimal response:

**Proposition 2:** The agent’s recommendation and the seller’s best response

(a) The agent can do no better than to recommend acceptance of the first period offer if $p_1 > \bar{p} - c/\alpha$, and recommend rejection otherwise.

(b) If $\varepsilon > c/\alpha$, then the seller behaves as follows. First, she rejects any offer that the agent recommends rejecting. She also rejects some offers that the agent recommends accepting, namely those where $p_1 < \bar{p} - (\varepsilon - c/\alpha)$; $\bar{p}_s = \bar{p} + \hat{\varepsilon}$ is her signal of the mean of the offer distribution.

(c) If $\varepsilon < c/\alpha$, then the seller ignores the agent’s advice altogether.

*Proof.* See appendix.

The intuition for Proposition 2a is as follows. At $p_1 = \bar{p} - c/\alpha$, the agent is indifferent between the seller accepting and rejecting the first-period offer. When the agent makes a recommendation to the seller, the probability that the seller follows that course of action weakly increases. Thus, the agent’s optimal strategy is to truthfully report whether the first-period offer is above or below his threshold for accepting the offer. Note, however, that the price the agent is willing to accept is lower than the seller’s ideal period 1 price cutoff (if she had perfect information) for two reasons: (1) the agent pays the cost $c$ of marketing the house in period 2 so wants to sell in period 1, and (2) the agent receives only a small share $\alpha$ of any higher price obtained by waiting a second period. Empirically, the divergence in incentives for the agent and the seller may be large. The relevant $\alpha$ is typically around 0.015, and a plausible value of $c$ might
be $500. In this case an agent is willing to accept an offer \( \frac{c}{\alpha} = \frac{500}{0.015} = 33,000 \) below the seller’s optimal cutoff with full information.\(^8\)

In response to the agent’s recommendation, the seller updates her prior beliefs about the distribution of offers. Since the seller knows that the agent is more eager to sell than she is, any time the agent recommends rejecting an offer, the seller can be certain the offer must be below her optimal cutoff and so rejects it. When the agent’s advice is to accept the offer the seller must be more cautious, as she knows the agent has the incentive to recommend acceptance of offers below her own optimal cutoff level. If the first-period offer is sufficiently below the seller’s prior expectation of the expected offer price, the offer is rejected despite the agent’s recommendation.

For \( \varepsilon < \frac{c}{\alpha} \), as in part (c), the agent’s incentives are so distorted that the agent’s signal provides no useful information to the seller. The seller therefore makes her reject/accept decision based only upon her signal of the median of the offer distribution, \( \bar{\varepsilon} = \bar{p} + \hat{\varepsilon} \). Because \( \bar{\varepsilon} \) is unbiased, it is the seller’s best estimate of the actual median. Therefore a seller ignoring an agent’s advice accepts \( p_1 \) if \( p_1 > \bar{\varepsilon} \), and rejects lower offers.

The tradeoffs faced by a seller incorporating the agent’s recommendation are presented graphically in Figure 2. The horizontal axis of Figure 2 reflects the seller’s private signal as to the value of her house, \( \hat{\varepsilon} \). An overly pessimistic (optimistic) seller has a low (high) value of \( \hat{\varepsilon} \). The vertical axis represents the first-period price offer, \( p_1 \). The area in Figure 2 is divided into different regions corresponding to the seller’s optimal decisions (accept or reject) with and without the agent’s recommendation. In the upper-left-hand portion of the graph (regions Ia and Ib), the seller would accept the offer without any agent advice. The agent’s positive recommendation only serves to reinforce this action. This area corresponds to offers that are high in both absolute terms and relative to the seller’s private signal. (Region Ia differs from Ib in that

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\(^8\) Out of a total commission of six percent, the seller’s agent and the seller’s agent’s real estate company share three percent, which is typically evenly split. Given that properties stay on the market an average of
the seller would be better off rejecting offers in Ib, and would if she had full information about
the offer distribution, but does not in an incomplete information case because of private
pessimism about likely offers.) Offers in the lower-right-hand part of the graph (regions II and
III) are rejected with or without the agent’s advice. We separately categorize these two areas
because region II contains offers that the agent recommends rejecting, whereas region III are
offers that the agent recommends accepting, but that optimistic sellers reject nonetheless because
they know that the agent has a lower threshold for recommending acceptance. Note that all of the
offers is region II should be rejected (they are below the expected value of the second period
offer). In contrast, for some of the offers in region III (those above $\overline{p}$ in IIIa), the seller would
actually be better off accepting, but because of the seller is overly optimistic, sometimes rejects
nonetheless.

