Reputational Dynamics and Political Careers

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I study a model of repeated elections with both symmetric learning about candidate ability and moral hazard. In this model, candidates choose how to allocate their resources between constituency service and policy work. Early in their careers, they devote excessive time to constituency service in an attempt to manipulate voter learning. Since voters use elections to select better candidates, incumbents become more confident of reelection over time and reduce the distortion in their effort allocations. I embed the basic model in a common agency framework to study seniority norms in legislative organization. The model organizes many of the stylized facts about elections and congressional organization, including retrospective voting, the incumbency advantage, the dynamics of effort allocation over a career, the importance of constituency service, and seniority norms in committee assignments.

1. Introduction

Competitive elections are traditionally thought to be crucial for keeping public officials accountable to citizens. The observation that incumbent members of Congress have an electoral advantage worth 7% to 10% of the two-party vote (Gelman and King, 1990; Levitt and Wolfram, 1997) seems to threaten this role of elections. Ansolabehere, Snyder, and Stewart (2000) show that about half of this advantage is due to a “personal vote” rather than to structural advantages for incumbents, and they show that this personal advantage increases with tenure.

A traditional explanation for this “personal vote” is that incumbents provide extensive constituency service, which builds up their vote margins (see, e.g., Fiorina, 1977). Indeed, Levitt and Snyder (1997) and Fiorina and Rivers (1989) find evidence that incumbents who provide more pork-barrel spending or have greater district presences are more successful in their reelection contests, and King (1991) finds that states whose legislators have larger budgets for constituency service have greater incumbency advantages. Cain, Ferejohn, and

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Fiorina (1987) show that voters have more favorable evaluations of, and are more likely to vote for, incumbents who provide constituency service. Cain, Ferejohn, and Fiorina (1987: 92–96) also find that freshmen and electorally vulnerable congressmen engage in more of these casework activities. Fenno (1978) finds similar dynamics in the “career in the district”; congressmen begin their careers in an “expansionist” phase, moving over time into a “protectionist” phase.

Recent empirical work by Ansolabehere and Snyder (2002) calls this account into question. They show that the trend of the incumbency advantage is common to congressional elections and all state-wide offices, ranging from senator and governor all the way to minor offices like auditor. This pattern strongly suggests a common cause for the changes in the incumbency advantage in all of these offices, and that an explanation that is specific to legislative politics is not satisfactory. How can this conclusion be squared with the substantial evidence that members of Congress use constituency service to increase their vote margins?

I construct a simple model of incumbent decision making and reelection that reconciles these observations. The model is based on Holmström’s (1999) career concerns model. In each of three periods, an incumbent politician takes some action that (stochastically) increases the utility of a representative voter. The incumbency advantage arises because voters are learning about the ability of incumbents—since they only reelect members when they believe they have high ability, the distribution of abilities drifts up due to selection effects. [See Zaller (1998) for simulations of a related model of selection.] Incumbents attempt to manipulate the learning process through their choice of actions. In equilibrium, voters are not fooled, and the reelection decision depends only on an unbiased signal of ability. This means the incumbency advantage (which is an average across many elections) is unaffected by these actions. At the same time, given the voter’s equilibrium expectations, variations in constituency service do lead to different reelection outcomes. In the equilibrium, voters expect a certain level of constituency service, and if a member falls short of this expectation, voters attribute this to low ability. Thus, within the model, variations in constituency service do account for variations in reelection, but it is a fallacy of composition to go further and say that the incumbency advantage is caused by the high average level of constituency service provision.

The key trade-off incumbents face is how to allocate their resources, particularly staff time and effort. Cain, Ferejohn, and Fiorina, (1987: 73) describe the many activities congressional staffers perform on behalf of constituents, including help preparing grant applications, arranging meetings, and helping with appeals. The average congressional staff has between four and five people doing casework fulltime (Cain, Ferejohn, and Fiorina, 1987: 60). Sixty percent of members maintain at least two district offices and 27% maintain at least three. Furthermore, members actively solicit additional casework: 85% advertise for casework. These activities help in the reelection contest, but the opportunity cost is less staff work supporting the member’s quest for better policy or for power in the chamber. Looking more closely at this trade-off lets me derive additional insights from the model.
First, this trade-off is resolved differently by incumbents at different stages in their careers. These dynamic results can be understood in the context of what a statistician would observe in a cross-sectional analysis of many electoral districts, each of which had a voter learning about a sequence of incumbents. He would find that (i) the probability a candidate wins an election is increasing with her tenure in office, (ii) that new politicians devote more resources to constituency service than do veteran politicians, even when both face the same future of reelection contests, and (iii) that, conditional on the incumbent’s initial reputation, more constituency service leads to better electoral outcomes.

The probability of reelection increases with tenure because the voter is selecting the best candidate at each election, so the expected ability of a candidate increases with each victory. It is important to recognize that the voter is not using incumbency as a voting cue; he observes the entire history of signals about the candidate, so he learns nothing from incumbency status. The statistician, who does not observe the history of voter utilities, does revise his beliefs when he conditions on incumbency status. Since the voter corrects his signal for the equilibrium level of the action, this learning is not affected by the equilibrium allocation to constituency service.

Effort devoted to constituency service decreases with tenure for a more subtle reason, as it is the result of two different effects: (i) incumbents who are believed to have higher ability devote less effort to constituency service, and (ii) the expected ability of politicians is increasing with their tenure in office. Since constituency service incentives come from the possibility of fooling the voter into having more favorable beliefs, high-ability legislators will not devote as many of their resources to constituency service. Their constituents already think the incumbent is good, so the marginal benefit of constituency service is low. Because the expected ability of an incumbent increases with each reelection, this decrease in constituency service with ability implies a decrease (on average) with tenure. Although the attention to constituency service declines over the career, this does not mean that the voters get less from more senior members—the improvement in average ability more than makes up for it.

Second, an important feature of the model is that it can say why constituency service is the incumbent’s preferred way to try to manipulate learning. Because the voter can more easily observe these tasks, they are the most efficient way to influence the voter’s posterior beliefs. This observability result also provides a rationale for the seemingly paradoxical preference of voters to unilaterally impose term limits, even as they continue to reelect incumbents. (More than 20 states passed laws imposing term limits on their federal representatives. These laws were struck down by the Supreme Court.) This preference cannot be rationalized as a willingness to tie one’s own hands to make sure other states do not return incumbents, since other states are not affected by unilateral term limits. In my model, voters might want term limits to commit to not rewarding the distortion of effort in favor of campaign activities.

Finally, implicit in the opportunity cost of constituency service is the idea that elections are not the only source of incentives for politicians—they also get incentives from their party leadership, legislative leaders, and interest groups.
Because of their better reputations, politicians with longer tenure are more responsive to incentives from interest groups and the party leadership. I study this effect by embedding the reelection model in a common agency framework, so I can discuss the role of seniority in the design of informative committees.

Starting with Gilligan and Krehbiel (1987, 1990), a large amount of literature has developed around the idea that legislatures organize themselves to facilitate information collection about the effects of potential policies. One way to do this is to assign members to committees that specialize in gathering information and reward the members when they produce information.

My results imply that in a legislature with relatively homogeneous policy preferences\(^1\), the floor gets a higher expected payoff if it delegates information gathering to a more senior member. The senior member is likely to be relatively immune to electoral pressure and consequently will be cheaper for the floor to influence. The intuition is simple. A freshman faces overwhelming incentives to engage in casework, and this work partially crowds out information gathering. Veterans, on the other hand, can afford to shift time and staff resources to information gathering. This provides a possible explanation for the use of seniority in determining committee leadership in Congress.

After a brief review of related theoretical literature, the rest of the article is organized as follows. Section 2 describes the model. Section 3 derives results on retrospective voting, the incumbency advantage, and the dynamics of constituency service effort. Section 4 considers a version of the model with a richer specification of voter preferences and information. Section 5 discusses committee assignments.

