1. Suppose there are two goods. The consumer’s demand for good 1 is \( x_1^*(p_1, p_2, m) = \sqrt{mp_2(1 - \frac{1}{2}p_1)}. \)

(a) Write down an equation for the inverse demand function \( (p_1 = \ldots). \)
(b) Plot the (inverse) demand curve when \( p_2 = 1 \) and \( m = 1. \)
(c) For the rest of the problem, assume \( p_1 = \frac{1}{2}, p_2 = 1 \) and \( m = 1. \) What is the
own-price elasticity of good 1.
(d) What is the income elasticity for good 1?
(e) What is the income elasticity for good 2?

2. Suppose a consumer has preferences represented by the utility function \( u(x_1, x_2) = x_1^\alpha x_2^{1-\alpha}, \) where \( 0 < \alpha < 1. \) Let \( m \) be his income and \( p_1 \) and \( p_2 \) be the prices of good 1 and good 2 respectively.

(a) What do we know about the preferences represented by this utility function? Specifically, does he prefer more of good 1 to less of good 1? Does he prefer more of good 2 to less of good 2? How do bundles consisting of only good 1 or only good 2 compare to bundles with some of both goods?
(b) Write the consumer’s utility maximization problem.
(c) Solve for the demand functions, \( x_1^*(p_1, p_2, m) \) and \( x_2^*(p_1, p_2, m) \) (these will also depend on \( \alpha).\)
(d) What is the own-price elasticity for good 1?
(e) What is the cross-price elasticity for good 1 with respect to good 2? Is good 2 a substitute or complement for good 1, or neither?
(f) What is the income elasticity for good 1? Is good 1 an inferior good or a normal good?
(g) For the rest of the problem, assume \( \alpha = \frac{1}{2}. \) Suppose \( m = 10, p_1 = 2 \) and \( p_2 = 4. \) What is the demanded bundle? Graph the budget constraint and label the demanded consumption bundle.
(h) Suppose now the price of good 1 increases to \( p_1 = 4. \) What is the demanded bundle. Draw the new budget constraint and label the new demanded consumption bundle on the same graph from part (g).
(i) Calculate the substitution effect and income effect of this price change, and show them on your graph.

3. Suppose Quan has preferences represented by the utility function \( u(x_1, x_2) \), where \( x_1 \) denotes donations given (by Quan or anyone else) to his favorite charity, and \( x_2 \) denotes Quan’s own spending on all other goods. An anonymous donor is thinking about matching funds Quan donates to the charity at rate \( r \); for every dollar Quan donates to the charity, the anonymous donor will donate \( r \) dollars.

(a) Quan’s budget constraint can be written as \( px_1 + x_2 \leq m \). Write down an expression for \( p \) in terms of \( r \). Explain why this is the price to Quan of the charity getting an additional dollar donation.

(b) Suppose Quan’s demand for dollars given to his favorite charity is \( x_1^*(p, m) = 1000 - 1000p \). Write an equation for Quan’s inverse demand function and graph it.

(c) Consider the effect of raising the matching rate \( r \) from \( r = 1 \) to \( r = 2 \). Is this a price increase or price decrease of \( x_1 \)? By how much do total donations to the charity change? By how much do Quan’s donations to the charity change?

(d) What value of \( r \) would maximize the amount of money Quan himself donates to the charity? What is the elasticity of demand for \( x_1 \) with respect to \( p \) corresponding to this optimal value of \( r \)? Why must this be the case?

(e) If the charity wants to maximize total donations and the anonymous donor will only give through this matching rate, would the charity ever want to limit how high \( r \) is?

(f) If we no longer assume the demand function given, but preferences over \( x_1 \) and \( x_2 \) are complete, transitive, monotone and strictly convex, is it possible that the charity would ever want to limit how high \( r \) is? Why or why not?

4. Suppose Bethany has \( m \) to spend on alcoholic beverages, \( x_1 \), and nonalcoholic beverages, \( x_2 \), per week. The price of alcoholic beverages is \( p_1 \) per unit and the price of nonalcoholic beverages is \( p_2 \) per unit. The government is considering taxing her consumption of alcoholic beverages with a value tax \( t \geq 0 \). Consider the following utility functions:

(a) \( u(x_1, x_2) = x_1 + x_2 \)

(b) \( u(x_1, x_2) = \min\{x_1, x_2\} \)

(c) \( u(x_1, x_2) = x_1x_2 \)

For each utility function,

(i) Find the demand function for alcoholic beverages as a function of \( p_1, p_2 \), and \( m \).

(ii) Suppose \( m = 100, p_1 = 1 \) and \( p_2 = 2 \), and find the substitution and income effects for any value tax \( t \) on alcoholic beverages.
(iii) Find the revenue the government raises by taxing alcoholic beverages at rate \( t \) given the income and prices used above. What is the maximum amount of revenue the government can raise?

5. Suppose for a particular good \( i \) we know that the Slutsky own-price elasticity \( \epsilon_{i,i}^S = -1 \). Whether good \( i \) is a Giffen good will depend on the income elasticity of good \( i \), \( \epsilon_{i,m} \) (which we know must be negative to have a chance at a Giffen good) and the income share for good \( i \), \( S_i \).

(a) Write down an inequality that tells us for any given income share \( S_i \), what must be true of \( \epsilon_{i,m} \) for good \( i \) to be a Giffen good.

(b) Fill in the following table using your answer from part (a).

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<th>( S_i )</th>
<th>( \epsilon_{i,m} )</th>
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(c) Write down an example of a good you might expect to be inferior. Would you expect this good to have a large income share?