**Local Finite-Amplitude Rossby Wave Activity as a Diagnostic for Wave Breaking**

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**Introduction**

Synoptic wave breaking not only induces transport and irreversible mixing of materials in the mid-latitude atmosphere, but it also has a close association with blocking, a local and persistent large-amplitude anomaly of pressure that can divert weather systems and cause extreme weather. To improve weather prediction, we need a metric that can identify such phenomena from observations and provide insights into the governing large-scale dynamical processes. We generalize the notion of finite-amplitude wave activity proposed by Nakamura and Zhu (2010) [1] to a function of both longitude and latitude, which can diagnose such anomalous weather regimes.

**Local Finite-Amplitude Wave Activity**

We define the local finite-amplitude wave activity \( \tilde{A}^i(x, y, t) \) as

\[
\tilde{A}^i(x, y, t) = \int_{y'} q(x, y + y', t) - Q(y, t) \, dy'
\]

where \( q(x, y, t) \) is the instantaneous meridional displacement of a vorticity contour line through its corresponding equivalent latitude circle \( Q(y, t) = q(x, y, t) \). The line integral domain is shown by the dashed lines in the conceptual diagram.

The zonal average \( \overline{\tilde{A}^i} \) of \( \tilde{A}^i(x, y, t) \) recovers the hybrid Eulerian-Lagrangian finite-amplitude wave activity \( A^i(x, y, t) \) proposed in [1], which obeys the non-acceleration theorem.

In the conservative limit, the evolution of \( \tilde{A}^i \) can be expressed in terms of the advective flux \( \tilde{F}_{\text{advec}} \) and the generalized Eliassen-Palm (E-P) flux \( \tilde{F}_{\text{E-P}} \)

\[
\frac{\partial \tilde{A}^i}{\partial t} + \nabla \cdot (\tilde{F}_{\text{advec}} + \tilde{F}_{\text{E-P}}) = 0,
\]

where

\[
\tilde{F}_{\text{advec}} = (\nabla \times q) \tilde{A} + \frac{1}{2} \int_0^1 \nabla q \cdot \hat{n} \, dy'
\]

and

\[
\tilde{F}_{\text{E-P}} = \frac{1}{\nu} (\omega^2 - u \frac{\partial q}{\partial y} - v \frac{\partial q}{\partial x})
\]

with \( u \) and \( v \) being respectively the zonal and meridional velocities, \( q(x, y, t) = \delta(y, t) + A^i(x, y, t), u(x, y, t) = u(x, y, t) - u(y, t) \), \( v(x, y, t) = v(x, y, t) - v(y, t) \), and \( Q(y, t) = \int_{-L}^{0} q(y', t) \, dy' \).

At the onset of wave breaking, the local finite-amplitude wave activity \( \tilde{A}^i \) increases at both the regions of overturning and cut-off.

**Barotropic Decay Experiment**

A barotropic decay model prescribed in [3] with an additional single-mode wave \((m, n) = (2, 3)\), where \( m \) is the zonal wave number and \( n \) is the meridional wave number, produces a zonally asymmetric flow.

1. There is a substantial difference between the spatial distribution of \( \tilde{A}^i \) and Impulse-Casimir wave activity \( A_{IC} \) [4], as shown in the barotropic decay experiment, captures the wave breaking process.
2. The vorticity contours overturn cyclonically at the poleward flank and anticyclonically at the equatorward flank of the zonal mean jet. \( \tilde{A}^i \) has a large amplitude over the regions of contour overturning, while \( A_{IC} \) has finer-scale fragments that resemble the complicated structure of vorticity.
3. Merging vortices are displayed as a single structure in \( \tilde{A}^i \) but separate structure in \( A_{IC} \).
4. \( \tilde{A}^i \) grows subsequently where wave activity flux converges (\( \nabla \cdot \tilde{F} < 0 \)).

**Blocking Impact on Superstorm Sandy**

The blocking high that steered the Superstorm Sandy in-shore to New Jersey is shown to be associated with cyclonic wave breaking at the upper levels [5]. We generalize \( \tilde{A}^i \) to analyze waves on pressure surfaces by using the quasi-geostrophic potential vorticity (QGPV) instead of the absolute vorticity \( \omega \) as a tracer (read [6] for details).

The poleward intrusion of low QGPV vortex (at 300° E) and its split on Oct 30 that steered Sandy westward are well-captured by the distribution of \( \tilde{A}^i \).

**Conclusion**

1. The local finite-amplitude wave activity \( \tilde{A}^i \), as shown in the barotropic decay experiment, captures the wave breaking process.
2. The tendency of \( \tilde{A}^i \) is determined by the sum of advective flux and generalized E-P flux.
3. Compared with the Impulse-Casimir wave activity, the structure of \( \tilde{A}^i \) upon breaking vortices is less complex than that of the I-C wave activity \( A_{IC} \) is thus a better statistical tool for wave-breaking events.
4. We show that \( \tilde{A}^i \) based on QGPV on 24hPa-surface indicates the break-up of a persistent block that steered hurricane Sandy in-shore.

**Cited References**