The other three regions of the graph represent areas in which an agent’s advice leads the
seller to change her optimal decision. The triangle in the bottom-left-hand corner (region IV)
corresponds to low offers that would have been mistakenly accepted by overly pessimistic sellers,
but which are correctly rejected because the agent recommends rejection. This triangle represents
a gain in expected value to the seller from hiring the agent.

The parallelogram near the center of the figure (region V) represents offers that would
have been correctly rejected by a seller who did not consult an agent, but which are accepted
when the agent’s recommendations are considered. These offers are above the agent’s threshold
for acceptance, but below the optimal cutoff for the seller, and thus represent a cost to heeding an
agent’s advice. Because the agent has private information, the seller is unable to differentiate
between cases where the offer is bad and her private information is not too optimistic and the
situation in which the offer is good, but her private signal is overly optimistic.

approximately 1.5 periods in the model, $c = 500$ would imply that the costs of selling a typical house to the
agent (including time and out of pocket expenditures) is $750.
The final region of Figure 2 is the trapezoid in the upper-right-hand corner (region VI). These represent good offers wrongly rejected by overly optimistic owners who do not have the benefit of (or who do not listen to) an agent, but are rightly accepted when an agent’s advice is considered. This region thus represents a gain from listening to an agent.

Below we characterize the solution to the seller’s problem assuming that \( \varepsilon > c/\alpha \), that is, when it is in the seller’s advice to listen to the agent. This supposition is based on the fact that we build our model around the notion that sellers hire agents (at least in part) in order to gain information about the offer distribution. If agents’ advice is always worthless, and agents are hired for completely different reasons, then our research strategy is misguided.

**Proposition 3: Seller’s Outcome**

When the seller has imperfect information,

(a) The first period offer is accepted if and only if

\[
\max \left\{ \bar{p} - \frac{c}{\alpha}, \bar{p} + \hat{\varepsilon} - \left( \frac{c}{\alpha} \right) \right\} > \frac{1}{\theta} \left[ \frac{c}{\alpha} - \frac{1}{\varepsilon} \left( \frac{c}{\alpha} \right)^2 \right],
\]

and the probability an offer \( p_1 \) is accepted is equal to

\[
\frac{1}{2} + \frac{1}{\theta} \left[ \frac{c}{\alpha} - \frac{1}{\varepsilon} \left( \frac{c}{\alpha} \right)^2 \right], \text{ where } \theta = p_s - p_1.
\]

(b) The expected sale price is

\[
E[p_{sale}] = \bar{p} + \frac{1}{8} \theta - \frac{1}{2\theta} \left[ \left( \frac{c}{\alpha} \right)^2 - \frac{2}{3} \left( \frac{c}{\alpha} \right)^3 \right].
\]

(c) The number of periods that the house is on the market is

\[
E[t_{sale}] = 1.5 - \frac{1}{\theta} \left[ \frac{c}{\alpha} - \frac{1}{\varepsilon} \left( \frac{c}{\alpha} \right)^2 \right].
\]

**Proof:** See appendix.

Comparing parts (b) and (c) of Propositions 1 and 3, the expected sale price and time-to-sale-differential between agent-owned and non-agent-owned homes are:
The expected sale price is higher for agent-owned homes for reasonable values of $\alpha$. The expected time on the market will be higher for agent-owned homes as long as $\alpha < 1 - (c/\alpha)/\varepsilon$; in other words, as long as the agent’s incentives aren’t too distorted relative to the seller’s uncertainty about the offer distribution. While we can’t empirically measure this ratio directly, given that $\alpha$ typically is quite small, we expect that the expected time-on-market should be longer for agent-owned homes in the data. These two equations provide the primary testable implications of the model, but especially the former, because it holds more generally.