1.1 Related Literature

Lohmann (1998) and Persson and Tabellini (2000: chaps. 4 and 9) apply two-period versions of Holmström’s model. Lohmann shows that an incumbent has an incentive to distort policy choices in the direction preferred by a well-informed minority voting bloc. Persson and Tabellini consider an incumbent who must choose between using tax revenue to provide a public good or for private consumption. They show that reelection provides stronger incentives against corruption under district-based elections than under a party-list system. My work builds on theirs by adding richer dynamics, so that I can study political careers.

Several other articles study dynamic models of reelection incentives with pure moral hazard or moral hazard plus adverse selection. However, these previous agency-based models of reelection cannot provide a unified explanation of the stylized facts about incumbency and constituency service.

Austen-Smith and Banks (1989) and Ferejohn (1986) study pure moral hazard models of reelection. A critical assumption in these models is that all politicians are identical. This means that the voters can commit to arbitrary reelection rules. The analyst uses this freedom to specify a retrospective voting

\(^{1}\) By homogeneity, I have in mind the dimension of the issue space. For the “floor” to have well-defined preferences, we want to appeal to a median voter theorem.
rule that gives the incumbent maximal incentive to work. Since these rules are stationary, incumbent behavior is time invariant, and there are no dynamics of effort choices or of the incumbency advantage. In addition, Fearon (1999) points out that the optimal reelection rule in a model with a small amount of uncertainty about the incumbent’s type can be quite dissimilar to the optimal rule in the pure moral hazard case.

Banks and Sundaram (1998) consider a model with both adverse selection and moral hazard. In their model, each politician is term limited and can serve only two terms. They also find that effort declines with tenure, but in their model this is due entirely to the last period effect of the term limit. This two-period political lifetime also makes it impossible for their model to explain the empirically observed incumbency advantage, namely that incumbents do better than candidates who have the same future of electoral contests.

As Banks and Sundaram point out, there are technical problems in extending their work so politicians can serve more than two terms. In particular, there will typically not exist Markov equilibria with the property that incumbents are reelected exactly when the voters’ beliefs about the incumbent’s type exceed some cutoff. This failure of monotonicity makes the existence of any equilibrium problematic, and makes characterization of an equilibrium difficult. The main feature of my model that allows it to overcome this problem is that uncertainty is about productive ability rather than cost of effort, as in Banks and Sundaram. This allows the voter to expect a high payoff from an incumbent with a strong reputation, even though the cutoff reelection rule leads such an incumbent to exert little effort.

McKelvey and Riezman (1992) also study a model of a legislature with both an electoral stage and a committee system. In that article, a legislature designs a seniority-based committee system so that the voters will suffer a capital loss if they turn out an incumbent. That model is based on the distributive view of committees, while mine is based on the informational view. Since both approaches to committees are probably needed to understand actual legislatures, the approaches are complementary.

2. Model
2.1 Description of the Model
At each of three dates, \( t = 0, 1, 2 \), an incumbent politician holds office. She has total resources \( R \) that she must divide between constituency service, \( a \), and policy work, \( y \). (Think of these resources as staff time and effort.) The incumbent’s payoff for a period when she is in office and chooses the allocation \((a, y)\) is \( B + u(y) \). The benefit \( B > 0 \) represents all of the nonpolicy benefits of holding office—it could be an “ego rent,” that is, a direct, psychic benefit from holding office, or it could represent a monetary benefit from bribes or from the diversion of public funds. The policy utility is increasing, continuously
differentiable, strictly concave, and satisfies $\lim_{y \to 0} u'(y) = \infty$. The incumbent gets no direct utility from constituency service, but she may still engage in these activities to help her chances of reelection.

Since adding the constant $u(R)$ to these payoffs is an affine transformation, we can use the budget constraint to rewrite the incumbent’s payoff as $B - c(a)$, where $c(a) = u(R) - u(R - a)$ is the opportunity cost of devoting resources to constituency service. Our assumptions about $c$ imply that $c$ is increasing, continuously differentiable, strictly convex, and satisfies $c(0) = 0$ and $\lim_{a \to \infty} a c'(a) = \infty$. From now on, we will work directly with the opportunity cost function, and $u$ will not appear explicitly. Any politician not in office gets a payoff of zero for that date.

At the end of periods 0 and 1, a single voter chooses to reelect the incumbent or to elect a fresh candidate. The initial incumbent is called candidate 0, while the challenger at date $i$ ($i = 1, 2$) is called candidate $i$. Each politician has an unknown ability or talent, $\theta$. The prior beliefs are that $\theta \sim \mathcal{N}(0, \sigma_\theta^2)$, for both the date 0 incumbent and for any challenger. Uncertainty about $\theta$ will be symmetric throughout the game. The assumption of symmetric learning considerably simplifies the analysis, as there is no possibility of signaling. For many aspects of politician quality, this is a reasonable assumption. For example, a new legislator is unlikely to know how good she will be at negotiating with lobbyists and party leaders. The symmetry assumption is a bad one, however, for the candidate’s ideology, which is why I focus on issues related to ability.

In each period, the voter has utility $u_t = \theta + a_t + \epsilon_t$, where $\epsilon_t \sim \mathcal{N}(0, \sigma_\epsilon^2)$. The noise term, $\epsilon$, contains the effects on the voter’s utility of all factors not affected by the policymaker. This payoff is also the voter’s only signal about the incumbent. The $\theta$'s and $\epsilon$'s are mutually independent, and there is no discounting. Section 4 considers the more general case where the voter cares about both tasks.

I also assume that the true ability of an incumbent follows a random walk, whose innovations, $\eta$, have variance

$$\sigma_\eta^2 = \frac{(\sigma_\theta^2)^2}{\sigma_\theta^2 + \sigma_\epsilon^2}.$$  

This implies that the variance of beliefs about ability does not decline with tenure, highlighting the fact that the comparative statics results about effort and tenure are entirely due to the nonlinear relationship between reputation and reelection, and not due to increasing precision of the voter’s beliefs.

2.2 Information and Beliefs

Let $h_i^t$ be the history of signals produced by candidate $i$ up to date $t$. Write $\bar{\theta}_i^t$ for $\mathbb{E}(\theta_i^t|h_i^t)$, the mean of the voter’s posterior belief about the incumbent’s ability. Since information is symmetric at the start of the game, this also describes the evolution of the incumbent’s beliefs along the equilibrium path. However, off the path, the incumbent’s beliefs may differ, as she will condition on her actual
action rather than the one prescribed by the equilibrium. Denote the mean of candidate \( i \)'s belief about her own ability at date \( t \) by \( m_i^t \). When there is no chance of confusion, I omit the \( i \) and \( t \).

The standard formula for updating in the normal learning process [see DeGroot (1970: 166–168)] gives us a recursion for the posterior mean of the voter's belief, \( \bar{\theta}_i \), as a function of the signal and the expected action \( a^* \),

\[
\bar{\theta}_i = \lambda (u_{i-1} - a^*_{i-1}) + (1 - \lambda) \bar{\theta}_{i-1},
\]

(1)

where \( \lambda = \sigma_{\overline{\theta}}^2 / (\sigma_{\overline{\theta}}^2 + \sigma_{\overline{\theta}}^2) \). This equation says that the (public) estimate of the expected ability of an incumbent is a weighted average of the old estimate, \( \bar{\theta}_{i-1} \), and the new observation,

\[
u_{i-1} - a^*_{i-1} = \theta_{i-1} + \epsilon_{i-1}.
\]

Similarly, the incumbent's private belief is updated as

\[
m_i = \lambda (u_{i-1} - a_{i-1}) + (1 - \lambda) m_{i-1},
\]

where the action \( a_{i-1} \) is the one actually taken. The weights in these averages depend on the variances in an intuitive way: the more informative the new signal (low \( \sigma_{\overline{\theta}}^2 \)) is, the greater the weight on the new observation, and the more precise the old estimate of ability (low \( \sigma_{\overline{\theta}}^2 \)) is, the greater the weight on the old belief.

