Deioces had always been a man of note, and now he set himself to practice justice ever more and more keenly...
The Medes in his own village, seeing the manner of the man's life, chose him to be a judge among them.
And he, since it was power he was courting, was always straight and just... so much so, indeed, that people in other villages learned that Deioces was the one man for judging according to the rule of right... and at last they would entrust their suits to none but him...
Deioces came to realize that now everything hung on himself. Whereupon he refused to sit as judge any more and said he would serve no longer... So robbery and lawlessness grew even more in the villages than before.
The Medes all came to a meeting place and... they persuaded one another to be ruled by a king.
Then at once the question was proposed as to whom to make king. Deioces was so much in everyone's mouth... that all ended by agreeing that he should be their king. For his part, he bade them to build him houses worthy of royalty and to strengthen him with a bodyguard... He compelled the Medes to make one great fortress...
When he had ordered these matters and had strengthened himself in royal power, he was very exact in his observance of justice.

Herodotus, The History c. 430 BCE

Almost all the governments which exist at present, or of which there remains any record in story, have been founded originally, either on usurpation or conquest, or both, without any presence of a fair consent or voluntary subjection of the people. When an artful and bold man is placed at the head of an army or faction, it is often easy for him, by employing, sometimes violence, sometimes false presences, to establish his dominion over a people a hundred times more numerous than his partisans. He allows no such open communication, that his enemies can know, with certainty, their number or force. He gives them no leisure to assemble together in a body to oppose him. Even all those who are the instruments of his usurpation may wish his fall; but their ignorance of each other's intention keeps them in awe, and is the sole cause of his security. By such arts as these many governments have been established; and this is all the original contract which they have to boast of.

David Hume, Of the Original Contract 1752.
The analysis of proposals for institutional reform is central to the mission of economics as a profession. Economists study how changing structure of social institutions could affect individual behavior and resulting welfare for people in society: structure, conduct, performance. We view institutions as the rules of the game that people play. We probe the consequences of institutional reform by analyzing how changing the game changes the set of equilibria.

But when we ask how an institution is established, we must embed it somehow in a larger game. A chess player is deterred from illegal moves (grabbing king) by fear of a bad reputation in the larger game of life. So the chess game seems supported by some kind of reputational equilibrium in a larger more fundamental game. But saying "games are equilibria of larger games" cannot be right, because if chess were embedded as an equilibrium in the game of life, that equilibrium would specify each player's strategy in the chess game itself.

Following Hurwicz (1998), we may say that an institution or legal game $G = (N, (C_i)_{i \in N}, (u_i)_{i \in N})$ is embedded in some true game $H$ when $H = (N, (D_i)_{i \in N}, (u_i)_{i \in N})$ has larger strategy spaces $D_i \supset C_i \forall i \in N$ and has utility functions $u_i$ that extend those of the legal game $G$ to the larger domain $D = \times_{j \in N} D_j$.

When the strategy sets of $G$ are curb sets in the true game $H$ (closed under rational behavior) in the sense that $\forall i \in N, \forall c \in \times_{j \in N} C_j, \forall d_i \in D_i \setminus C_i, u_i(c_{-i}, d_i) < u_i(c)$ (Basu Weibull, Econ Letters 1991), then we may say that the institution $G$ is (weakly) enforceable in the true game $H$.

(So each player has an incentive to stay in his part of the curb set when all others are expected to stay in theirs.)

A big true game $H$ may contain many different minimal curb sets, which might seem an annoying indeterminacy. But the right mathematical model of institutions must admit such multiplicity of solutions, because real institutions are manifestly determined by cultural norms and concepts of legitimacy, which would have no scope for effect if the economic structure of the true game $H$ admitted only one solution. Schelling’s focal-point effect can be extended to questions of selecting among multiple curb sets, just as among multiple equilibria.
A fable about the foundations of institutions.
Consider an island where, every day, all people are matched into pairs that play following rival-claimants game:

<table>
<thead>
<tr>
<th>Player 1 claims</th>
<th>Player 2 claims</th>
<th>Player 2 defers</th>
</tr>
</thead>
<tbody>
<tr>
<td>-t, -t</td>
<td>r, 0</td>
<td></td>
</tr>
<tr>
<td>0, r</td>
<td>0, r</td>
<td>0, 0</td>
</tr>
</tbody>
</table>

Here r > t > 0. (Say r=20, t=1.) Any one match has 3 possible equilibria: (claim, defer) → (r, 0), (defeer, claim) → (0, r), and a symmetric equilibrium where each randomizes \((r/(r+t))[\text{claim}]+(t/(r+t))[\text{defer}] \rightarrow (0, 0)\).

