THEORETICAL COMPARISONS OF ELECTORAL SYSTEMS

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These notes are available in Adobe PDF format at

http://home.uchicago.edu/~rmyerson/research/electsys.pdf
1. Invitation to political economics

The scope of economics has changed. From an initial goal of explaining the production and allocation of material goods, economic theorists developed analytical tools to predict how changes in market structure may affect rational behavior of producers and consumers. Today we can extend these tools to predict how changes in the structure of any social institution may affect the rational behavior of individuals. With game-theoretic models, we can now analyze institutions of civilized society with the same breadth of vision that characterized the ancient Greek social philosophers who gave economics its name.

Among these new applications of economic analysis, none are more important than analysis of incentives in political institutions. Markets and politics are substantively interconnected systems. Logical similarities between political competition and market competition. Understanding political failures, to guide constitutional reform, is not less socially important than understanding market failures, to guide regulatory reform and fiscal policy.

Traditional political scientists have lacked the skills to build analytical theories of comparative incentives in political institutions.

The basic constitutional structures of any democratic system defines the rules of the game played by politicians. Consists of structures on two levels. **Governmental structures**: definitions of political offices and their powers. **Electoral structures**: procedures for electing candidates to these offices. A complete theory of politics requires analysis of both levels: electoral competition, interactions among elected officials in government. To keep things tractable, most game-theoretic work focuses on one of these levels, ignoring or oversimplifying the other. Here I emphasize the electoral level, to introduce elements of a theory of electoral systems and how they may affect competitive political behavior.
2. Variety of electoral systems

We have a wide variety of ballot types in different voting rules. Let $K$ denote the number of candidates (or party-lists) in the election. In a scoring rule, the ballot is a $K$-vector, giving points to each candidate, which are summed across all voters to determine outcome. Non-scoring rules include single-transferable vote (STV).

Determination of winners in scoring rules.

Single winner: Candidate with highest total point score wins.

$M$ winners: $M$ highest scorers are winners.

Proportional representation: $M$ seats are allocated to party lists in proportion to their scores (using some rounding formula).

Single winner with quota: Candidate with highest total point score wins if her score exceeds some quota (50%), otherwise a second election is called.

Ballot types in scoring rules.

In a rank-scoring rule, permissible vote-vectors are the permutations of one vector $(s_1, s_2, ..., s_{K-1}, s_K)$ where (w.l.o.g) $1 = s_1 \geq s_2 \geq ... \geq s_{K-1} \geq s_K = 0$.

Single-positive vote: $(1, 0, ..., 0, 0)$ (with single-winner = plurality, with $M$ winners = SNTV, with proportional representation = list PR)

Single-negative vote: $(1, 1, ..., 1, 0)$.

$V$-positive votes: $1 = s_1 = ... = s_V, s_{V+1} = ... = s_K = 0$.

$V$-negative votes: $1 = s_1 = ... = s_{K-V}, s_1+K-V = ... = s_K = 0$.

One-positive-and-one-negative vote: $s_1 = 1, s_2 = ... = s_{K-1} = 1/2, s_K = 0$.

Borda voting: $s_j = (j-2)/(K-1)$ for $j = 1, 2, ..., K$.

Nonrank scoring rules may permit the permutations of several vectors. Approval voting allows any number of positive votes (the union of permissible ballots under all $V$-positive-vote rules, for $V = 1, ..., K-1$).

Ballots like single-positive voting ask voters to reward the best candidate; ballots like single-negative voting ask voters to punish the worst candidate. With large $K$, $\sum_j s_j/K \approx 0$ if best-rewarding, $\sum_j s_j/K \approx 1$ if worst-punishing.
3. Symmetric equilibria and incentives for diversity.

Hotelling (1929) posed the question of rational competitive diversity both in oligopolistic markets and in politics. But in politics, each voter's utility is from the policy position of the winner, not the candidate that he supported. No costs of "excessive sameness."


Example: single winner, K candidates, \{policy alternatives\}=\{Left, Right\}. Q denotes the fraction of voters expected to prefer Left; 1−Q prefer Right. First, each candidate independently chooses a policy position in \{Left,Right\}. Assume symmetry: candidates at same position are treated same by voters. Given K, Cox's threshold of diversity $Q^*$ is the greatest value of Q such that there is a symmetric equilibrium in which all K candidates choose Right.

- **Fact.** $Q^* = \sum_j s_j / K$ for a rank-scoring rule with points $1 = s_1 \geq s_2 \geq ... \geq s_K = 0$.

