1. Consider the three-state Potts model

\[ H = -J \sum_{ij} S_i \cdot S_j \]  

(1)

where the spin at each site \( S_i \) can point to one of three directions in the two-dimensional plane: \((1,0), (-\frac{1}{2}, \frac{\sqrt{3}}{2}),\) and \((-\frac{1}{2}, -\frac{\sqrt{3}}{2})\).

Use the molecular-field (Bragg-Williams) approximation, find the temperature of the phase transition and the order of the phase transition.

2. In Landau theory, the tricritical point is the point \((T_0, P_0)\) where two coefficients \( a \) and \( b \) in the expansion of the free energy

\[ F(m) = F_0(T, P) + a(T, P)m^2 + b(T, P)m^4 + c(T, P)m^6 + \cdots \]  

(2)

vanish: \( a(T_0, P_0) = b(T_0, P_0) = 0 \). Expanding \( a \) and \( b \) in Taylor series around \((T_0, P_0)\), assuming \( c(T_0, P_0) > 0 \), show that there is a line of first-order phase transition, which terminates at \((T_0, P_0)\) and becomes a second order phase transition.

Across the first-order phase transition, \( m \) has a finite jump. Investigate the behavior of the jump when one approach the tricritical point along the line of the first order phase transition (i.e., how does the jump scales as a function of the distance to the tricritical point)

3. Compute the specific heat of the one-dimensional Ising chain

\[ H = -J \sum_{i=1}^{N} S_i S_{i+1}, \quad S_i = \pm 1, \quad N \gg 1 \]  

(3)

Determine the asymptotics of the specific at low and high temperatures.