Estimating Sequential Search Models using MPEC

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Abstract

The dynamic programming approach of Weitzman (1979) has recently been used in several papers estimating consumer sequential search (Kim et al., 2010; Ghose et al. 2012; Honka and Chintagunta, 2014; Chen and Yao, 2014). The popularity of this method owes to its ability to exactly characterize the sequence of choices made by consumers. However, in order to characterize this sequence, knowledge of reservation values is required. Reservation values are values that make consumers indifferent between searching and stopping at each stage in their search process and are solutions to structural equations relating consumer utilities and their search costs. Inverting these equations to solve for reservation values for each draw of the structural parameters makes estimation of the model using nested fixed point (NFXP) methods computationally challenging. In this paper, we follow Su and Judd (2012) and show how to recast the optimal sequential search problem as a mathematical program with equilibrium constraints (MPEC). Instead of computing reservation values for each draw, this method augments the likelihood function with these reservation values and imposes the structural equation defining reservation values as a constraint. The main advantage of this method is that it only requires checking this equation once, without solving it, thereby significantly reducing computational time.

Method (preliminary)

In the optimal sequential search model of Weitzman (1979), Pandora faces a set of closed boxes and must decide whether or not to continue searching, and if so, which box to open next. By opening a box \( i \), Pandora observes a reward \( u_i \sim F_i(u_i) \) and must pay a search cost \( c_i \). Let Pandora’s preferences \( u(\theta) \) and search costs \( c(\theta) \) be parameterized by the vector \( \theta \). A fallback

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option $u_0$ is available that Pandora can collect if she decides not to search or if no sampled reward exceeds $u_0$. Weitzman (1979) provides the solution to the sequential search problem that Pandora faces. This solution dictates that Pandora will first rank the set of closed boxes in order of their reservation values and then proceed to sample them in this order. Reservation values are defined as the reward level that the consumer would have to have in hand before opening box $i$ to make her indifferent between searching it or not. The reservation value of box $i$ is defined as the solution to the following structural equation

$$c_i = \int_{z_i}^{\infty} (u_i - z_i)dF(u_i)du_i$$

(1)

At some stage in Pandora’s search process, let $S$ denote the set of opened boxes, and $\tilde{S}$ the set of closed boxes. When Pandora stops searching, suppose the set of opened boxes is given by $S^e$ and the set of closed boxes by $\tilde{S}^e$. Since the fallback option is always available, assume that \{0\} $\in S$ and \{0\} $\in S^e$, so that it is always searched. Given a reward structure and reservation values, the following strategy due to Weitzman (1979) characterizes her optimal search:

1. (Selection Rule): If a search is to be made, the box with the highest reservation value should be searched next.

   *Formally:* When Pandora searches box $i \in \tilde{S}$, it must be that
   $$z_i \geq \max_{j \in \tilde{S}} z_j$$
   (2)

2. (Stopping Rule): Search should terminate when the maximum sampled reward exceeds the reservation value of any closed box.

   *Formally:* When Pandora searches box $i \in \tilde{S}$, it must be that
   $$z_i \geq \max_{j \in S} u_j$$
   (3)

   When Pandora stops searching, for all boxes $i \in \tilde{S}^e$ that Pandora does not open,
   $$z_i \leq \max_{j \in S^e} u_j$$
   (4)

3. (Choice Rule): Once the consumer stops searching, she will choose the box that revealed the highest reward.

   *Formally:* If Pandora chooses box $i^* \in S^e$, then
   $$u_{i^*} \geq \max_{j \in S^e} u_j$$
   (5)

These search rules characterize Pandora’s conditional choice probability. Putting all of these conditions together, the probability $P_{i^*|S^e}$ that Pandora opens set $S^e$ and chooses box $i^*$ is given by
The log-likelihood function is given by

\[ LL(\theta) = \sum_{i^*} \sum_{S^e} d_{i^* S^e} \log P_{i^* S^e} \]  \hspace{1cm} (6)

where \( d_{i^* S^e} = 1 \) if Pandora opened boxes in \( S^e \) and chose \( i^* \). Recall that \( P_{i^* S^e} \) depends on \( \theta \) since rewards and search costs are parameterized by \( \theta \).

Next, we describe the optimization procedure following NFXP, and then contrast it with the MPEC method.

**Nested Fixed Point (NFXP)**

The parameter \( \theta \) is estimated by maximizing the log-likelihood function

\[ \max_{\theta} LL(\theta) \]  \hspace{1cm} (7)

Maximizing the likelihood to determine \( \theta \) involves evaluating \( P \), which requires knowledge of the reservation value of each closed box faced by Pandora. To compute these reservation values, for each draw of \( \theta \), one needs to solve for reservation utilities by inverting equation (1). Using these reservation utilities, the conditional choice probability can be evaluated and the log-likelihood computed. The maximization thus consists of an outer loop and an inner loop. In an outer loop, the optimization package searches over the space of \( \theta \) to maximize \( LL(\theta) \). In the inner loop, for a given draw of the parameter vector \( \theta \), one solves the structural equation (1) for \( z \), in order to evaluate the choice probability \( P \) and compute \( LL(\theta) \). In other words, the structural equation in (1) has to be solved accurately for each draw of the parameters, which may be computationally demanding. This observation motivates our extension of the MPEC method to the estimation of sequential search models.

**Mathematical Program with Equilibrium Constraints (MPEC)**

We follow Su and Judd (2012) and augment the log-likelihood function with the reservation value \( LL(\theta, z) \) and use the structural equation in (1) as a constraint. More formally,

\[ \max_{\theta, z} LL(\theta, z) \]

\[ \text{s.t. } c = \int_{z}^{\infty} (u - z) dF(u) du \]  \hspace{1cm} (8)

By rewriting the maximization problem as a constrained optimization problem, for each draw of \( \theta \), one only needs to evaluate the structural equation (1) once and not solve it. This feature of
the optimization problem may significantly reduce the computational demands of search model estimation. In addition, Su and Judd (2012) show that the two problems are mathematically equivalent and thus will lead to the same parameter estimates.
References

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