When we focus on statements about the equilibrium path, we will suppress the distinction between \( \bar{\theta} \) and \( m \), as they are equal on the path.

### 2.3 Definition of Equilibrium

We focus on symmetric Markov sequential equilibria, so we can write the incumbents’ effort choices as a sequence \( \{a^*_0, a^*_1(\cdot), a^*_2(\cdot)\} \), where \( a^*_i : \mathbb{R}^2 \rightarrow [0, \infty) \) gives the incumbent’s action as a function of her own and the voter’s posterior beliefs about ability for dates \( t = 1, 2 \). Similarly, the voter’s strategy is given by two functions, \( \rho_0 \) and \( \rho_1 \), where \( \rho_t : \mathbb{R} \rightarrow \{0,1\} \). The incumbent is reelected if and only if \( \rho_t(\bar{\theta}_{t+1}) = 1 \).

The incumbent’s decisions must maximize her payoff given the voter’s equilibrium election rules. Formally, this means that the action decisions must satisfy

\[
a^*_0 = \arg \max_a \mathbb{E} \left( \rho^*_0(\bar{\theta}) V(\bar{m}, \bar{\theta}) | a^*_0, a \right) - c(a),
\]

\[
a^*_1(m, \theta) = \arg \max_a \mathbb{E} \left( \rho^*_1(\bar{\theta}) \left( B - c \left( a^*_2(\bar{m}, \bar{\theta}) \right) \right) | a^*_1(\theta, \theta), m, \theta, a \right) - c(a),
\]

\[
a^*_2(m, \theta) = \arg \max_a -c(a),
\]

where

\[
V(m, \theta) = \max_a \mathbb{E} \left( B + \rho^*_1(\bar{\theta}) \left( B - c \left( a^*_2(\bar{m}, \bar{\theta}) \right) \right) | a^*_1(\theta, \theta), m, \theta, a \right) - c(a).
\]
Each of these expressions says that the incumbent politician maximizes the probability of getting reelected times the expected reward from returning to office minus the opportunity cost of the action. At date 2, there is no chance of getting reelected, so this simplifies to minus the cost of effort.

Given these action choices, the reelection rules must maximize voter utility, so $\rho_1^*(\theta) = 1$ if and only if
\[
\mathbb{E}(\theta + a_2^*(\theta, \theta)) \geq \mathbb{E}(a_2^*(0, 0)),
\]
and $\rho_0^*(\theta) = 1$ if and only if $W(\theta) \geq W(0)$, where
\[
W(\theta) = \theta + a_1^*(\theta, \theta) + \mathbb{E}(\rho_1^*(\tilde{\theta})(\tilde{\theta} + a_2^*(\tilde{\theta}, \tilde{\theta})) + (1 - \rho_1^*(\tilde{\theta}))(a_2^*(0, 0))).
\]

Both of these expressions say that the voter chooses whichever candidate offers the most (expected) output. The function $W$ accounts for the option values of candidates in the date 0 election; the voter knows he will get another chance to review whomever he elects.

Say that the voter uses a cutoff strategy if each $\rho_i$ is monotone, so $\theta > \theta'$ and $\rho_i(\theta') = 1$ imply that $\rho_i(\theta) = 1$.

3. Main Results

We begin by showing that there is an equilibrium in which the voter uses a simple retrospective voting rule. This is a crucial step in the development, as the voting rule will induce a strongly nonlinear incentive scheme.

The development is complicated by the presence of the normal cumulative distribution function (cdf) in the incumbent’s objective. Because this cdf is a convex function on part of its domain, we must be careful about second-order conditions. To ensure that the incumbent’s equilibrium action is characterized by first-order conditions, we assume the following:
\[
B \leq (\sigma_\theta^2 + \sigma^2_e) \cdot \sqrt{\frac{\pi e}{2}} \cdot c''(0) \quad \text{and} \quad c''' \geq 0.
\]

Substantively, this assumption says that the per period benefit to holding office is not too large relative to the voter’s uncertainty about the incumbent’s ability. The second derivative of the cost function appears in the upper bound for $B$ because “more” convexity of $c$ can help compensate for failure of concavity elsewhere in the objective. The proof of Proposition 1 shows that this assumption not only guarantees concavity of the incumbent’s objective function, but it also ensures that the incumbent’s action declines slowly enough for the voter to find a cutoff strategy optimal for the date 0 election.

**Proposition 1.** There is an essentially\(^3\) unique pure strategy equilibrium of the game. In that equilibrium, the voter uses a cutoff strategy, and, along the

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\(^3\) Equilibrium does not pin down the voter’s action when his beliefs make him indifferent between candidates. This event has probability zero.
equilibrium path, the action is a decreasing function of the posterior expectation of ability.

Proofs of all propositions are in the appendix, but it will be instructive to go over the main points here. Since period 2 ends the game and candidates cannot commit to effort levels before the election, the last incumbent will set \( a^*_2 \) to zero no matter what beliefs are. Given this, the voter will choose whichever candidate has higher \( \bar{\theta} \) in the date 1 election.

Next, we can characterize the incumbent’s action in period 1. She will choose the action that maximizes her utility, given the voter’s reelection rule. We first derive an expression for the probability of reelection given the incumbent’s action, which we then use to find a simple form for the incumbent’s first-order condition.

Given the voter’s belief about ability, \( \bar{\theta} \), the action expected by the voter, \( a^* \), the incumbent’s belief about ability, \( m \), and the actual action taken by the incumbent, \( a \), the incumbent expects the voter’s posterior will be normal with mean

\[
\lambda (\theta + a + \epsilon_1 - a^*) + (1 - \lambda) \bar{\theta}.
\]

By the law of iterated expectations, the expected value of this posterior mean is

\[
\bar{\theta} + \lambda (a - a^*) \equiv \lambda m + (1 - \lambda) \bar{\theta} + \lambda (a - a^*). \tag{3}
\]

The prior distribution of the posterior mean is normal, with the mean given by Equation (3); call its variance \( \sigma^2 \). Since the incumbent is reelected if and only if the posterior mean is greater than zero, the reelection probability is

\[
1 - \Phi \left( \frac{-\bar{\theta} - \lambda (a - a^*)}{\sigma} \right).
\]

This probability is increasing in \( a \). Intuitively, the incumbent’s action is a substitute for ability. Since the voter thinks he has corrected for this by subtracting \( a^* \), increases in the action fool the voter into thinking ability is high.

The incumbent solves

\[
\max_a B \left( 1 - \Phi \left( \frac{-\bar{\theta} - \lambda (a - a^*)}{\sigma} \right) \right) - c(a).
\]

The first-order condition for this problem is

\[-B \frac{\phi \left( \frac{-\bar{\theta} - \lambda (a - a^*)}{\sigma} \right)}{\sigma} - \lambda \phi \left( \frac{\lambda}{\sigma} \right) - c'(a) = 0.
\]

In equilibrium, the voter’s forecast of the incumbent’s action is correct and the voter’s belief agrees with the incumbent’s, so (using the symmetry of the normal density), we have the equilibrium condition

\[
\frac{\lambda B}{\sigma} \phi \left( \frac{\bar{\theta}}{\sigma} \right) = c' \left( a^*_1 \right).
\]
The appendix proves that the above analysis in fact describes equilibrium behavior.\footnote{For the most part, we focus cases where the equilibrium is interior. This does not affect the results, except that comparative statics results are weak inequalities when the equilibrium action is on the boundary of the feasible set.}

We can use the equilibrium condition to derive a crucial comparative statics result. Since the cost function is strictly convex, the marginal cost is invertible and we can write

$$a^*_i(\bar{\theta}) = (c^*)^{-1}\left(\frac{\lambda B}{\sigma} \phi\left(\frac{\bar{\theta}}{\sigma}\right)\right).$$

Since $\phi(\bar{\theta}/\sigma)$ is decreasing in the absolute value of $\bar{\theta}$, the equilibrium action in decreasing in the absolute value of the posterior mean. Since an incumbent is reelected only if her expected ability is nonnegative, the action is decreasing in expected ability along the equilibrium path.