  Single-positive voting yields $Q^* = 1/K$, and so small minority positions can win when K is large. Best-rewarding creates incentives to diversify, low $Q^*$. Single-negative voting yields $Q^* = (K-1)/K$, so a majority can be neglected when K is large! Worst-punishing creates incentives to cluster, high $Q^*$. Approval voting and Borda voting yield $Q^* = 1/2$ for any K. Majoritarian.

A related model: The winner of the election will get a budget of €1 per voter to distribute as cash or to spend on a public good worth €B to every voter. An equilibrium where the public good is guaranteed exists only if $B \geq 1/Q^*$. When $B < 1/Q^*$, if all other candidates promised the public good, then a candidate could win by promising €1/$Q^*$ to a $Q^*$ minority of the voters. (Myerson, Amer.Poli.Sci.Rev. 1993; Lizzeri and Persico, 1998.)

These results depend only on ballot type; extend to multi-winner elections. Consider open list PR, where supporters of a party vote to order its list. Single-positive vote for candidates in Brazil induces separate constituencies (Ames, J. of Politics, 1995). Approval voting would induce broader appeals.
4. Nonsymmetric equilibria and Duverger's law.


A candidate is in a **close race** when a small number of votes could change her from winner to loser or from loser to winner.

A rational voter knows that his vote matters only in the event of a close race, and so should vote in equilibrium to maximize expected utility in this event. A candidate is a **serious contender** if, conditional on a close race existing, there is a substantial positive probability of her being in the close race. A **stronger** candidate has greater probability of winning; **weaker**, less. A **likely winner** has probability of winning close to 1; **likely loser**, close to 0.

- In an election with K candidates for M seats, serious candidates normally include the strongest likely loser and the weakest likely winner.
- With best-rewarding votes, being a serious contender strengthens a candidate.
- With worst-punishing votes, being a serious contender weakens a candidate.

These propositions yield generalizations of Duverger's law. Consider an election with M winners, single-positive voting for candidates (SNTV). Becoming weaker than the strongest likely loser makes a candidate less serious, which weakens her further until she gets almost no votes. Among likely winners however, being perceived stronger tends to weaken. Such a tendency towards M+1 serious candidates, with relatively level scores for top M candidates was found by Steven Reed in Japan (*Brit.J.P.S.*, 1991). Unstable non-Duvergerian equilibria have ties for strongest likely loser. (Cox) (Single-negative voting? M–1 likely winners, others all serious contenders!)

- In list-PR elections, any party likely to win a seat can seriously contend for more or less; but only strongest of likely losers can be a serious contender. So list PR in M-seat districts should yield at most M+1 serious parties.
5. Electoral barriers to entry and incentives to reduce corruption.

Interpretation of Duverger's law:
Plurality voting creates a barrier to entry against new "third" parties. Economists should recognize importance of barriers to entry, which may be crucial determinants of long-run profit. Politicians' profits = corruption. So Duverger's law suggests that electoral systems may differ in their effectiveness at deterring political abuse of power, a basic goal of democracy.

Simple theoretical model (Myerson, *G.E.B.*, 1993). Each party publicly chooses a corruption level in $\mathbb{R}_+$ and a policy position in \{Left, Right\}. A party's profit is its seats multiplied by its corruption level; max $E(profit)$. A voter gets his utility of legislative majority's policy, minus parties' profits. Will multiparty democratic competition force party profits to 0?

4-party example: L1 and L2 are leftist parties, R1 and R2 are rightist parties. L1 and R1 are similarly corrupt, L2 and R2 are noncorrupt. (Fixed, known.) Half of voters prefer left policy, half prefer right, all prefer noncorrupt.

Plurality voting admits a Duvergerian equilibrium where only L1 and R1 are serious contenders. A leftist voter deviating from L1 to L2 is more likely to affect a close L1-R1 race unfavorably, than a close L2 race favorably. This equilibrium may be focal when L1 and R1 are established parties.

Coordination game. Corrupt parties hold left/right policy question as hostage. Shift to less corrupt equilibrium would require coordinating leaders before the election; but elections are our institution for choosing our leaders! (Single-positive voting for candidates in multi-seat districts demands even more coordination of voters to maximize seats for a bloc; reforms in Japan.)

With approval voting, such a corrupt equilibrium does not exist. Even if a close L1-R1 race is most likely, voters approve L2 or R2 as well as their preferred serious contender. So a perception of weakness cannot become a self-fulfilling prophecy against the noncorrupt parties.

List PR (pure, assuming large M) also has no corrupt equilibrium in this model, because small parties can contend seats as seriously as large parties.

Worst-punishing ballots creates barriers to exit that protect corrupt parties. Under single-negative voting, or even Borda voting, a perception that "only L2 or R2 can win" would encourage voting against the less-preferred among L2 and R2, which strengthens L1 and R1 until they are serious contenders.
6. Other perspectives on democratic incentives to reduce corruption.