There can be many kinds of general equilibria on the island:
1. All pairs play the symmetric randomized eqm, and so everyone gets \(E_{u_i} = 0\) (anarchic state of nature).
2. In some matches, symmetry may be broken by a shared understanding of who should claim (property rights, law).
    In an assembly, players could establish rules that define claiming rights in more situations (legislation).
    Shared understanding of these rules gives them force.
3. A generally recognized leader or judge may allocate claiming rights when traditional rights are unclear.
    People's shared recognition of his authority gives his rulings force. (Herodotus's Deioces).
    Leaders may be identified by any generally accepted selection procedure (political institutions).
    The scope and duration of their authority could have any limits recognized by the players (constitutionalism).

The leader's authority is established, not by common consent (choice), but by common recognition (belief). [Hume]

Costs of anarchy make the acceptance of political leadership a coordination game with multiple equilibria. (Hardin)
Here, establishment of political leadership is the eqm-selection problem to solve all other eqm-selection problems.
If payoff is reproductive fitness, then cultures with systems of rights and authority for effective coordination should have spread over the world, by Darwinian dynamics.
In common games, we may expect to find culture and authority selecting the focal eqm (not mathematical refinements).
Let us now extend this fable to say more about how political institutions are actually established. We may discover something more about the true nature of the "original contract" that Hume overlooked.

New political regimes are established by people: by political leaders with supporters. What can we say about them? In any system, political leaders can get power only with help of active supporters, whose acts of support precede the leader's capacity to reward it.

Like a banker, a leader's promises of future credit must be trusted and valued as rewards for current service. So a strong leader needs some way to credibly commit himself to reward his supporters as promised.

Let's start with a island where peasants play the rival-claimants game in agricultural fields every period. In each part of the island, the island's ruling prince, or his authorized representative, can allocate claiming rights, and the peasants will focus on the eqm where the one who has claiming rights takes the benefits r of the field. Payoff is transferable, and so the ruler can charge up to r as rent per period, in exchange for rights to claim benefits. So the ruling prince may be able to extract rental income without any substantial exercise of force. But force may be needed to become the ruler of the island. Suppose the ruler is identified as the leader whose supporters have won the most recent battle on the island. (Natural law?)
**The model.** The total revenue (per unit time) from the rents paid by peasants in the island is $R$.
At any time, the ruler can consume any part of $R$ and reallocate the rest among his supporters any way he chooses.
To become ruler, a prince must first defeat a rival army.
Then, to stay in power, the prince must defeat similar challenges from invaders that arrive at a Poisson rate $\lambda$.
(In any short time period of length $\varepsilon$, the probability of a challenger arriving is approximately $\lambda \varepsilon$.)
To defeat any rival or challenger, the prince needs captains to form his army.
For each captain, the cost of supporting the prince in a battle against a rival or challenger is $c$.
The prince and the captains are assumed to be risk neutral and have discount rate $\delta$.
The prince's probability of winning when he is supported by $n$ captains is $p(n)$,
where $0 \leq p(n) \leq 1$, $p(0) = 0$, and $p(n)$ increases as $n$ increases ($p'(n) > 0$).
The win probability $p(n)$ may depend on the anticipated size of rivals' armies, which we may denote by $M$.
To be specific, let's assume that $p(n) = p(n|\ M) = n^s/(n^s+M^s)$ for some $s>1$.
We will consider stationary symmetric equilibria where each contender has the same number of supporters $n=M$.

To induce captains to support him, the prince must promise to pay them some part of his future revenue if they win.
Consider a captain who expects the prince to grant him some income $y$ as long as the prince rules and captain supports.
When there is no current challenger, a captain's expected discounted payoff is $U(n,y) = (y-\lambda c)/[\delta+\lambda(1-p(n))]$.
When a new challenge arrives, a captain's expected payoff before the battle is $pU-c$.
So for captains to rationally give support in battle, we need $p(n)U(n,y)-c \geq 0$.
The lowest income $y$ that satisfies this participation constraint is $Y(n) = (\delta+\lambda)c/p(n)$.
$Y$ defines a downward-sloping ($Y'<0$) supply curve for captains, who would accept a lower wage in a larger army.