This comparative statics result is quite intuitive—it says that the incumbent supplies more effort to constituency service the greater is the chance that a small increase in effort will change the election’s outcome. To see this, consider a small increase in effort, $\Delta a$. This change is unexpected by the voter, so he attributes the higher signals to high ability. This shifts up the distribution of the voter’s posterior beliefs by $\lambda \Delta a$. The change in the probability the incumbent wins is just the amount of probability mass moved across zero by this shift. This probability mass is approximately $\lambda \Delta a$ times the density at zero.

Thus the marginal benefit of effort is just the right-hand side of the equilibrium condition. This marginal benefit is decreasing in $|\bar{\theta}|$ because the normal density decreases as we move away from the mean, so less mass is moved across zero by a small shift in the distribution.

It turns out that the voter’s value of a candidate in the date 0 election is an increasing function of the candidate’s expected ability. This means that the voter again finds it optimal to reelect the date 0 incumbent if and only if her expected ability is greater than zero. Given this cutoff rule, an analysis just like the one above holds for the date 0 incumbent.

Now that we have a description of the equilibrium, we can present the main results of this section. The first result describes the dynamics of action choices.

**Proposition 2.** (i) At date 1, the incumbent’s action is decreasing in her tenure. (ii) Actions are declining over time: the date 0 incumbent takes a greater action than any date 1 incumbent, and every date 1 incumbent takes a greater action than any date 2 incumbent.

The action decreases over time because of the fixed time horizon. The decline of the date 1 action with tenure is more interesting. Even though they both face one reelection contest, veteran politicians devote fewer resources to constituency service (on average). Two effects contribute to this result. First, for a fixed
cutoff point in terms of the signal, an incumbent has a smaller marginal benefit from increasing the action. Second, the voter sets a lower signal cutoff point for veterans. Unlike the decline of all the actions over time, the decline with tenure is robust to the time horizon. In an infinite-horizon model, actions will decrease with tenure so long as the voter uses a cutoff reelection rule in every election.\(^5\)

This model fits well with the story told by Cain, Ferejohn, and Fiorina (1987), who argue that constituency service accounts for a substantial part of the incumbency advantage. They find that voters who have benefited from casework are more likely to have a high opinion of the congressman and are more likely to vote for him in the next election. They also find that incumbents who face poorer reelection prospects devote more time and staff resources to casework.

This relationship between tenure and the expected action would not hold in a model without learning. With only moral hazard, the voter is indifferent between the candidates in every election. This allows the voter to commit to any reelection rule he wants. He will use this freedom to induce the same action from every date 1 incumbent. In my model, in contrast, the voter has beliefs about the abilities of the two candidates and cannot commit to any reelection rule that does not optimally reflect these beliefs—in particular, he cannot commit to the rule that is optimal for controlling pure moral hazard.

Although attention to constituency service declines with tenure, the voter is willing to retain the incumbents because the increase in talent more than compensates for the reduced action. Because of this selection effect, the distribution of abilities among winners of the date 0 election is “better” than the distribution of abilities among challengers. This is the source of the incumbency advantage. Since candidates are ex-ante identical, we use a 50–50 chance of reelection as a baseline for measuring the incumbency advantage.

**Proposition 3.** At date 1, there is an incumbency advantage: The incumbent wins reelection with probability strictly greater than 1/2.

This incumbency advantage arises because the voter censors the distribution of abilities every time he selects the best candidate. Without this selection, the mean value of expected ability in a cross section of many districts would be zero. (This is just the law of iterated expectations.) With selection, however, this mean will drift up over time. This is because the voter is more likely to remove a low ability candidate than a high ability one.

Equilibrium action choices play no role in the incumbency advantage. The voter correctly forecasts the effort that the incumbent takes along the equilibrium path, so reelection is determined entirely by the evolution of expected

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5. This result is also robust to the assumed normality of beliefs about ability and noise. As long as signals and abilities satisfy the monotone likelihood ratio property, better incumbents will face a lower hurdle for reelection. Furthermore, increases in the expected ability lead to lower marginal benefits of effort (for fixed reelection hurdles) under the relatively mild assumption of log-concavity of the unconditional signal density.
ability, $\bar{\theta}$. This means that we could have derived the incumbency advantage in a pure learning model, without effort choices. However, such a model would be unable to explain the observed covariation between electoral vulnerability and effort in constituency service activities.

The best way to understand how these results interact is to think about the differences we would observe between a cross-sectional and a longitudinal data analysis. In each case the actual actions would decline with tenure. Of course, outcomes are more likely to be observed than are the actual actions. When only outcomes are observed, we would see a difference between the cross section and the panel. Because the increase in expected ability is greater than the decrease in effort, we should see a positive relationship between outcomes and tenure in cross-sectional data. Indeed, this is what leads to the incumbency advantage. On the other hand, in a panel setting, we see a fixed talent added to declining effort, so the outcomes produced by a fixed candidate will decrease with tenure. Finally, a dynamic analysis that ignored the fact that voters are selecting which incumbents to retain would find that outcomes were increasing with tenure.

4. Service Activities and Term Limitation

The previous section showed that incumbents will use constituency service provision to attempt to manipulate voter learning and that this activity is more prevalent from members with weaker reputations. However, the deck was stacked in favor of this conclusion, since voters only observed and cared about constituency service. In this section I show that the results do not depend on these simplifications, but also arise in a more general model in which the voter observes signals about, and cares about, both constituency service and policy work. This model also helps identify why constituency service activities are the efficient way for members to try to affect beliefs.

The central result is that incumbents have incentives to bias their effort allocations toward tasks that voters observe with relatively little noise. This is intuitive; the optimal updating rule puts more weight on a signal if its conditional variance is smaller, so the voter is more sensitive to easily monitored tasks. In this light, it makes perfect sense for incumbents to spend excessive time on casework. These tasks reveal more about the politician because they can be done independently, so there is no noise from the legislative bargaining process, and these tasks get results right away, while policy pays off in the future.

We will make these points in a generalization of the model of the previous sections. Assume that the incumbent takes two actions, $a_1$ and $a_2$. The voter’s current period utility is increasing in both actions, and he wants to reelect the incumbent only if he believes that her expected ability is greater than zero, the expected ability of a challenger. (The results extend easily to imperfectly correlated abilities for the two tasks, but the notation becomes more complex.)

The voter gets signals $s_1$ and $s_2$, where $s_i = \theta + a_i + \epsilon_i$, and $\sigma_{\epsilon_1}^2 < \sigma_{\epsilon_2}^2$. If the voter expects $(a_1^*, a_2^*)$ and the incumbent takes action $(a_1, a_2)$, then the prior distribution of the posterior mean is normal with mean
\( \theta + \lambda_1 (a_1 - a_1^*) + \lambda_2 (a_2 - a_2^*), \)

where

\[
\lambda_i = \frac{\sigma_i^2 \sigma_i}{\sigma_i^2 \sigma_i + \sigma_i^2 \sigma_i + \sigma_i^2 \sigma_i}.
\]

Since the noise term for signal 1 has lower variance than the noise for signal 2, \( \lambda_1 > \lambda_2 \). This means that shifting effort from task 2 to task 1 will raise the voter’s estimation of the incumbent’s talent. Since effort incentives come from the incumbent’s desire to fool the voter about ability, the incentive to work on task 1 is stronger than the incentive to work on task 2.