Barriers to entry can depend on many other factors in political systems. A list-PR system with a national 5% quota to win seats (Germany) may create higher barriers to entry than a list-PR system with 20-seat districts. Freedom of the press, freedom of assembly, and restrictions on patronage are important in democratic theory because they lower political barriers to entry.

Recent Italian electoral reforms to reduce corruption went from PR to a system where most seats are allocated by plurality voting. In my model, such a reform decreases effectiveness against corruption! Sharpened question: what assumption of my model may be wrong, causing it to overestimate the effectiveness of multiparty PR against corruption? (Difficulties identifying corruption should not affect electoral comparisons.)

Critical assumption: government policy (left/right) is determined by a majority coalition in which noncorrupt parties can participate as well as corrupt parties that advocate the same left/right policies. But if most parties are corrupt, a small anticorruption party would be very unlikely to be invited into the governing coalition. So a vote that wins an extra seat for a small anticorruption party is as wasted as if it got no seats. Then only a leader of a major party that is a serious contender for a legislative majority can gain support by pledging to fight corruption. This shows advantage of a political system dominated by two large parties. With corruption as politicians' profit, we must be concerned about collusion among the leaders of the two big parties on this question, but their independent incentive is to undercut each other on the corruption dimension.

A related argument for presidentialism: Voters know that a committed anti-corrupt winner of a presidential election would have power to reduce corruption, So presidential candidates can effectively compete against corruption.
7. Legislative bargaining and party structure.

Rational voters must take account not only probability of affecting winners, but also probability that winners will be able to affect government policy. Voting equilibria cannot be fully analyzed without a theory of post-election bargaining among elected officials. But modeling both is difficult.

Austen-Smith and Banks (A.P.S.R. 1988) consider a parliament where the largest party gets first chance to form a government; expected to succeed. So voters care about which party is largest, and may rationally (single-positive) vote for more preferred among two serious contenders for largest, rather than for most preferred among all parties, even in a pure PR system. AS&B find an equilibrium where leftist and rightist parties contend for largest; a small centrist party loses votes from center until there is a serious question of it falling below minimal quota for seats, in which case leftist or rightist party would govern alone without a moderating coalition partner.

Now consider a constitutional change that eliminates any special role for the largest party in coalitional bargaining. As a result, more voters should sincerely vote for most-preferred parties, and so largest two parties should lose seats to smaller parties, as recently in Israel.

\[
\begin{array}{ccc}
\text{Parties} & L & C & R \\
\text{Voters} & 1111111111111111111 & crrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr
\end{array}
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(Proliferation of small parties could also be inhibited by smaller districts, according to the ≤M+1 law for list PR.)

In any parliamentary system, the anticipated process of post-election bargaining to form a governing coalition creates a kind of increasing returns to scale for large national parties that can be serious contenders for leadership of the governing coalition. This may account for big differences between cross-district coherence of legislative parties in presidential and parliamentary countries that use the same electoral system (Britain, USA).

Shugart and Carey (Presidents and Assemblies, 1992) compare different presidential systems and find that greater legislative powers of the president tend to decrease party coherence in legislatures. (Russia as extreme case.)
8. Conclusions
Assuming a given number of equally serious candidates, best-rewarding ballot => more differentiated specialization of candidates worst-punishing ballot => candidate clustering, similar (broad) appeals Intermediate majoritarian properties of Borda and approval voting.

Dropping symmetry assumption, number of serious candidates (parties) may depend on ballot structure and counting rule. Duverger's law, (≤)M+1 law. Electoral barriers to entry (and exit) of serious candidates, and democratic incentives to reduce political corruption.

Importance of post-election bargaining among elected officials. Initiatives against widespread corruption can be usefully promised only by candidates who are contenders for high office (majority leader or president). Voters' desire to influence post-election coalition formation can create a kind of increasing returns to scale for larger national legislative parties in parliamentary systems, different from presidential.

Economic analysis aims to expose the workings of incentives in complex social institutions, so that institutional reforms can be debated with a better understanding of their long-term consequences. In a world of increasing democracy, questions of constitutional design in new and reforming democracies have generated a great need for economic analysis of political institutions.

When objective academic expertise is lacking, the principal voices in constitutional debates are likely to be those of professional politicians, whose constitutional preferences may depend mostly on their short-term perceptions of how the political game might be best restructured to their own advantage. (Analogy to classic tariff debates in international trade, where economists have helped to express the interests of large unfocused groups.) Economic analysis of constitutional structures should serve to expand the scope of public constitutional debates, by offering more perspectives on how a change in the rules of the political game may affect rational political behavior and the resulting performance of democratic government for the welfare of its citizens.