If the prince has $n$ captains who each get income $y$ then, when no challenge is impending,
the expected discounted value of the prince's payoffs is $V(n,y) = [R - ny]/[\delta+\lambda(1-p(n))]$.
The prince's expected payoff on the eve of battle against a challenger is $p(n)V(n,y)$.
To formalize Hume's image of an **absolute ruler who does not allow his supporters to communicate**, let us say that a leader is **absolute** if his agents have no communication with each other, only with the leader. So a captain does not observe or respond to any change in the absolute prince's relationship with other captains. We distinguish two cases: whether the prince can recruit from a bounded or unbounded set of potential captains.

In the case of **bounded recruiting**, the prince is trusted only by captains in some given finite group or tribe. Any captain in this group believes the prince will pay him $y$ as long as he serves in battles and the prince retains power. Individuals outside this group would expect never to be paid by the prince, and so would never support him in battle. After winning, prince wants to pay $y$ to the $n$ members of his trusted group only if $(n,y)$ satisfies the demand constraint $V(n,y) = \max_{k \leq n} V(k,y)$.

So we can define the prince's demand curve by $N(y) = \arg\max_{n \geq 0} V(n,y)$. The demand constraint is satisfied on this curve and at some $(n,y)$ such that $n \leq N(y)$.

A **local equilibrium** $(n,y)$ must satisfy the demand constraint and supply constraint $y \geq Y(n)$. There always exists a local equilibrium at $n=0$, $y=\infty$, which represents distrust.

In the case of **unbounded recruiting**, an unbounded supply of potential captains are equally ready to trust the prince. But the prince would not pay captains for past service if he could freely recruit new captains before next battle. Any $n$ which maximizes $p(n)V(n,y) - n(y/\lambda)$ for the prince before battle also maximizes $V(n,y)$ after battle, because $V(n,y) = \left[R - ny\right]/\left[\delta + \lambda - \lambda p(n)\right] = \left[R - \lambda n(y/\lambda) + \lambda p(n)V(n,y)\right]/(\lambda + \delta)$. So an absolute prince with unbounded recruiting can sustain an eqm $(n,y)$ with $n=N(y)$ and $y \geq Y(n)$ if earning trust of a new recruit requires an advance payment of $y/\lambda$ or more. Advance pay $y/\lambda$ is a costly signal that the prince has not hired more captains than he will want to retain after battle.

**Fact.** If $(n,y)$ is any local equilibrium for an absolute prince and $n > 0$, then $\exists k > n$ such that $V(k,Y(k)) > V(n,y)$. So an absolute prince could always gain from relaxing absolutism and committing himself to retain more captains.
With a win-probability function $p(n) = p(n|M)$ that depends on the anticipated size of challengers' armies, let us consider stationary symmetric scenarios in which all challengers will resemble our prince.

With such a stationarity assumption, a local eqm is also a global equilibrium iff $n = M$.

To see how intense competition for power can become, we focus on global equilibria with maximal $M$.

**A numerical example** Consider $R=90$, $\lambda=0.2$, $\delta=0.05$, $c=5$, $s=1.5$, so $p(n|M) = n^{1.5}/(n^{1.5} + M^{1.5})$.

For this example, the maximal global absolutist eqm is at $n=M=12$ and $y=2.5$. A new recruit would cost $y/\lambda=12.5$.

Calculations at [http://home.uchicago.edu/~rmyerson/research/prince.xls](http://home.uchicago.edu/~rmyerson/research/prince.xls)

In this eqm, the captains' portion of the principality's income is $ny/R = 0.333$.

With $M=12$: $Y(12)=2.5$, $N(2.5) = \text{argmax}_{n \geq 0} V(n,2.5) = 12$, yielding $p=0.5$, $V=400$.

But $\text{argmax}_{n \geq 0} V(n,Y(n)) = 25.5$, which would yield $p=0.756$, $Y=1.65$, $V=484$.

Before battle, $\text{argmax}_{n \geq 0} p(n)V(n,Y(n)) = 31.1$, yielding $p=0.807$, $Y=1.55$, $pV=380$. 
**Introducing the prince's court**

In the context of a global absolutist equilibrium, a prince would do better if he could credibly recruit more captains. The prince can make this commitment credible by creating a court where the captains meet regularly and share any complaints that they may have against him. Such courts or high councils of government seem universal in political systems (Finer, 1997).

In a **weak court**, complaints could shift the expected equilibrium to one where nobody trusts the leader, so that his expected value would drop to \( V_0 = \frac{R}{\delta + \lambda} \).

Captains who have not been cheated by the prince get expected payoff \( U(n,y) > 0 \) in equilibrium, so they have no incentive to complain unless the prince actually deviated. Following Aumann, a repeated-game folk thm can be applied in the weak court, with minimax=\( V_0 \) for prince. So a local eqm for the prince with a weak court can be any \((n,y)\) satisfying \( y \geq Y(n) \) and \( V(n,y) \geq V_0 \).