Additional predictions of the model arise out of the comparative statics of the $p_{dif}$ and $t_{dif}$ expressions. For example, it is clear from the above that the greater is the informational advantage of the agent (i.e., the higher is $\varepsilon$), the larger the distortions when an agent is hired. One might expect the agent’s informational advantage to be greater when there is large heterogeneity in the housing stock or few neighboring houses have recently sold. More experienced agents might also hold a greater advantage relative to agents who have sold fewer homes. Technological advances that lower the cost of a seller obtaining accurate house price estimates (e.g. internet web sites that provide records of recent home sales free of charge), on the other hand, should expectedly decrease agent-induced distortion.

\[
E[p_{dif}] = \frac{1}{2\varepsilon} \left[ (1 - \alpha)^2 \left( \frac{c}{\alpha} \right)^2 - \frac{2}{3\varepsilon} \left( \frac{c}{\alpha} \right)^3 \right]
\]
\[
E[t_{dif}] = \frac{1}{\theta} \left[ (1 - \alpha) \frac{c}{\alpha} - \frac{1}{\varepsilon} \left( \frac{c}{\alpha} \right)^2 \right]
\]

9 Specifically, $E[p_{dif}] > 0$ for $\sigma^2 < 1 - 2c/3\alpha\varepsilon$. The largest relevant value of $c/\alpha\varepsilon$ is one, implying this condition holds in all cases for $\alpha < 0.577$, a threshold well above observed commission rates.

10 The intuition for the uncertain sign in $E[t_{dif}]$ is that at one information extreme, completely informed sellers ($c/\alpha = 0$) would accept half of the first-period offers (those above the median), while at the other extreme, sellers with valueless agent information ($c/\alpha \geq \varepsilon$) would also accept half of the first-period offers on average. Thus the seller’s expected time on the market when an agent is hired must be nonlinear in $c/\alpha$. 


References


Appendix

Proof of Proposition 1.

(a) If the agent-as-seller rejects $p_i$, he will incur the cost of keeping the house on the market for an additional period and will expectedly receive a second period offer of $\bar{p}$. Clearly, then, the agent-owner will accept any $p_i > \bar{p} - c$ and reject lower offers.

(b) Denoting an accepted offer as $p_{\text{sale}}$, the expected sales price, the above acceptance rule implies that the expected (before the first-period offer is observed) sales price is:

$$E(p_{\text{sale}}) = E(p_{\text{sale}} | p_i > \bar{p} - c) \Pr(p_i > \bar{p} - c) + E(p_{\text{sale}} | p_i < \bar{p} - c) \Pr(p_i < \bar{p} - c)$$

$$E(p_{\text{sale}}) = \frac{p_h + (\bar{p} - c)}{2} \left( \frac{p_h - (\bar{p} - c)}{p_h - p_i} \right) + \frac{\bar{p}(\bar{p} - c - p_i)}{p_h - p_i}$$

Noting that $\bar{p} = 0.5(p_i + p_h)$, some algebra yields

$$E(p_{\text{sale}}) = \frac{1}{4}(5p_h^2 - 2p_hp_i - 3p_i^2) - \frac{c^2}{2(p_h - p_i)}$$

Factoring the polynomial, using $\bar{p} = 0.5(p_i + p_h)$ again, and denoting $\theta = p_h - p_i$ yields the expression in the text:

$$E(p_{\text{sale}}) = \bar{p} + \frac{1}{8} \theta - \frac{c^2}{2\theta}$$

(c) The expected time-on-market $t_{\text{sale}}$ for an agent-as-owner can be expressed as

$$E(t_{\text{sale}}) = 1 \cdot \Pr(p_i > \bar{p} - c) + 2 \cdot [1 - \Pr(p_i > \bar{p} - c)] = 2 - \Pr(p_i > \bar{p} - c)$$

$$E(t_{\text{sale}}) = 2 - \left( \frac{p_h - (\bar{p} - c)}{p_h - p_i} \right)$$

Simplifying generates the equation in the text:

$$E(t_{\text{sale}}) = \frac{3}{2} - \frac{c}{\theta}$$

(d) Because the agent accepts with certainty offers above his indifference first-period offer and rejects those below, the first-best outcome is achieved.
Proof of Proposition 2.

