The **final examination on May 31** may test topics from any part of the course, but the emphasis will be on topic after the first three homework assignments, which were covered in the midterm.

**Topics from the latter part of the course (covered in homework assignments 4-7) include:**

*Subgame-perfect eqms in extensive games with perfect info where players choose a number* (first-order conditions, boundary conditions, discontinuities).

*Finding equilibria in 2×3 strategic-form games*, searching among the possible supports.

Osborne section 7.6: allowing chance moves (0="chance"). Osborne section 7.1: simultaneous moves in extensive games.

**Repeated games:** Discounted Average Value, recursive characterization of a subgame-perfect equilibrium on social states.

**Imperfect information extensive games:** information sets (or information states) to indicate decision nodes that the player cannot distinguish; strategies, normal representation in strategic form, belief probabilities and move (action) probabilities, Bayes consistency of beliefs, sequential rationality of strategies, sequential equilibrium.

**Bayesian games** with two actions and utility functions that satisfy increasing differences, equilibrium strategies that switch at a cutoff type (or pivotal type), discrete & continuous uniform examples.

You might be asked to do sensitivity analysis in a parameter of the game. (E.g.: "How would your answer change if this number were changed to ...?")

There will probably be four questions on the exam. You might be asked to put your answers to questions 1&2 in one blue book and your answers to questions 3&4 in another blue book.

You are allowed to bring one sheet of notes (8.5"x11") for use in the exam.

Numerical answers may be expressed as ratios of whole numbers, without reducing.
Econ 20700. Assignment 7. (not to be handed in)

1. In Chapter 9, do exercise 282.1.

2. Consider a two-person game where player 1 chooses T or B, and player 2 chooses L or R. When they play this game, player 2 also knows whether her type is A or B. The players' utility payoffs \((u_1, u_2)\) depend on their actions and on player 2's type as follows:

<table>
<thead>
<tr>
<th>2'sType = A</th>
<th>2'sType = B</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>R</td>
</tr>
<tr>
<td>T</td>
<td>4,0</td>
</tr>
<tr>
<td>B</td>
<td>0,4</td>
</tr>
<tr>
<td>L</td>
<td>0,2</td>
</tr>
</tbody>
</table>

(a) Suppose first that, in this game, player 1 thinks that 2's type is equally likely to be A or B. Find a Bayesian equilibrium.

(b) How would the Bayesian equilibrium change in a game where player 1 thinks that player 2 has probability 1/6 of being type A, and has probability 5/6 of being type B?

3. Players 1 and 2 each must decide whether to fight for a valuable prize. If both players decide to fight, then they both lose $1, and nobody gets the prize (it is destroyed). If one player decides to fight but the other does not, then the player who is willing to fight gets the prize. A player who does not fight is guaranteed a payoff of 0. Everybody knows that the prize is worth \(V_2 = $2\) to player 2. But the prize may be worth more to player 1. Let \(V_1\) denote the value of the prize to player 1. In terms of \(V_1\), the players' payoffs \((u_1, u_2)\) will depend on their actions as follows:

<table>
<thead>
<tr>
<th>Player 2</th>
<th>NotFight</th>
<th>Fight</th>
</tr>
</thead>
<tbody>
<tr>
<td>NotFight</td>
<td>0, 0</td>
<td>0, 2</td>
</tr>
<tr>
<td>Fight</td>
<td>(V_1, 0)</td>
<td>(-1, -1)</td>
</tr>
</tbody>
</table>

Let us explore some different assumptions about this value \(V_1\).

(a) Suppose first that the value of the prize to player 1 is \(V_1 = $3\), and everybody knows this. Find all equilibria of this game, including a mixed-strategy equilibrium in which both players have a positive probability of fighting.

(b) Suppose next that the value of the prize to player 1 is either \(V_1 = $2\) or \(V_1 = $3\). Player 1 knows his actual value, but player 2 thinks each of these possibilities has probability 1/2. Find a Bayesian equilibrium of this game where player 2 randomizes between fighting and not fighting.

(c) Finally, suppose that the value of the prize to player 1 is \(V_1 = $2 + \tilde{t}_1\) where \(\tilde{t}_1\) can be any number between 0 and 1. Player 1 knows its actual value, but player 2 thinks of \(\tilde{t}_1\) as a uniform random variable on the interval from 0 to 1. Find a Bayesian equilibrium of this game where player 2 randomizes between fighting and not fighting.