To see this more formally, consider the incumbent’s maximization problem:

\[
\max_{a_1, a_2} B \left( \frac{1 - \Phi \left( \frac{-\theta - \lambda_1 (a_1 - a_1^*) - \lambda_2 (a_2 - a_2^*)}{\sigma} \right)}{\sigma} \right) + v(a_1) + u(a_2)
\]

st \( a_1 + a_2 \leq R \).

The equilibrium actions must satisfy

\[
v'(a_1) = \mu - \frac{\lambda_1 B}{\sigma} \phi \left( \frac{\theta}{\sigma} \right)
\]

\[
u'(a_2) = \mu - \frac{\lambda_2 B}{\sigma} \phi \left( \frac{\theta}{\sigma} \right),
\]

where \( \mu \) is the Kuhn-Tucker multiplier on the resource constraint. Since \( \lambda_1 > \lambda_2 \), these imply that \( u'(a_2) > v'(a_1) \), which means that the incumbent has distorted her actions in favor of the task that is observed with less noise.

These results suggest that the voter may prefer not to use electoral incentives at all. This will be the case if the incumbent and the voter have common preferences over \( a_1 \) and \( a_2 \), and if the distortions in effort allocation due to different monitoring intensities are large. This fact can help us understand the demand for term limits.

To see how term limits might be valuable, consider a setting in which the incumbent must divide her time between campaign activity and policymaking. Assume that the incumbent’s preferences over effort allocations are exactly the same as the voter’s. This means there is no agency problem and optimal effort can be elicited by providing no electoral incentives at all. Unfortunately the only way to implement this incentive scheme is to commit not to reelect the incumbent—and a rational voter cannot commit to ignore anything he learns in the campaign. A term limit is an institutional mechanism for achieving

---

6. This logic is familiar from the multitask moral hazard model of Holmström and Milgrom (1991), which was extended to the career concerns setting by Dewatripont, Jewitt, and Tirole (1999).
low-powered incentives. Because incumbents cannot commit to ignore their reelection chances, voters must tie their own hands. The common criticism that term limits are unnecessary, since voters can always vote out an incumbent, has no force here. Even if voters announced that they would never reelect an incumbent, they would not get undistorted effort choices. The incumbent would know the plan was not credible.

It is important to note that term limits are a second-best policy, even with perfect incentive alignment. Providing correct effort incentives also means giving up the ability to select good types.

5. Committee Assignments

The implicit incentives from reelection concerns are only a part of any politician’s incentives. Both organized interest groups and the leaders of political parties try to influence the behavior of politicians. In this section I study the interaction of these incentives and reelection incentives. For concreteness, I focus on the case of a legislature delegating the responsibility to acquire information about the effects of a proposed policy change. My results provide one reason for legislatures to choose more senior members for such information gathering roles. The key to this application is that constituency service and information gathering are substitutes in the incumbent’s cost function. This is a quite reasonable assumption: if the incumbent is using her resources for casework, then she can’t use those resources to find out the effects of policies.

All of the results in this section are based on the following idea: The action for the voter (think of constituency service) competes with some other action (think of information gathering) for the incumbent’s time and resources. This competition means that an increase in the level of either activity raises the marginal cost of performing the other. Since we have seen above that veteran incumbents do less of the service activity than new members, they have a lower marginal cost of effort for the second activity. In turn, this makes the veteran a more attractive choice to be an agent for the party leadership or for the floor.

Gilligan and Krehbiel study a cheap-talk game in which a committee is the sender and the floor is the receiver. They find conditions under which information can be shared efficiently under different rules for voting and amendments on the floor and they analyze the incentives for the committee to invest in the skills needed to acquire information. Because I want to focus on the provision of incentives to acquire the information, rather than the efficiency with which it is transmitted, I analyze a simpler setting than Gilligan and Krehbiel. In particular, I assume that the information is verifiable, so talk is not cheap.

Consider the median voter in a legislature who must decide to pass or reject a piece of legislation. The effect of the bill depends on an unknown state of the world, \( \omega \). A member of the legislature can be assigned the task of trying to determine \( \omega \). Let \( V \) denote the unconditional expected utility of the median voter when he votes optimally, and let \( V(\omega) \) be the expected utility of the median voter when he knows the state is \( \omega \) and votes optimally. Assume that \( \mathbb{E}(V(\omega)) > V \).

A member can discover the true state of the world with probability \( p \) by exerting costly effort. Let \( b = \mathbb{E}(V(\omega)) - V \) be the benefit of information to the floor.
To formalize the intuition about substitute efforts, we let the date 1 incumbent take two actions: $a$, which enters the voter’s utility as before, and $p \in [0, 1]$, which is the probability that information is collected. To simplify the algebra, we specify the cost function directly as

$$c(a, p) = \frac{1}{2}a^2 + \frac{1}{2}p^2 + \gamma ap.$$  

Assume that $0 < \gamma < 1$, so a higher level of one action increases the marginal cost of the other action. The incumbent values money linearly, has limited liability, and has zero initial wealth.

If the floor offers a payment of $\beta$ conditional on receiving the information, then the incumbent maximizes

$$B(1 - \Phi\left(\frac{-\bar{\theta} - \lambda(a - a^*)}{\sigma}\right)) + \beta p - c(a, p).$$  

The solution to this problem is $(a^*(\beta, \bar{\theta}), p^*(\beta, \bar{\theta}))$, the *effort supply functions*. Assume that the floor always finds it optimal to induce a positive amount of information gathering, so $p^* > 0$.

We can take advantage of the specific form of the cost function to derive formulas for the optimal efforts, given any values for $\bar{\theta}$ and $\beta$. To conserve on notation, write $\psi(\bar{\theta})$ for $(\lambda B / \sigma)\phi(\bar{\theta} / \sigma)$. Consider first the case where the incumbent chooses $a^* > 0$ in equilibrium. The results of Section 3 immediately imply that, in the continuation game following the committee assignment, the equilibrium conditions are

$$\psi(\bar{\theta}) = a + \gamma p$$

$$\beta = p + \gamma a.$$  

Solve this system of equations to get

$$a^*(\beta, \bar{\theta}) = \frac{1}{1 - \gamma^2}\psi(\bar{\theta}) - \frac{\gamma}{1 - \gamma^2}\beta$$

$$p^*(\beta, \bar{\theta}) = \frac{1}{1 - \gamma^2}\beta - \frac{\gamma}{1 - \gamma^2}\psi(\bar{\theta}).$$  

At an interior solution, the probability that the member finds information for the floor, $p^*$, is increasing in the beliefs about ability, $\bar{\theta}$. This monotonicity result follows from the interaction of $a$ and $p$ in the cost function: higher ability means that the incumbent has less incentive to expend effort to impress the voter, so she is more willing to substitute toward effort for the floor.

Of course, these solutions are not the only ones we need to consider: for great enough $\bar{\theta}$, these equations imply that $a^*$ is negative, which is impossible. Thus we must also consider corner solutions. If the incumbent chooses $a^* = 0$, the equilibrium conditions are
These first-order conditions imply that \( p^*(\beta, \bar{\theta}) = \beta \). For this case to hold, we must have \( \psi(\bar{\theta}) \leq \gamma \beta \). The left-hand side of this inequality is decreasing, so the inequality is satisfied on some interval \([\hat{\theta}(\beta), \infty)\). (If the inequality holds for all \( \hat{\theta} \), then set \( \hat{\theta}(\beta) = 0 \).) This result tells us that incumbents with sufficiently strong reputations, \( \hat{\theta} \geq \hat{\theta}(\beta) \), will specialize completely in information gathering. The incentive to impress the voters is so weak that she completely ignores them.

