Consider a homeowner/worker/consumer whose mortgage obligates him to pay \( x \) per year for \( T > 5 \) years in order to occupy his house. Denote time \( t = 0, 1, 2, \ldots \), in years, and assume that this homeowner has incomes \( \{y_0, y_1, \ldots \} \). Let \( c \) denote the present value of what the homeowner can spend on things other than his mortgage.

Using a constant annual real interest rate \( r \) to future discount cash flows, the present value budget constraint for the consumer equates the combination of non-mortgage spending \( c \) and mortgage spending \( x \) to the present value of income:

\[
c + x + \frac{x}{1+r} + \frac{x}{(1+r)^2} + \ldots + \frac{x}{(1+r)^T} = y_0 + \frac{y_1}{1+r} + \frac{y_2}{(1+r)^2} + \ldots
\]

(1) Draw the above budget constraint in the \([y_0, c]\) plane, holding future incomes \( \{y_1, y_2, \ldots \} \) constant.

(2) The U.S. government has a “mortgage modification” program that reduces homeowners’ mortgage payments during years 1 – 5 to be 31 percent of their income \( y_0 \). If the original mortgage payment \( x \) were already less than 0.31\( y_0 \), then mortgage payments are left at \( x \). Write down a formula for the modified mortgage payment \( m \) in years 1 – 5.

(3) Write down the present value budget constraint for a homeowner that recognizes the possibility of such modifications. [Hint: replace some of the \( x \)’s in the consumer budget constraint with the formula for \( m \) you found above]

(4) Draw the budget constraint in the \([y_0, c]\) plane, holding future incomes constant. Is it possible for a household to reduce its income and increase its spending?

(5) Suppose that homeowners work only for the purpose of having money to spend \( c \). What do you think the mortgage modification program does to their income in period zero?

(6) Without government programs like this, would mortgage lenders modify loans (that is, reduce the borrowers payment below the originally contracted amount \( m \))? If so, would the lenders modify differently than your derived in part (2)?