Problem Set 8
Intertemporal Choice and Review

Due Tuesday, May 29 at 1:30pm in class

1. Milton is a young boy whose parents will give him an allowance of $5 today and an allowance of $5 tomorrow. Suppose Milton’s preferences over spending today, \( x_1 \), and spending tomorrow, \( x_2 \), are represented by the utility function \( U(x_1, x_2) = \ln x_1 + \frac{1}{2} \ln x_2 \). For parts (a) through (c), do the following: (i) find how much money Milton spends in each period, (ii) find how much money Milton saves or borrows in the first period, and (iii) graph Milton’s budget set, his endowment bundle, his demanded bundle, and a rough sketch of the indifference curve containing his demanded bundle.

(a) Suppose there is no credit market, and also, that Milton’s parents take back any money that Milton doesn’t spend in each period (that is, he cannot save any money).

(b) Continue to assume there is no credit market, but now suppose that Milton’s parents allow Milton to save some or all of his allowance from period 1 if he wishes.

(c) Now suppose that Milton’s parents decide to set up a credit market with interest rate \( r = 0.1 \) so that he can borrow or lend as much as he wants at this interest rate.

(d) What interest rate \( r \) should Milton’s parents pick if they want Milton to spend the same amount of money in period 2 as he spends in period 1? Is Milton strictly better off under this interest rate than he was in part (a)?

2. Consider an infinitely lived consumer who makes 1 dollar each year, with preferences over spending in each year represented by the utility function

\[
U(x_0, x_1, ...) = \sum_{t=0}^{\infty} \beta^t \ln(x_t)
\]

Suppose there is a constant interest rate \( r \) at which the consumer can lend or borrow. Suppose \( \beta = \frac{9}{10} \) and \( r = \frac{1}{9} \).

Hint: \[
\sum_{t=0}^{\infty} \frac{a}{(1+r)^t} = a \left( 1 + \frac{1}{r} \right)
\]

(a) What is the present value of the consumer’s lifetime wealth? Your answer should be a number (use the hint).
(b) What is the present value of the consumer’s lifetime expenditure? Your answer should include $x_t$ for all $t = 0, 1, 2, \ldots$

(c) Write down the consumers budget constraint (Hint: It will be “answer for (b)” $\leq$ “answer for (a)”).

(d) Write down the consumer’s utility maximization problem.

(e) Find an equation that relates $x_t^*$ to $x_{t+1}^*$ for any $t \geq 0$.

(f) Is consumption increasing, decreasing or constant over time?

(g) Find $x_t^*$ for all $t = 0, 1, 2, \ldots$. Your answer should be a number for each $t$. (Hint: use your answers to parts (c) and (e) and the hint at the beginning of the question.)

(h) Suppose the consumer is promised $\$100$ in 38 years from today. How does the consumer’s consumption change in each year? (You don’t need to solve for the consumer’s new consumption in each year, just say whether it increases, decreases or stays the same for each period $t$...absolutely no math is necessary).

3. Define and/or explain the significance of each of the following terms. Shoot for about a sentence per term. (You may find it useful to think about each term more in depth than what you actually write and to think of other important terms not on this list as a way to prepare for the final exam. Also, if you mindlessly copy definitions from the book or your notes this exercise will probably be a waste of time.)

- Budget set
- Opportunity cost
- Monotone preferences
- Strictly convex preferences
- Marginal rate of substitution
- Utility function
- Optimal choice
- Corner solution
- Demand
- Income elasticity
- Price elasticity
- Income effect
- Substitution effect
- Law of demand
- Equivalent variation, compensating variation or consumer surplus (pick one)
- Market demand
- Endowment economy
- Feasible allocation
• Pareto efficient allocation
• Walrasian equilibrium
• First Theorem of Welfare Economics
• Second Theorem of Welfare Economics
• Externality
• Coase Theorem
• Endowment effect
• Anchoring, i.e., coherent arbitrariness
• Insurance market
• Risk averse
• Diminishing marginal utility of wealth/income
• Credit market
• Discount factor
• Interest rate
• Present value
• Sophisticated/naive