Combining these results, we see that

\[
p^*(\beta, \bar{\theta}) = \begin{cases} 
\frac{1}{1-\gamma^2} \beta - \frac{\gamma}{1-\gamma^2} \psi(\bar{\theta}) & \text{if } \bar{\theta} < \hat{\theta}(\beta) \\
\beta & \text{otherwise.}
\end{cases}
\]

Notice that the partial derivative of \( p^* \) with respect to \( \beta \) is greater at an interior solution than it is at a corner solution. In both cases, an increase in \( \beta \) raises the incumbent’s marginal benefit for \( p \). The induced increase in \( p \), in turn, raises the marginal cost of \( a \). At an interior solution, \( a \) can be decreased, which in turn makes \( p \) even more attractive. At a corner solution, \( a \) cannot be decreased, so this multiplier effect is absent.

Given this function describing the incumbent’s behavior, the floor will choose \( \beta \) to maximize

\[
W(\beta, \bar{\theta}) = (b - \beta)p^*(\beta, \bar{\theta}),
\]

where \( b \) is the monetary value of the benefit. If there are multiple solutions to this problem, assume that the floor chooses the largest one. This corresponds to choosing the most informative equilibrium. Let \( \beta^*(\bar{\theta}) \) be the solution to this program, and let

\[
W^*(\bar{\theta}) = (b - \beta^*(\bar{\theta}))p^*(\beta^*(\bar{\theta}), \bar{\theta})
\]

be the maximized value function. The main result of this section is that \( W^* \) is increasing in \( \bar{\theta} \)—the floor gets a higher payoff when it deals with a higher ability incumbent. This is easy to see when the optimal incentive scheme induces an interior effort allocation. In that case, the first-order condition for the floor’s problem is

\[
\frac{1}{1-\gamma^2} (b - 2\beta + \gamma \psi(\bar{\theta})) = 0.
\]

Solve this to get

\[
\beta^*(\bar{\theta}) = \frac{1}{2}(\gamma \psi(\bar{\theta}) + b).
\]
Since $\psi(\bar{\theta})$ is decreasing in $\bar{\theta}$, the optimal incentive pay is decreasing in the incumbent’s reputation. The next result says that this intuition holds in the more general case.

Proposition 4. The floor’s equilibrium payoff, $W^*$, is nondecreasing in the incumbent’s expected ability, $\bar{\theta}$, and the optimal incentive pay, $\beta^*$, is nonincreasing in the expected ability. If $a^* > 0$ on a neighborhood of $\bar{\theta}$, then both functions are strictly monotone on that neighborhood.

In light of the earlier results, this means that the floor is better off dealing with more senior incumbents. This is because, when dealing with a more senior member, the floor can induce any fixed level of effort at less cost. This effect comes entirely from the fact that the incumbent is less concerned with impressing the voter, so she exerts less effort in that task. Because of the interaction of the two tasks in the cost function, this lowers her marginal cost of work for the principal. This means that the floor can offer less powerful explicit incentives to induce a fixed level of effort. This higher payoff is the key to the value of seniority in my model. The floor delegates tasks to members who can take the necessary time away from their constituents without putting their seats at too much risk.

Let me point out some implications for comparisons of senior and junior members. The floor prefers to deal with senior members because, all else being equal, they are cheaper. Being cheaper does not mean that senior members end up with fewer resources, however. If there is only one task, a senior ends up with something, while juniors are not dealt with at all. With multiple tasks to assign, the optimal assignment by the floor will have the property that seniors and juniors are equally costly. This will entail giving more tasks to the seniors.

6. Conclusion

In this article, I study a simple model of the dynamics of electoral careers with learning and moral hazard. Although constituency service does not cause the incumbency advantage, both phenomena arise as equilibrium responses to the possibility of voter learning. The incumbency advantage arises in the model because the voter censors the candidate’s distribution of ability at each election, and constituency service activities are used by incumbents in an attempt to manipulate voter learning. Although the model explains various stylized facts about congressional careers, there are more questions a model of these phenomena should be able to address. In particular, can this type of model explain why there is more constituency service and a greater incumbency advantage in the United States than in the United Kingdom (Cain, Ferejohn, and Fiorina 1987)? Can it explain why both have increased dramatically in the United States over the past half-century?

Ashworth and Bueno de Mesquita (2006, 2005) derive some comparative statics about the forces studied in this article, and argue that they do shed light...
on these comparative facts. In comparing the United States and the United Kingdom, the key idea is that U.K. voters cannot separate a vote for a favored member of Parliament (MP) from a vote for her party. A voter who likes his MP but does not want that MP’s party to form the government must choose which of these preferences to express with his ballot. Since many voters will go with their party preference, the link between an MP’s own reputation and her reelection chances is weaker than it is for a member of the U.S. House. Consequently the incumbency advantage is smaller (the censoring is less extreme) and there is less incentive for constituency service (a given improvement in reputation has less impact on the chance of reelection).

For the United States, we identify a potential explanation in the relationship between the partisan nature of district electorates and the importance of incumbent reputation in elections. If a district-level pivotal voter is biased toward one party, the link between incumbent reputation and reelection will again be weakened. Thus a move toward more competitiveness (on average) will lead to increased incumbency advantages and increased provision of constituency service. Ansolabehere and Snyder (2002) summarize empirical evidence suggesting that this kind of change has in fact occurred in American politics, which suggests that the model in this article may account for the correlated changes in the incumbency advantage and in constituency service.

Appendix A
A.1 Proof of Proposition 1

We prove the proposition by constructing an equilibrium with the desired properties, working back from the end of the game. We saw in the text that a date 2 incumbent takes zero effort and the voter elects the date 1 candidate with the best posterior.

Next we show that the date 1 action satisfies

$$\frac{\lambda B}{\sigma} \phi \left( \frac{\theta}{\sigma} \right) \leq c^\prime (a_1^*) \quad \text{and} \quad a_1^* \geq 0, \quad (A.1)$$

with complementary slackness. Since $c$ is strictly convex, this system has at most one solution, and it has at least one since $c^\prime (a) \to \infty$ as $a \to R$. The argument in the text shows that $a_1^*$ is the only candidate for a pure strategy equilibrium action at date 1. Now we show that it is in fact an equilibrium action by showing that, if the voter expects $a_1^*$, then $a_1^*$ is a best response by the incumbent.

The incumbent’s effort satisfies the following first-order condition

$$-B \phi \left( -\theta - \frac{\lambda (a - a^*)}{\sigma} \right) \left( \frac{\lambda}{\sigma} \right) - c^\prime (a) \leq 0 \quad \text{and} \quad a \geq 0, \quad (A.2)$$

with complementary slackness. The following lemma shows that the objective function is concave, which implies that the first-order condition derived above is sufficient for a maximum. Let the voter expect effort $a^*$. Then we have
Lemma 1. The function

\[ a \mapsto B \left( 1 - \Phi \left( \frac{-\theta - \lambda(a - a^*)}{\sigma} \right) \right) - c(a) \]

is concave.

Proof. Recall that \( d/(dx) \phi(x) = -x \phi(x) \) (Greene, 1990: 685). Using this, the second derivative is

\[ \frac{\lambda^2 B}{\sigma^2} \phi(-\alpha) - c''(a), \]

where

\[ \alpha = \frac{\theta + \lambda(a - a^*)}{\sigma}. \]

Straightforward calculation shows that \( \sigma^2 = \lambda \sigma^2_\theta \), so we can rewrite the second derivative as

\[ \frac{B}{\sigma^2_\theta + \sigma^2_\epsilon} \phi(-\alpha) - c''(a). \]

I claim the function

\[ x \mapsto x \phi(x) \]

is bounded between \(- (2\pi e)^{-1/2}\) and \((2\pi e)^{-1/2}\), so the upper bound for \( B \) in Equation (2) implies that

\[ \frac{B}{\sigma^2_\theta + \sigma^2_\epsilon} \phi(-\alpha) \leq \frac{1}{\sigma^3_\theta + \sigma^3_\epsilon} (\sigma^2_\theta + \sigma^2_\epsilon) \sqrt{\frac{\pi e}{2}} \frac{1}{\sqrt{2\pi e}} c''(0) \]

\[ = \frac{1}{2} c''(0) \]  

\[ \leq c''(a), \]

where the last inequality follows from \( c'' \geq 0 \). Thus

\[ \frac{B}{\sigma^2_\theta + \sigma^2_\epsilon} \phi(-\alpha) - c''(a) \leq 0, \]

and the function is concave.