1. Consider the three-player game in Osborne's Figure 331.2 (exercise 331.1).
   (a) Show its normal representation in strategic form.
   (b) Show that this strategic game has a pure-strategy Nash equilibrium that corresponds to a sequential equilibrium of the given extensive-form game. Be sure to indicate what beliefs would make this a sequential equilibrium.
   (c) Show that this game also has a pure-strategy Nash equilibrium that does not correspond to a sequential equilibrium of the given extensive-form game.

2. Consider the following extensive-form game, where player 1 observes the chance move, but player 2 does not observe it. If 2 gets to move, she only knows that 1 chose either x1 or z1.
   (a) Find a sequential equilibrium in which the (prior) probability of player 2 getting to move is 1.
   (b) Find a sequential equilibrium in which the probability of player 2 getting to move is 0. (You must describe what player 2 would believe and do if she got to move.)
   (c) Find a sequential equilibrium in which the probability of player 2 getting to move is strictly between 0 and 1.
   (d) Show the normal representation of this game in strategic form.

3. Consider the following extensive game, which begins with a chance move that can be observed by player 1 but not by player 2.
   (a) Find a sequential equilibrium of this game. Show clearly all move probabilities for each player at each information set, and show what player 2 would believe to be the probability of player 1 having done x1 at 1.1 if 2 got to move at a 2.3 node.
   (b) Show the normal representation of this game in strategic form.
   (c) Find a Nash equilibrium of this game that does not correspond to a subgame-perfect or sequential equilibrium.

1. Consider a repeated game where 1 and 2 repeatedly play the game below infinitely often.

\[
\begin{array}{c|cc}
    & a_2 & b_2 \\
\hline
a_1 & 8, 8 & 1, 2 \\
b_1 & 2, 1 & 0, 0 \\
\end{array}
\]

The players want to maximize their $\delta$-discounted average value of payoffs, for some $0<\delta<1$. Consider the following state-dependent strategies: The possible states are state 1 and state 2. In state 1, we anticipate that player 1 will play $b_1$ and player 2 will play $a_2$. In state 2, we anticipate that player 1 will play $a_1$ and player 2 will play $b_2$. The game begins at period 1 in state 1. The state of the game would change after any period where the outcome of play was $(a_1,a_2)$, but otherwise the state always stays the same. What is the lowest value of $\delta$ such that these strategies form an equilibrium?

2. Consider a repeated game where 1 and 2 repeatedly play the game below infinitely often.

\[
\begin{array}{c|cc}
    & a_2 & b_2 \\
\hline
a_1 & 3, 3 & 0, 5 \\
b_1 & 5, 0 & -4, -4 \\
\end{array}
\]

The players want to maximize their $\delta$-discounted average value of payoffs, for some $0<\delta<1$. (a) Find the lowest value of $\delta$ such that you can construct an equilibrium in which the players will actually choose $(a_1,a_2)$ forever, but if any player i ever chose $b_i$ at any period then they would play the symmetric randomized equilibrium of the one-stage game forever afterwards. (b) What is the lowest value of $\delta$ such that you can construct an equilibrium in which the players will actually choose $(a_1,a_2)$ forever, but if some player i unilaterally deviated to $b_i$ at any period then that player i would get payoff 0 at every round thereafter? Be sure to precisely describe state-dependent strategies that form this equilibrium.

3. Consider a repeated game where 1 and 2 repeatedly play the game below infinitely often.

\[
\begin{array}{c|cc}
    & a_2 & b_2 \\
\hline
a_1 & 0, 8 & 2, 0 \\
b_1 & 8, 0 & 0, 2 \\
\end{array}
\]

Each player i wants to maximize his or her $\delta_i$-discounted average value of payoffs, for some $\delta_1$ and $\delta_2$, where each $0<\delta_i<1$. Find the lowest values of $\delta_1$ and $\delta_2$ such that you can construct an equilibrium in which the players will actually alternate between $(a_1,a_2)$ and $(b_1,a_2)$ forever, but if any player ever deviated then they would play the randomized equilibrium of the one-stage game forever afterwards.

1. Do exercises 59.1 and 189.1 in Osborne.

2. Player 1 chooses a number $a_1$ between 0 and 1 ($0 \leq a_1 \leq 1$), and player 2 also chooses a number $a_2$ between 0 and 1 ($0 \leq a_2 \leq 1$). Their payoffs $(u_1, u_2)$ depend on the chosen numbers $(a_1, a_2)$ and a known parameter $\gamma$ as follows:
   \[ u_1(a_1, a_2) = \gamma a_1 a_2 - (a_1)^2, \]
   \[ u_2(a_1, a_2) = 2a_1 a_2 - a_2. \]

   (a) Given $\gamma = 1.5$, find all (pure) Nash equilibria of this game if the players choose their numbers independently.

