Endogenous Sectoral Reallocation, Balance Sheet Quality, and Active Learning

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Outline

Model

Primitives

Active Learning
The economy is populated with a representative agent

The agent has utility over consumption and human capital

\[ u(C_t) = \rho \log(C_t) \]

There are 2 firms with capital accumulation processes

\[
\frac{dK_t^A}{K_t^A} = \left( \frac{I_t^A}{K_t^A} - \frac{\theta_t}{2} \left( \frac{I_t^A}{K_t^A} \right)^2 \right) dt - \delta dt + \sigma dZ_t^A
\]

\[
\frac{dK_t^B}{K_t^B} = \left( \frac{I_t^B}{K_t^B} - \bar{\theta} \left( \frac{I_t^B}{K_t^B} \right)^2 \right) dt - \delta dt + \sigma dZ_t^B
\]

\[
\frac{d\theta_t}{\theta_t} = \varphi_0 (\bar{\theta} - \theta_t) dt + \varphi_1 dZ_t^\theta
\]

\[
\text{corr} \left( dZ_t^A, dZ_t^\theta \right) = \rho \leq 0
\]

The mean-reverting \( \theta_t \) captures firm/balancesheet quality and will mechanically yield that firm A will rotate from “Value” to “Growth”
There are 2 state variables, \( x, \theta \). Value function

\[
V = \log(K_A + K_B) + F(x, \theta)
\]

\[
x = \frac{K_A}{K_A + K_B}
\]

HJB

\[
0 = \max_{i_t^A, i_t^B} \log(C) + V_A E(dK^A) + V_B E(dK^B)
\]

\[
+ \frac{1}{2} V_{A,A} (dK^A)^2 + \frac{1}{2} V_{B,B} (dK^B)^2 + V_\theta E(d\theta)
\]

\[
+ \frac{1}{2} V_{\theta,\theta} d\theta^2 + V_{\theta,A} d\theta dK^A
\]
\( \theta_t \) is not known but the agent forms expectations \( \tilde{\theta}_t = E_t(\theta_t) \). The agent learns about \( \theta_t \) using \( K_t^A \) as well as a signal \( e_t \); where \( \frac{de_t}{e_t} = \theta_t dt + \sigma_e d\tilde{Z}_t^e \). The agent uses the Kalman filter. Given \( \{\tilde{\theta}_0, \Theta_0\} \), the updating equations are

\[
\frac{d\tilde{\theta}_t}{\tilde{\theta}_t} = \vartheta_0 \left( \theta - \tilde{\theta}_t \right) dt + \Theta_t \left( \frac{1}{2} \left( \frac{I_t^A}{K_t^A} \right)^2 \right) d\tilde{Z}_t^A + \frac{1}{\sigma_e} d\tilde{Z}_t^e + \rho \vartheta_1 \frac{1}{2} \left( \frac{I_t^A}{K_t^A} \right)^2 d\tilde{Z}_t^A
\]

\[
d\tilde{Z}_t^A = -\frac{1}{\sigma} \left( \frac{dK_t^A}{K_t^A} - \left( \frac{I_t^A}{K_t^A} - \tilde{\theta}_t \left( \frac{I_t^A}{K_t^A} \right)^2 - \delta \right) dt \right)
\]

\[
d\tilde{Z}_t^e = \frac{1}{\sigma_e} \left( \frac{de_t}{e_t} - \tilde{\theta}_t dt \right)
\]

\[
\Theta_t = E \left[ (\theta_t - \tilde{\theta}_t)^2 \right] = E \left[ (E(\tilde{\theta}_t) - \tilde{\theta}_t)^2 \right] = E \left[ (\tilde{\theta}_t - E(\tilde{\theta}_t))^2 \right]
\]

\[
d\Theta_t = \left[ \psi_1^2 - 2\vartheta_0 \Theta_t - \left( \frac{1}{2} \left( \frac{I_t^A}{K_t^A} \right)^2 \left( \Theta_t \frac{1}{\sigma} + \rho \vartheta_1 \right)^2 + \frac{1}{\sigma_e^2} \Theta_t^2 \right) \right] dt
\]

\( \Theta_t \) decreases with \( I_t^A / K_t^A \) this may lead to over-investment as in Johnson (2007).