Notice that the upper bound on \( B \) appears overly restrictive: we could have multiplied it by two and still carried out the analysis above. However, we will use the full force of the bound when we consider the date 0 action.
Finally, we prove the claim. Consider the function $x \mapsto x\phi(x)$. Clearly it is zero at $x = 0$, and it approaches zero as $x$ tends to either $\infty$ or $-\infty$, since the Gauss kernel tends to zero faster than any polynomial. The derivative is $\phi(x) - x^2\phi(x)$, so the critical points are 1 and $-1$. At each of these points, the absolute value of the function is

$$\frac{1}{\sqrt{2\pi}} e^{-(1/2)}.$$  

Now consider the problem of the voter in the date 0 election. It is clear that a date 0 incumbent whose date 1 posterior is less than zero is never reelected. This follows easily from the fact that effort, ability, and the option value for the date 1 election all decline as $\theta$ decreases from zero. Thus we need to determine the voting rule only when the posterior is nonnegative.

I next show that, under the assumption that $B$ has the upper bound from Equation (2), the voter will use a cutoff rule at date 0. Recall that $\rho_0(\tilde{\theta}) = 1$ if and only if $W(\tilde{\theta}) \geq W(0)$, where

$$W(\theta) = \theta + a^*_1(\theta) + \mathbb{E}\left(\rho_1^*(\tilde{\theta})(\tilde{\theta} + a^*_2(\tilde{\theta}, \tilde{\theta})) + (1 - \rho_1^*(\tilde{\theta}))(a^*_2(0, 0))\right).$$

The voter’s expected value of a candidate with expected ability $\theta$ is the sum of three terms: the expected ability, the date 1 action, and an option value for the date 1 election. The difference in date 1 efforts is given by

$$(c')^{-1}\left(\frac{\lambda B}{\sigma} \phi\left(\frac{\tilde{\theta}}{\sigma}\right)\right) - (c')^{-1}\left(\frac{\lambda B}{\sigma} \phi(0)\right),$$

which, by the fundamental theorem of calculus and the inverse function theorem, is

$$\int_0^\theta \frac{\lambda B}{\sigma} \phi(x) \frac{dx}{c''}.$$  

The analysis from Equation (A.5) shows that the integrand has an absolute value less than one. Thus the integrand is greater than $-1$, so

$$\int_0^\theta \frac{\lambda B}{\sigma} \phi(x) \frac{dx}{c''} > -\int_0^\theta dx,$$

or

$$a^*(\theta) - a^*(0) > -\theta,$$

which says the voter’s date 1 expected payoff is increasing in $\theta$. Since the option value is clearly increasing in $\theta$, the voter’s value of the incumbent is increasing in the posterior on the incumbent’s ability. This implies the voter will return the incumbent if and only if the posterior is greater than zero.
Finally, we show that there is an equilibrium for the date 0 action. Recall that $V(m, \bar{\theta})$ is the value of being a date 1 incumbent when beliefs are $m$ and $\bar{\theta}$. Since $\bar{\theta} = m + \lambda(a - a^*)$, the date 0 incumbent solves

$$\max_a \int \rho_0^*(\tilde{\theta}) V(\tilde{\theta} - \lambda(a - a^*), \tilde{\theta}) \phi\left(\frac{\tilde{\theta} - \lambda(a - a^*)}{\sigma}\right) d\tilde{\theta} - c(a).$$

The first-order condition for this maximization is

$$\int -\rho_0^*(\tilde{\theta}) V_1(m, \bar{\theta}) \phi\left(\frac{\tilde{\theta} - \lambda(a - a^*)}{\sigma}\right) d\tilde{\theta}$$

$$+ \int \rho_0^*(\tilde{\theta}) V(m, \bar{\theta}) \phi\left(\frac{\tilde{\theta} - \lambda(a - a^*)}{\sigma}\right) \left(\frac{\tilde{\theta} - \lambda(a - a^*)}{\sigma}\right) d\tilde{\theta} \leq c'(a)$$

and $a \geq 0$, with complementary slackness. We make two observations. First, notice that $V(m, \bar{\theta}) \leq 2B$, since the maximum utility in each period is $B$. Second, the envelope theorem implies that

$$V_1(m, \bar{\theta}) = \frac{\lambda B}{\sigma} \phi\left(\frac{\theta_1 + \lambda(a_1 - a_1^*)}{\sigma}\right) > 0.$$

With these observations, we can mimic the proof of Lemma 1 to show that the first-order condition characterizes the optimum. Notice that, for any $x = (\tilde{\theta} - \lambda(a - a^*))/\sigma$,

$$\frac{\lambda}{\sigma} \int_0^\infty (V(m, \bar{\theta}) \phi(x) x - \lambda V_1(m, \bar{\theta}) \phi(x)) d\tilde{\theta} \leq \frac{\lambda}{\sigma} \int_0^\infty V(m, \bar{\theta}) \phi(x) x d\tilde{\theta}$$

$$\leq 2\frac{\lambda B}{\sigma} \int_0^\infty x \phi(x) d\tilde{\theta} = 2\frac{\lambda B}{\sigma} \phi\left(\frac{-\lambda(a - a^*)}{\sigma}\right).$$

Now an argument like that of Lemma 1 implies the objective is concave.

A.2 Proof of Proposition 2

(i) Since the voter will never reelect an incumbent whose ability is less than zero, every reelected candidate will have expected ability at least zero, and almost all will have ability strictly greater than zero. Thus Proposition 1 implies that almost every reelected incumbent will exert less effort than a new candidate.

(ii) Recall that the date 1 action is maximal for an incumbent with $\bar{\theta} = 0$, who takes an action equal to

$$(c')^{-1}\left(\frac{\lambda B}{\sigma} \phi(0)\right),$$
while the date 0 incumbent takes action equal to

\[
(c')^{-1} \left( \frac{\lambda}{\sigma} \int_0^\infty V(\tilde{\theta}) \phi \left( \frac{\tilde{\theta}}{\sigma} \right) \cdot \frac{\tilde{\theta}}{\sigma} d\tilde{\theta} \right).
\]

Since the inverse marginal cost function is increasing, we need only show that

\[
\frac{\lambda}{\sigma} \int_0^\infty V(\tilde{\theta}) \phi \left( \frac{\tilde{\theta}}{\sigma} \right) \cdot \frac{\tilde{\theta}}{\sigma} d\tilde{\theta} > \frac{\lambda B}{\sigma} \phi(0).
\]

I claim that \(V(\tilde{\theta}) > B\) for all \(\tilde{\theta}\). To see that this will prove the proposition, notice that

\[
\frac{\lambda}{\sigma} \int_0^\infty V(\tilde{\theta}) \phi \left( \frac{\tilde{\theta}}{\sigma} \right) \cdot \frac{\tilde{\theta}}{\sigma} d\tilde{\theta} > \frac{\lambda B}{\sigma} \int_0^\infty \phi \left( \frac{\tilde{\theta}}{\sigma} \right) \cdot \frac{\tilde{\theta}}{\sigma} d\tilde{\theta} = \frac{\lambda B}{\sigma} \phi(0).
\]

Finally, I prove the claim. Consider an incumbent at date 1. One feasible strategy is to set \(a = 0\), for an immediate payoff of \(B\). Because the normal distribution gives every nondegenerate interval positive probability and there is always a sufficiently high signal that will lead to reelection, this do-nothing strategy has a payoff strictly greater than \(B\). The optimal strategy must have a payoff of at least this.