(a) Given an observed first-period offer \( p_1 \), the expected payout to the agent if the seller accepts the offer is \( \alpha p_1 - c \). The expected second-period payout to the agent if the seller rejects \( p_1 \) is \( \alpha \bar{p} - 2c \). Thus the first-period offer at which the agent is indifferent between these payouts is \( p_1 = \bar{p} - c / \alpha \). The agent prefers that the seller always reject offers below this level and accept those above. Furthermore, the agent cannot affect the expected second period offer (and hence his own payout conditional upon rejection of the first-period offer) with his recommendation. Clearly, then, if there is a (weakly) higher probability that a seller would accept offers \( p_1 < \bar{p} - c / \alpha \) if the agent recommended acceptance, the agent is no worse off, and sometimes better off, recommending rejection. Likewise, a (weakly) increased chance that a seller rejects offers \( p_1 \geq \bar{p} - c / \alpha \) upon an agent’s rejection recommendation implies that the agent will optimally recommend only acceptance for all offers in this range.

We characterize below the seller’s optimal response in two cases. The first is when \( \varepsilon > c / \alpha \), i.e., when the difference between the agent’s and the seller’s optimal cutoff offers is smaller than the seller’s uncertainty about the offer distribution. In this case, the agent’s information can be valuable to the seller. The second case involves instances when \( \varepsilon < c / \alpha \)—the agent’s distorted incentives are simply too large for the seller to learn anything valuable from his advice. We characterize the seller’s reaction in the former case assuming that the agent is being truthful. As will be seen, given that the agent is truthful, the seller’s probability of offer acceptance/rejection is weakly increasing with the agent’s recommendation to do either. In the latter case, we show that the seller’s best response is to ignore the agent. Because the seller can always ignore the agent if she wishes, which would result in a lower payout to the agent in the former case than if the seller listened, it is in the interest of the agent to truthfully reveal his preferences. In the latter
case, the seller will ignore the agent’s advice regardless, so the agent is indifferent between truthful and untruthful revelation.

(b) The seller knows that when \( \varepsilon > c/\alpha \), the agent has the incentive to truthfully reveal his preference regarding acceptance or rejection of the first-period offer. Any offer that the seller recommends rejecting, therefore, is no greater than \( \bar{p} - c/\alpha \). The seller follows such rejection advice because it implies the offer must be less than the her optimal cutoff price.

Likewise, the seller knows that any offer the agent recommends accepting must be at least \( \bar{p} - c/\alpha \), so an acceptance recommendation for a given \( p_1 \) implies \( \bar{p} \leq p_1 + c/\alpha \). Now, prior to observing either \( p_1 \) or the agent’s suggested action, the seller knows that \( \bar{p} - \varepsilon < \bar{p} < \bar{p} + \varepsilon \), because \( \varepsilon \in [-\varepsilon, \varepsilon] \). After observing an offer that the agent recommends accepting, then, the seller updates her beliefs over the value of the mean/median of the distribution to \( \bar{p} - \varepsilon < \bar{p} < \min[\bar{p} + \varepsilon, p_1 + c/\alpha] \). A uniform offer distribution implies that the seller’s best estimate of the mean offer (which is her optimal cutoff value with imperfect information) is as follows

\[
E\bar{p} = \begin{cases} 
\bar{p}, & \text{if } \bar{p} + \varepsilon < p_1 + \frac{c}{\alpha} \\
\frac{1}{2} \left[ \bar{p} - \varepsilon + p_1 + \frac{c}{\alpha} \right], & \text{if } \bar{p} + \varepsilon > p_1 + \frac{c}{\alpha} 
\end{cases}
\]