It may be strange to assume that challengers only arrive at rate \( \lambda \) when all captains know that nobody trusts the prince. So we define a **strong court** to be one where complaints could shift the expected equilibrium to distrust \((n=0, y=\infty)\) and cause a new challenger to emerge from the court, so that the incumbent's expected value would drop to 0. So a local eqm for the prince with a strong court can be any \((n,y)\) satisfying \( y \geq Y(n) \) and \( V(n,y) \geq 0 \).

Before battle, the prince would want to negotiate an optimal local equilibrium that maximizes \( p(n)V(n,y) \) subject to these constraints. Because \( p(n) \) is increasing in \( n \), \( \operatorname{argmax}_{n \geq 0} p(n)V(n,Y(n)) \geq \operatorname{argmax}_{n \geq 0} V(n,Y(n)) \).

As before, a local equilibrium is **global** when \( n = M \).

**Fact** With \( s > 1 \), if \((M,Y(M))\) is a global equilibrium with weak courts, then \( \operatorname{argmax}_{n \geq 0} V(n,Y(n)) > M \).
Consider again our **numerical example** with \( R = 90, \ \delta = 0.05, \ c = 5, \ s = 1.5, \ \lambda = 0.2. \)

The maximal global eqm with weak courts has \( M = 14.4 = n, \ \ y = Y = 2.5, \ \ ny/R = 0.4, \ V = 360 = R/(\delta + \lambda). \)

But with \( M = 14.4: \ \argmax_{n \geq 0} V(n,Y(n)) = 25.7, \ \text{yielding} \ p = 0.705, \ Y = 1.77, \ V = 407; \)
before battle, \( \argmax_{n \geq 0} p(n)V(n,Y(n)) = 32.1, \ \text{yielding} \ p = 0.769, \ Y = 1.63, \ pV = 302. \)

With strong courts, \( \exists \) global eqm such that even before battle the prince would not want to commit to a larger army.

In this eqm, \( M = 30 = n, \ y = Y(n|M) = 2.5, \ \ny/R = 0.833, \ V(n,y) = 100, \ p(n)V(n,y) = 50. \)

Consider alternative local equilibria that are feasible for a prince against \( M = 30. \)
As we vary \( n, \) prince's total wage bill \( nY(n) = n(\delta + \lambda)c/p(n) \) is u-shaped, going above \( R \) when \( n < 6.24 \) or \( n > 48.4. \)

When \( n < 6.24, \) the small \( p(n) \) makes \( Y(n) \) so large that \( nY(n) > R. \)
So a captain will not support a leader who is considered unlikely to get support from many other captains.
The captains are in a coordination game, where nobody wants to support a leader whom nobody else is supporting.
Xenophon's Cyrus the Great recruited the largest army in Asia by an equilibrium like the high end \( (n \approx 48, V \approx 0), \)
invoking the possibility of a behavioral type (Persian virtue) to make this eqm focal. (Xenophon, *Educ. of Cyrus.*)
For our example with $R=90$, $\delta=0.05$, $c=5$, $s=1.5$, $\lambda=0.2$, the **maximal global equilibrium with strong courts** has $M = 36 = n$, $y = Y = 2.5$, $ny/R = 1$, $V = 0$.

Against $M=36$, $n=23.7$ would maximize $V(n,Y(n))$. But princes could be deterred from reducing $n$ by fear that a deviation from the virtuous (high-$n$ low-$V$) strategy would cause their court members to distrust and desert them.

A strong court also allows a **nonsymmetric** stationary global eqm where challengers cannot get any support and so are all defeated by the ruler with some $m$ captains, where $\bar{m} = R/[c(\lambda+\delta s(s-1)^{(1/s-1)})] < m \leq R/[c(\lambda+\delta)]$.

A new ruler with such prospects and no past debts could get the expected payoff $\hat{V}(m) = (R-\lambda mc)/\delta$.

If the challenger got initial support from $n>0$ captains, he would have to promise them each rewards worth $c/p(n|M)$, and so he would get the expected payoff value $p(n|m)[\hat{V}(m) - nc/p(n|m)]$.

But $m > \bar{m}$ implies that $\hat{V}(m) < nc/p(n|m)$ $\forall n>0$, so a challenger could not credibly pay any positive $n$ against $m$. Such a nonsymmetric global equilibrium requires a strong court to enforce captains' rights.

