Midterm Exam: Mortgage Modification Arithmetic

Consider a homeowner/worker/consumer whose mortgage obligates him to pay \( x \) per year for \( T > 5 \) years in order to occupy his house. Denote time \( t = 0, 1, 2, \ldots \), in years, and assume that this homeowner has incomes \( \{y_0, y_1, \ldots\} \). Let \( c \) denote the present value of what the homeowner can spend on things other than his mortgage.

Using a constant annual real interest rate \( r \) to future discount cash flows, the present value budget constraint for the consumer equates the combination of non-mortgage spending \( c \) and mortgage spending \( x \) to the present value of income:

\[
\frac{c}{1 + r} + \frac{x}{(1 + r)^2} + \ldots + \frac{x}{(1 + r)^T} = y_0 + \frac{y_1}{1 + r} + \frac{y_2}{(1 + r)^2} + \ldots
\]

(1) Draw the above budget constraint in the \([y_0, c]\) plane, holding future incomes constant.

**Budget Set without Modification**

The horizontal axis measures borrower income during the base year, from highest to lowest.

Note that the budget constraint has slope equal to 1 (slope equal to -1 in the \([-y_0, c]\) plane).

(2) The U.S. government has a “mortgage modification” program that reduces
homeowners’ mortgage payments during years 1 – 5 to be 31 percent of their income \( y_0 \).
If the original mortgage payment \( x \) were already less than \( 0.31y_0 \), then mortgage payments are left at \( x \). Write down a formula for the modified mortgage payment \( m \) in years 1 – 5.

\[
m = \min\{x, 0.31y_0\}
\]

This says that mortgage payments are 31% of \( y_0 \) or \( x \), whichever is less. Thus, if \( x < 0.31y_0 \), then \( m = x \) and the payment is unmodified.

(3) Write down the present value budget constraint for a homeowner that recognizes the possibility of such modifications.

Only the payments for years 1 – 5 are modified, so we adjust our notation to separate those payments from the payment for years 0, 6, 7, 8, …, \( T \):

\[
c + x + \left[ \frac{x}{1+r} + \frac{x}{(1+r)^2} + \ldots + \frac{x}{(1+r)^5} \right] + \frac{x}{(1+r)^6} + \ldots + \frac{x}{(1+r)^T} = y_0 + \frac{y_1}{1+r} + \frac{y_2}{(1+r)^2} + \ldots
\]

Now replace the \( x \)’s in brackets with the modified payment \( m \), using the formula from (3):

\[
c + x + \left[ \frac{1}{1+r} + \frac{1}{(1+r)^2} + \ldots + \frac{1}{(1+r)^5} \right] \min\{x, 0.31y_0\} + \frac{x}{(1+r)^6} + \ldots + \frac{x}{(1+r)^T} = y_0 + \frac{y_1}{1+r} + \frac{y_2}{(1+r)^2} + \ldots
\]

Finally, collect the \( c \) and \( y_0 \) terms together:

\[
c + \left[ \frac{1}{1+r} + \frac{1}{(1+r)^2} + \ldots + \frac{1}{(1+r)^5} \right] \min\{x, 0.31y_0\} - y_0 = \frac{y_1}{1+r} + \frac{y_2}{(1+r)^2} + \ldots - x - \frac{x}{(1+r)^6} - \ldots - \frac{x}{(1+r)^T}
\]

The slope of this constraint depends on whether \( x < 0.31y_0 \). If it is, then the slope in the \([y_0, c]\) plane is one – as it is without modification – because no modification occurs with incomes that large. If \( x > 0.31y_0 \), then “min” term is \( 0.31y_0 \) and the slope is

\[
1 - \left[ \frac{1}{1+r} + \frac{1}{(1+r)^2} + \ldots + \frac{1}{(1+r)^5} \right] 0.31
\]

(4) Draw the budget constraint in the \([y_0, c]\) plane, holding future incomes constant. Is it possible for a household to reduce its income and increase its spending?
For large incomes \( y_0 \), the slope is 1 (slope equal to -1 in the \([-y_0,c]\) plane) as it is without modification. At small incomes, the slope is

\[
1 - \left[ \frac{1}{1+r} + \frac{1}{(1+r)^2} + \ldots + \frac{1}{(1+r)^k} \right] 0.31
\]

which depends on the value for \( r \). At \( r = 0 \), the slope is \( 1 - (5*0.31) = -0.55 \). At \( r = 0.05 \), the slope is \(-0.33\). At \( r = 0.10 \), the slope is \(-0.18\). For all of these \( r \) values, the slope is negative. So we conclude that mortgage modification changes the slope from positive to negative, and a negative slope means that a household can spend more by earning less!

(5) Suppose that homeowners work only for the purpose of having money to spend \( c \). What do you think the mortgage modification program does to their income in period zero?

A number of homeowners have a strong incentive to earn nothing for just one year, so that they can live in their house for free for the next 5!

(6) The labor income tax discourages work effort.

Uncertain. Effort produces income, which is taxed, and therefore the substitution effect involves less effort. But, depending on the supply properties of effort and, more important, what the
government does with the money, the wealth effect can go in the opposite direction.

(7) *If President Obama were to increase the labor income tax, that would reduce the “Compensation of Employees” shown in the National Accounts.*

“Compensation of employees” is the employee income *before* taxes are deducted. Employee income tends to fall from the substitution effect, although it might not fall if the scarcity of employees sufficiently increased wage rates. Employee income tends to increase from the wealth effect (of which there would be one only to the extent that Obama used the revenue to buy goods and services rather than transfers), although it might not if employers had to reduce wages a lot to taken on the extra employees.