The seller’s optimal strategy is to accept any offers above this updated belief about the mean offer. Note first that when \( \varepsilon > c/\alpha \),

\[
\bar{p} + \varepsilon < p_1 + \frac{c}{\alpha} \Leftrightarrow p_1 > \bar{p} + \varepsilon - \frac{c}{\alpha} > \bar{p}.
\]

That is, if the first period offer is large enough so that an acceptance recommendation does not change the seller’s prior beliefs about the median of the offer distribution (because \( \bar{p} + \varepsilon < p_1 + c/\alpha \)), the offer must be above the seller’s corresponding cutoff level. The seller therefore accepts all such offers. For the case when a seller’s beliefs are affected by the
agent’s acceptance recommendation, the corresponding updated belief about the mean offer
implies the following decision rule for this case:

\[
\text{If } \bar{p} + \varepsilon > p_1 + \frac{c}{\alpha}, \text{ then accept } p_1 \text{ if } p_1 > E\bar{p} = \frac{1}{2} \left[ \bar{p} + \varepsilon + p_1 + \frac{c}{\alpha} \right].
\]

\[\Rightarrow \text{If } p_1 > \bar{p} + \hat{\varepsilon} - \left( \varepsilon - \frac{c}{\alpha} \right)\]

The seller rejects first-period offers below this level, despite the agent’s recommendation to accept.

Now when \( \varepsilon < c/\alpha \), the information distortion is larger in magnitude than is the sellers’ uncertainty over the true distribution. To show that it is optimal in this case for the seller to ignore the agent’s advice, we first characterize a seller’s optimal response when no agent is preset.

The seller receives her signal of the mean of the offer distribution \( \bar{p}_s = \bar{p} + \hat{\varepsilon} \) and then observes \( p_1 \). Given that \( \bar{p}_s \) is unbiased, it is the seller’s best guess as to the actual mean. Therefore, in the absence of credible information from the agent, the seller will want to accept \( p_1 \) if \( p_1 > \bar{p}_s \) and reject offers below this level.

Consider now the seller’s decision whether to listen to the agent when \( \varepsilon < c/\alpha \). As in the above case when \( \varepsilon > c/\alpha \), the seller will want to reject any offers that the agent recommends rejecting. In this case, however, the seller would always do that even if there were no agent. Why is this so? We showed above that with no agent input the seller will reject \( p_1 < \bar{p}_s \). The lowest possible realization of \( \bar{p}_s \) is \( \bar{p} - \varepsilon \), but this is still greater than \( \bar{p} - c/\alpha \) when \( \varepsilon < c/\alpha \).

Therefore, the agent’s rejection recommendations yield no benefit to the seller beyond what is contained in the seller’s own information. (This was not true when \( \varepsilon > c/\alpha \), because following the agent’s advice sometimes keeps an overly pessimistic seller from accepting too low an offer.) Furthermore, the agent will recommend accepting some offers that the seller would reject (and
rightly so) if she paid no attention to the agent. Thus, in terms of rejection recommendations, the seller is better off ignoring the agent’s advice.

A similar argument holds for agents’ acceptance recommendations. Intuitively, if agents’ rejection recommendations are of no value, their positive counterparts should be as well. To show this more formally, consider again a seller who ignores agent advice. Because she will accept first-period offers \( p_i > \overline{p} \), the agent could make two types of mistakes: accepting too low an offer (if \( \hat{\epsilon} < 0 \)), and rejecting a high offer (if \( \hat{\epsilon} > 0 \)). We already know that an agent’s rejection advice conveys no information to the seller, so any advice in the former case would only reinforce the seller’s mistaken acceptance. Thus they only way an agent’s advice could still be helpful is if \( \hat{\epsilon} > 0 \) and \( p_i > \overline{p} \), and an acceptance recommendation causes the over-optimistic seller to accept an offer she would have otherwise rejected. So let us consider the impact of an acceptance recommendation when \( p_i > \overline{p} \). Recall that such a recommendation only conveys information to the seller about the offer distribution if \( p_i + c/\alpha < \overline{p} + \epsilon \). Therefore the agent’s advice will not affect the behavior of any seller with private signal draws \( \hat{\epsilon} < p_i - \overline{p} + c/\alpha - \epsilon \). For those with higher draws, the updated beliefs about segment containing the median offer is \([\overline{p} - \epsilon, p_i + c/\alpha]\). This in turn leads to a cutoff rule of an seller accepting only those offers \( p_i > \overline{p} + c/\alpha - \epsilon \). Because \( \epsilon < c/\alpha \), however, such a cutoff rule would result in already over-optimistic sellers wrongly rejecting even more offers than they would if they ignored the agent and used the \( p_i > \overline{p} \) cutoff rule. Therefore it is not in the seller’s interest (regardless of her type) to listen to the agent when \( \epsilon < c/\alpha \).