In our example: $R/[c(\lambda+\delta)] = 72 > \bar{m} = 61.1$, $\bar{mc}(\delta+\lambda)/R = 0.849$, $\hat{V}(\bar{m}) = 272 < R/(\delta+\lambda) = 360$. 
Leadership, as a reputational equilibrium, depends on a multiplicity of equilibria

We have found two reasons why captains might not support a prince in equilibrium:

**Unreliability**  A captain should not support when he fears that he would not be rewarded, which would be rational for the prince if it would not adversely affect the prince's ability to recruit other supporters in the future.

So there can always be an eqm where people outside any given group would not trust or support the prince.

**Weakness**  A captain should not support when he fears that other captains are also unlikely to support the prince, making prince's probability of success so small that even credible promises to share R would not be worth the cost c.

So this coordination game always has equilibria where the prince gets no support.

Implicit threats of switching to such bad equilibria sustain the good equilibria where supporters trust their leader.

A reliable reputation with many active supporters is the rare asset that defines a leader.

Factors that focus eqm-coordination on one leader may be called legitimacy (extrinsic) or charisma (intrinsic).

The charisma to gather a confident army may be bestowed on individuals by random (Poisson λ) events.

To maintain reliability for a larger force, the prince's supporters need a court or forum to communicate grievances, and they need a shared sense of group identity so that they will all react to a breach of trust against any one captain. There can be new recruiting, but any new captain must be accepted into this group, to be assured that his mistreatment by the prince would cause them all to distrust the prince.

Participation in the prince's court can be required for good relationship with prince (feudal oath of aid and counsel). To have a strong court, the prince must at least allow that, if he denied a captain's appropriate rewards, then his other supporters should not be ignorant of each other's intention to withdraw support from him. (pace Hume)

These rules, which define what a leader must do to maintain his supporters' trust, may be considered as a kind of personal constitution for the leader, even if he is a monarch who is not formally constrained by any other constitution. In this personal constitution, the leader is constrained to share benefits of power with a privileged group of supporters. To them at least he must give a kind of justice.  This may be the true original contract that Hume overlooked.
We have not yet asked why the prince should give justice to others who are not active supporters. But his personal constitution could also commit the leader to comply with other norms and promises, if his violation of them would be treated like his cheating a supporter. A political leader may fear to violate an established constitution when his relationships with supporters were developed in its context, so that violating the constitution would seem to his supporters like cheating one of them. Thus, constitutional restraints on political leaders may be based on the fragility of their supporters' trust.

Many great institutions of civilization were initially developed to unite several rival princely courts, such as the English common law under Henry II (c1160), and the Chinese civil-service system under the Song (c980). Rights and privileges of these institutions were later extended to broader groups of the population, but their enforcement relied on an understanding that a ruler who violated them could lose elite supporters' trust.

In democracy, leaders are supposed to extend their base of support to include voting masses. Campaigns and elections are the extended court where leaders are constrained and judged by the voters. But a core of active supporters, small enough to monitor, is essential for any political leader in any system. A democratic constitution would be imperiled if its most powerful office were won by a politician who could be confident that his active supporters would still trust him after he openly violated the constraints of the constitution. So survival of democracy may depend on the personal constitutions that bind its political leaders with supporters.

http://home.uchicago.edu/~rmyerson/research/stonyb06.pdf
Appendix: Monarchy and oligarchy.

Starting with an assumption of absolute monarchy, we showed that, to credibly motivate his captains into battle, a prince can gain by establishing a court where his captains can remove him from power if he breaks promises to them. But the prince's court can yield other equilibria that are worse for the prince.

When succession disputes are resolved in such a court, rival heirs may compete by promising more than the minimal \( y = (\delta + \lambda) c / p(n) \) that motivates captains into battle.

With Bertrand price-competition among heirs, the court could eventually convert the monarchy into an oligarchy where each of \( n \) captains gets income \( Y = R / n \). Each captain's utility is then \( W(n) = (R / n - \lambda c) / [\delta + \lambda (1 - p(n))] \).

In our example with \( R = 90, \delta = 0.05, c = 5, \lambda = 0.2, s = 1.5 \) and \( M = 30 \), this \( W(n) \) is a monotone decreasing function of \( n \).

\[
\lim_{n \to 0} W(n) = +\infty. \quad \text{So oligarchs prefer to reduce the size } n \text{ of their oligarchy, even though } p(n) \to 0 \text{ as } n \to 0.
\]

In the oligarchy, any captain must become an equal partner who gets \( R / n \); but a monarch can hire captains at the wage \( Y(n) \) that motivates them in battle. Transformation of monarchies into contracting oligarchies could be one explanation of dynastic decline.