   (b) Given $\gamma = 1.5$, find a subgame-perfect equilibrium of this game if player 1 chooses $a_1$ first, and then player 2 chooses $a_2$ after observing $a_1$.

   (c) Given $\gamma = 0.8$, find all (pure) Nash equilibria of this game if the players choose their numbers independently.

   (d) Given $\gamma = 0.8$, find a subgame-perfect equilibrium of this game if player 1 chooses $a_1$ first, and then player 2 chooses $a_2$ after observing $a_1$.

3. Find all Nash equilibria (pure and mixed) of the following $2 \times 3$ game:

   \[
   \begin{array}{ccc}
   & L & M & R \\
   T & 0, 4 & 5, 6 & 8, 7 \\
   B & 2, 9 & 6, 5 & 5, 1 \\
   \end{array}
   \]

4. Do exercise 145.1 in Osborne. (Hint: In the symmetric equilibrium, each of the n players is willing to bid any amount between 0 and K. The probability distribution for a player's bid can be characterized by its cumulative distribution function.)
The **midterm examination on April 26** will test topics that were in the first three homework assignments, and may include questions like the following:

*Strategic-form games*
Find pure and randomized equilibria in 2x2 games, and show the resulting expected utility payoffs.
Find symmetric equilibria of symmetric games, including games where \( n \geq 2 \) players each have 2 possible actions (2x2x...x2 games), or 3x3 symmetric games.
Find best-response functions and equilibria in games where each player chooses a number within some given bounds.
Find dominated actions in games that are 2x3 or larger, and find randomized strategies that dominate a given action.
Find equilibria in larger games by eliminating dominated actions and then analyzing the smaller game that remains.

*Extensive-form games with perfect information*
Find subgame-perfect equilibria in perfect-information extensive games (finite tree, or choose-a-number).
For a finite extensive game with perfect information (tree diagram), find all strategies and construct the normal representation in strategic form. Find Nash equilibria that are not subgame-perfect.

You might be asked to do sensitivity analysis in a parameter of the game. (E.g.: "How would your answer change if this number were changed to ...?")
There will probably be four questions on the exam. You may be asked to put your answers to questions 1&2 in one blue book and your answers to questions 3&4 in another blue book.
You are allowed to bring one sheet of notes (8.5"x11") for use in the exam.
Numerical answers may be expressed as ratios of whole numbers, without reducing.
In Chapter 5, do exercises:
156.2ac,
173.2 and show the strategic form that represents Figure 173.1,
176.1 simplify the problem by assuming that a player who does not pass must bid exactly 1 more
than the other player's most recent bid (say the other's "most recent bid" is 0 at the start of
the game), analyze the cases (v,w)=(2,3) and (v,w)=(3,4) and (v,w)=(3,5),
177.1.
Econ 20700. Assignment 2. Due April 12, 2017.
In Osborne chapter 4, do exercises:
114.1, 114.3, 120.2 (only consider mixed strategies that randomize among M and B),
128.1, 130.2, 141.2b, 142.1 (look for a symmetric randomized equilibrium).
*34.3(a) with randomization: For the first game in 34.3 (without the middle road), find a
symmetric randomized equilibrium in which each driver independently chooses between the X
route, with some probability p, and the Y route, with probability 1-p. What is p in equilibrium?
In Osborne chapter 2, do exercises:
42.1,
42.2 (notice that each player gets half of f(x_1,x_2), and each x_i satisfies 0 ≤ x_i ≤ 1),
47.1,
52.2,
34.3 (just find one equilibrium for each of the two games),
and the following small attrition game:

Small Attrition game. There are two players numbered 1 and 2. Each player i must choose a
number c_i in the set {0, 1, 2}, which represents the number of days that player i is prepared to
fight for a prize that has value V=$9. A player wins the prize only if he is prepared to fight
strictly longer than the other player. They will fight for as many days as both are prepared to
fight, and each day of fighting costs each player $1. Thus, the payoffs for players 1 and 2 are as
follows:
Player 1’s payoff is u_1(c_1,c_2) = 9-c_2 if c_1 > c_2, but u_1(c_1,c_2) = -c_1 if c_1 ≤ c_2.
Player 2’s payoff is u_2(c_1,c_2) = 9-c_1 if c_2 > c_1, but u_2(c_1,c_2) = -c_2 if c_2 ≤ c_1.
Show a 3×3 matrix that represents this game.
What dominated strategies can you find for each player in this game?
What pure-strategy equilibria can you find for this game?