Proof of Proposition 3.

(a) From Proposition 2 we know that given an agent’s advice, sellers will reject two sets of offers. The first is the set that the agent recommends rejecting, those \( p_i < \overline{p} - c/\alpha \). The second set is comprised of those offers that the agent recommends accepting, but where the
sellers’ private signal draws are high enough so that \( p_i < \bar{p} + \hat{\varepsilon} - \left( \varepsilon - 2c/\alpha \right) \). Any accepted offer must be greater than or equal to the larger of these two cutoff levels, otherwise it would have been either recommended for rejection by the agent or too low relative to the seller’s signal of the offer distribution.

Note that the private signal draw at which these two bounds are equal is \( \hat{\varepsilon} = \varepsilon - 2c/\alpha \).

We can write the probability that \( p_i \) is accepted as:

\[
\Pr(p_i \text{ accepted}) = \Pr(p_i \text{ accepted} | \hat{\varepsilon} \leq \varepsilon - \frac{2c}{\alpha}) \Pr(\hat{\varepsilon} \leq \varepsilon - \frac{2c}{\alpha}) + \Pr(p_i \text{ accepted} | \hat{\varepsilon} > \varepsilon - \frac{2c}{\alpha}) \Pr(\hat{\varepsilon} > \varepsilon - \frac{2c}{\alpha})
\]

Sellers with draws \( \hat{\varepsilon} \leq \varepsilon - 2c/\alpha \) reject \( p_i < \bar{p} - c/\alpha \) and accept all others. Thus such sellers accept \( p_i \) with probability

\[
\Pr(p_i \text{ accepted} | \hat{\varepsilon} \leq \varepsilon - \frac{2c}{\alpha}) = \frac{p_h - \left( \bar{p} - \frac{c}{\alpha} \right)}{p_h - p_i} = \frac{0.5(p_h - p_i) + \frac{c}{\alpha}}{p_h - p_i} = \frac{1}{2} + \frac{c/\alpha}{\theta}
\]

And the fraction of sellers with such draws is

\[
\Pr(\hat{\varepsilon} \leq \varepsilon - \frac{2c}{\alpha}) = \frac{\varepsilon - 2c/\alpha - \varepsilon}{2\varepsilon} = \frac{\varepsilon - c/\alpha}{\varepsilon}
\]

Now, sellers with \( \hat{\varepsilon} \in (\varepsilon - 2c/\alpha, \varepsilon] \) reject \( p_i < \bar{p} + \hat{\varepsilon} - (\varepsilon - c/\alpha) \). Evaluating the probability of acceptance at the endpoints of this region yields

\[
\Pr(p_i \text{ accepted} | \hat{\varepsilon} = \varepsilon - \frac{2c}{\alpha}) = \frac{p_h - \left[ \bar{p} + (\varepsilon - 2c/\alpha) - (\varepsilon - c/\alpha) \right]}{p_h - p_i} = \frac{1}{2} + \frac{c/\alpha}{\theta}
\]

\[
\Pr(p_i \text{ accepted} | \hat{\varepsilon} = \varepsilon) = \frac{p_h - \left[ \bar{p} + \varepsilon - (\varepsilon - c/\alpha) \right]}{p_h - p_i} = \frac{1}{2} - \frac{c/\alpha}{\theta}
\]