**A.3 Proof of Proposition 3**

The date 1 incumbent wins reelection if and only if the posterior mean of the voter’s assessment of her ability is at least zero. There are two cases to consider: the incumbent won office as a challenger in the date 0 election, or the incumbent was the date 0 incumbent. In the first case, the incumbent wins with probability 1/2, by the martingale property of beliefs. We need only show that the probability of reelection in the second case is greater than 1/2.

So consider an incumbent who won the date 0 election as an incumbent. If there were no date 0 election, the date 0 incumbent would win the date 1 election with probability 1/2. (This is the martingale property of beliefs again.) We will show that the distribution of date 1 expected abilities for the date 0 incumbent, conditional on her winning the date 0 election, strictly first-order stochastically dominates the unconditional distribution of her date 1 expected abilities. Since the reelection rule is monotone, this will prove the proposition.

We will use the following concepts. Let \(x\) and \(y\) be two random variables that have a strictly positive, smooth joint density function \(f\). Say \(x\) and \(y\) are affiliated\(^7\) if \(\partial \log f(x, y)/\partial x \partial y \geq 0\). Affiliation is a strong form of positive correlation—it means that \(x\) and \(y\) are positively correlated conditional on

---

\(^7\) See Milgrom and Weber (1982) for a treatment of affiliated random variables.
any rectangle. If \( x \) is a random variable with density \( f \) and \( z \) is a random variable with density \( g \), then \( x \) dominates \( z \) in the monotone likelihood ratio (MLR) order\(^8\) if, for all \( x > z \),

\[
\frac{f(x)}{g(x)} > \frac{f(z)}{g(z)}.
\]

This is a strong notion of “larger” for distributions; in particular, it implies that the distributions conditional on any interval are ordered by first-order stochastic dominance.

**Theorem 1.** (Milgrom, 1981). Let \( x, y \) be affiliated random variables, and let \([a, b]\) and \([c, d]\) be intervals with \( a \geq c \) and \( b \geq d \), at least one strict. Then the distribution of \( x \) conditional on \( y \in [a, b] \) MLR dominates the distribution of \( x \) conditional \( y \in [c, d] \).

Since likelihood ratio dominance implies first-order stochastic dominance, this theorem will prove the proposition, once we show that \( \theta_1 \) and \( \theta_2 \) are affiliated.

The date 0 posterior mean is

\[
\bar{\theta}_1 = \lambda(\theta + \epsilon_0),
\]

and the date 1 posterior (ignoring the election) is

\[
\bar{\theta}_2 = \lambda(\theta + \eta + \epsilon_1) + (1 - \lambda)(\theta + \epsilon_0).
\]

Both of these distributions are mean 0 Gaussians, and their covariance is

\[
\text{cov}(\bar{\theta}_1, \bar{\theta}_2) = \lambda^2(1 - \lambda)(\sigma_\theta^2 + \sigma_\epsilon^2).
\]

Thus their covariance matrix is strictly positive and they are affiliated. \( \blacksquare \)

### A.4 Proof of Proposition 4

We will use the following results about optimization problems.\(^9\) Consider the problem

\[
\max_{x \in \mathcal{X}} f(x, \theta),
\]

where \( \theta \) is a parameter drawn from some parameter space \( \Theta \subset \mathbb{R} \). Consider two possible choices, \( x > x' \). The incremental return to increasing the choice from \( x' \) to \( x \) is \( f(x, \theta) - f(x', \theta) \). Say that \( f \) has increasing differences in \( (x, \theta) \) if, for all \( x > x' \), the incremental return is increasing in \( \theta \).

---

8. See Milgrom (1981) for technical details and several applications to economics.
9. See Milgrom and Roberts (1990) for a discussion of these results.
Theorem 2. (i) (Topkis, 1978) Assume that $f$ has increasing differences in $(x, \theta)$, and that there exists at least one solution to Equation (A.6) for all $\theta$. Then, for each $\theta$, the set of solutions to Equation (A.6) has a greatest element, $x^*(\theta)$, and $x^*$ is an increasing function of $\theta$.

(ii) (Edlin and Shannon, 1998) Fix $\theta > \theta'$. Assume in addition that $f$ is differentiable in $x$ at $(x^*(\theta), \theta)$ and $(x^*(\theta'), \theta')$, at least one of these points is interior, and that $\partial f/\partial x(x^*(\theta), \theta) > \partial f/\partial x(x^*(\theta'), \theta')$. Then $x^*(\theta) > x^*(\theta')$.

A measurable function $f$ is absolutely continuous\(^{10}\) if $f$ is differentiable almost everywhere, and $f$ can be written as the definite integral of its almost-everywhere derivative. We need the following version of the envelope theorem. Let $V(\theta) = \max_{x \in X} f(x, \theta)$ be the value function.

Theorem 3. (Milgrom and Segal, 2002) Suppose that $f(x, \cdot)$ is absolutely continuous for all $x \in X$. Suppose also that there exists an integrable function $b : \Theta \to \mathbb{R}_+$ such that $|\partial f/\partial \theta(x, \theta)| \leq b(\theta)$ for all $x \in X$ and almost all $\theta$. Then $V$ is absolutely continuous. Suppose, in addition, that $f(x, \cdot)$ is differentiable for almost all $x \in X$. Then

$$V(\theta) = V(0) + \int_0^\theta \frac{\partial}{\partial \theta} f(x^*(s), s) \, ds.$$  

We first prove the results about $W^*$. For any value of $\beta$, $p^*$ is continuous in $\theta$, and it has a uniformly bounded right-hand derivative, so it is absolutely continuous. Furthermore, the almost-everywhere derivative of $p^*$ is absolutely integrable. Thus the envelope theorem implies that $W^*$ is absolutely continuous and that, for $\bar{\theta} > \bar{\theta}'$,

$$W^*(\bar{\theta}) - W^*(\bar{\theta}') = \int_{\theta'}^{\bar{\theta}} (b - \beta^*) \frac{\partial}{\partial \theta} p^*(\beta^*, \tilde{\theta}) \, d\tilde{\theta}.$$  

Now the result follows since the integrand is nonnegative, and is strictly positive on any neighborhood where $a^* > 0$.

Next, we prove the results about $\beta^*$. We must show that $W(\beta, \bar{\theta}) = (b - \beta)p^*(\beta, \bar{\theta})$ has increasing differences in $(\beta, -\bar{\theta})$. Fix $\beta > \beta'$. By the fundamental theorem of calculus for absolutely continuous functions,

$$W(\beta, \bar{\theta}) - W(\beta', \bar{\theta}) = \int_{\beta'}^\beta -p^*(\tilde{\beta}, \bar{\theta}) + (b - \tilde{\beta}) \frac{\partial}{\partial \beta} p^*(\tilde{\beta}, \bar{\theta}) \, d\tilde{\beta}.$$  

The first term of the integrand is strictly decreasing in $\tilde{\beta}$. The second term is constant in $\tilde{\beta}$ except for points where $\tilde{\theta} = \tilde{\theta}(\tilde{\beta})$, at which it jumps down. Thus the incremental return is strictly decreasing in $\tilde{\theta}$. This means that $\beta^*(\bar{\theta})$ is decreasing in $\bar{\theta}$.

\(^{10}\) See Section 5.4 of Royden (1988) for an introduction to absolute continuity.
Finally, if $a^*$ is strictly positive on some open set, then $p^*$ and $W$ are differentiable on that set, so the comparative statics conclusions are strict.

References