Because \( \hat{\varepsilon} \) is uniformly distributed and the probability of acceptance increases linearly in \( \hat{\varepsilon} \), the average probability of acceptance over the region is simply the average of the two probabilities at the endpoints, i.e., \( \frac{1}{2} \). The fraction of sellers with noise draws in this region is

\[
\Pr(\hat{\varepsilon} > \varepsilon - \frac{2c}{\alpha}) = \frac{\varepsilon - (\varepsilon - 2c/\alpha)}{2\varepsilon} = \frac{c/\alpha}{\varepsilon}
\]

Placing these components into the above expression yields:
\[
\Pr(p_i \text{ accepted}) = \left( \frac{1}{2} + \frac{c}{\alpha} \theta \right) \left( \frac{\varepsilon - c/\alpha}{\varepsilon} \right) + \frac{1}{2} \left( \frac{c/\alpha}{\varepsilon} \right)
\]

or simplifying,

\[
\Pr(p_i \text{ accepted}) = \frac{1}{2} + \frac{1}{\theta} \left[ \frac{c}{\alpha} - 1 \left( \frac{c}{\alpha} \right)^2 \right]
\]

(b) We derive the expected sales price of for a seller who hires an agent by again breaking the region of possible draws of the private signal into the two sections that depend on the accept/reject rule.

First, we analyze draws where \(\hat{\varepsilon} \leq \varepsilon - 2c/\alpha\). Any seller with draws in this range rejects any \(p_i < \bar{p} - c/\alpha\) and accepts any other offers. Thus the expected sales price for this range of noise draws, conditional upon receiving such a draw, is as follows.

\[
\begin{align*}
E\left(p_{\text{sale}} \mid \hat{\varepsilon} \leq \varepsilon - \frac{2c}{\alpha}\right) &= E\left(p_{\text{sale}} \mid p_i > \bar{p} - \frac{c}{\alpha}\right) \Pr\left(p_i > \bar{p} - \frac{c}{\alpha}\right) + E\left(p_{\text{sale}} \mid p_i < \bar{p} - \frac{c}{\alpha}\right) \Pr\left(p_i < \bar{p} - \frac{c}{\alpha}\right) \\
&= \frac{p_h + \left( \bar{p} - \frac{c}{\alpha} \right)}{2} \left( \frac{p_h - \left( \bar{p} - \frac{c}{\alpha} \right)}{p_h - p_i} \right) + \bar{p} \left( \frac{\bar{p} - \frac{c}{\alpha} - p_i}{p_h - p_i} \right)
\end{align*}
\]

Tedious algebra reveals

\[
E\left(p_{\text{sale}} \mid \hat{\varepsilon} \leq \varepsilon - \frac{2c}{\alpha}\right) = \bar{p} + \frac{1}{8} \theta - \frac{(c/\alpha)^2}{2\theta}.
\]

Now for \(\hat{\varepsilon} > \varepsilon - 2c/\alpha\): sellers with these draws reject any \(p_i < \bar{p} + \hat{\varepsilon} - (\varepsilon - c/\alpha)\) and accept higher offers. Because this cutoff offer depends on \(\hat{\varepsilon}\), we must first compute an expression for the expected sales price conditional on a particular private signal:

\[
\begin{align*}
E\left(p_{\text{sale}} \mid \hat{\varepsilon} > \varepsilon - \frac{2c}{\alpha}\right) &= E\left(p_{\text{sale}} \mid p_i > \bar{p} + \hat{\varepsilon} - \left( \varepsilon - \frac{c}{\alpha} \right) \right) \Pr\left(p_i > \bar{p} + \hat{\varepsilon} - \left( \varepsilon - \frac{c}{\alpha} \right) \right) \\
&\quad + E\left(p_{\text{sale}} \mid p_i < \bar{p} + \hat{\varepsilon} - \left( \varepsilon - \frac{c}{\alpha} \right) \right) \Pr\left(p_i < \bar{p} + \hat{\varepsilon} - \left( \varepsilon - \frac{c}{\alpha} \right) \right).
\end{align*}
\]

Similarly to above, we derive the expression for this expected price.
\[
E \left( p_{\text{sale}} \mid \hat{e} > e - \frac{2c}{\alpha} \right) = \bar{p} + \frac{1}{8} \theta - \frac{\left( e - \hat{e} - \frac{c}{\alpha} \right)^2}{2\theta}.
\]

We must integrate this expression over \( \hat{e} \in (e - 2c/\alpha, e] \) to compute the expected sales price (unconditional on \( \hat{e} \)) over this range.

\[
E \left( p_{\text{sale}} \mid \hat{e} > e - \frac{2c}{\alpha} \right) = \int_{e - \frac{2c}{\alpha}}^{e} \left[ \bar{p} + \frac{1}{8} \theta - \frac{\left( e - \hat{e} - \frac{c}{\alpha} \right)^2}{2\theta} \right] d\hat{e} = \bar{p} + \frac{1}{8} \theta - \frac{\left( e - \frac{c}{\alpha} \right)^2}{6\theta}.
\]

Evaluation of this integral yields

\[
E \left( p_{\text{sale}} \mid \hat{e} > e - \frac{2c}{\alpha} \right) = \bar{p} + \frac{1}{8} \theta - \frac{\left( \frac{c}{\alpha} \right)^2}{6\theta}.
\]

Using the expected sales prices from these two ranges, we can write the following expression for the expected sales price when a seller hires an agent.

\[
E(p_{\text{sale}}) = E \left( p_{\text{sale}} \mid \hat{e} \leq e - \frac{2c}{\alpha} \right) \Pr(\hat{e} \leq e - \frac{2c}{\alpha}) + E \left( p_{\text{sale}} \mid \hat{e} > e - \frac{2c}{\alpha} \right) \Pr(\hat{e} > e - \frac{2c}{\alpha})
\]

\[
E(p_{\text{sale}}) = \left( \bar{p} + \frac{1}{8} \theta - \frac{\left( \frac{c}{\alpha} \right)^2}{2\theta} \right) \left( - \frac{2c}{\alpha} - (e) \right) + \left( \bar{p} + \frac{1}{8} \theta - \frac{\left( \frac{c}{\alpha} \right)^2}{6\theta} \right) \left( \frac{e - \frac{2c}{\alpha}}{2e} \right).
\]

Simplifying gives the expression in the text.

(c) It is easy to compute the expected time-on-market using the probability that \( p_1 \) is accepted calculated above.

\[
t_{\text{sale}} = 1 \cdot \Pr(p_1, \text{accepted}) + 2 \cdot \left[ 1 - \Pr(p_1, \text{accepted}) \right] = 2 - \Pr(p_1, \text{accepted})
\]

\[
t_{\text{sale}} = 2 - \left\{ \frac{1}{2} + \frac{1}{\theta} \left[ \frac{c}{\alpha} - \frac{1}{e} \left( \frac{c}{\alpha} \right)^2 \right] \right\}.
\]

\[
t_{\text{sale}} = \frac{3}{2} \cdot \frac{1}{\theta} \left[ \frac{c}{\alpha} - \frac{1}{e} \left( \frac{c}{\alpha} \right)^2 \right].
\]
Figure 1. Temporal Structure of the Model

Period 0: Seller hires agent, puts house on market

Period 1: $p_1$ observed, Agent recommends rejection or acceptance of offer

Seller Accepts $p_1$

- Seller keeps $(1 - \alpha)p_1$
- Agent gets $\alpha p_1 - c$

Seller Rejects $p_1$

- Period 2: $p_2$ observed; Seller keeps $(1 - \alpha)p_2$
- Agent gets $\alpha p_2 - 2c$
Figure 2. Seller’s Responses to First-Period Offer

\[ p_1 = \bar{p} + \hat{\epsilon} - (\varepsilon - c / \alpha) \]

Seller’s indifference line when ignoring agent’s advice

Seller’s indifference line when heeding agent’s advice