Spatial competition among financial service providers and optimal contract design

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Motivation

- Einav et al. (2010), Einav et al. (2013) on health care contracts
- Einav et al. (2012) on auto loans contracts
- We try to make few restrictions on contracts and we add a variety of different supply-side market structures
- Entry and dynamics of bank branches in Assuncao et al. (2012)
Agents of type $\theta$ have preferences

$$u(c, a|\theta),$$

where $c$ is consumption, and $a$ is hidden or observed action. Agents can borrow capital $k$ from the intermediaries to augment their labor effort. Investment can be observed or unobserved.

$\theta$ is a vector that allows to consider different risk aversion, different risk types and different aversion to effort

$P(q|k, a, \theta)$ is a probability to reach the output $q$ that depends on agent’s type $\theta$ and the effort $a$ exercised by an agent with borrowed capital $k$ that augments the agent’s labor effort.

The features of model contract can be related to empirically observed financial contracts with complex structure.
The agents are distributed uniformly in $\mathbb{R}^1 : [0; 1]$ with total market mass set to one. The household cost to access financial services is equal to

$$\bar{L} \ast |l - l_i|$$

where $l$ is location of the agent, $l_i$ is location of bank $i$, $\bar{L}$ is a market spatial cost. The agents at location $l$ choose to go to bank 1 if the offer of utility from bank 1 satisfies participation constraint

$$\omega_1(\theta) - \bar{L} \ast |l - l_1| \geq \hat{\omega}_0(\theta)$$

where $\hat{\omega}_0(\theta)$ is autarky value. Location can be a stand in for other characteristics that make a given contact more or less attractive. Simplification: additive disutility. Can allow promise to be location-specific, but Nash equilibrium might not exist. Or, one uniform promise, in which case bank 1 gets market share $\mu(\omega_1(\theta), l_1)$ which is a fraction of agents attracted by bank 1 offer.

Easily generalized in $\mathbb{R}^2$ to model realistic road structure and village network.
Building block - standard mechanism design problem

The optimal contract to maximize the bank surplus extracted from each agent:

\[
S\{\omega(\theta)\} := \max_{\pi(q,c,k,a|\theta)} \left[ \sum_{\theta} \rho(\theta) \sum_{q,c,k,a} \pi(q,c,k,a|\theta) [q - c - k] \right]
\]

where \( \pi(q,c,k,a|\theta) \) is a probability distribution over the vector \((q,c,k,a)\) given the agent's type \(\theta\).

Mother Nature/Technology Constraints:

\[
\forall \{\bar{q}, \bar{k}, \bar{a}\} \in Q \times K \times A and \forall \theta \in \Theta
\]

\[
\sum_{c} \pi(\bar{q}, c, \bar{k}, \bar{a}|\theta) = P(\bar{q}|\bar{k}, \bar{a}, \theta) \sum_{q,c} \pi(q, c, k, a|\theta)
\]

Incentive Compatibility Constraints for action variables:

\[
\forall a, \hat{a} \in A \times A and \forall k \in K and \forall \theta \in \Theta:
\]

\[
\sum_{q,c} \pi(q,c,k,a|\theta) u(c,a|\theta) \geq \sum_{q,c} \pi(q,c,k,a|\theta) \frac{P(q,k,\hat{a},\theta)}{P(q,k,a,\theta)} u(c,\hat{a}|\theta)
\]

Utility Assignment Constraints (UAC):

\[
\forall \theta \in \Theta
\]

\[
\sum_{q,c,k,a} \pi(q,c,k,a|\theta) u(c,a|\theta) = \omega(\theta); \omega(\theta) \geq \omega_0(\theta)
\]

can add other constraints (adv. selection, LC, unobserved \(k\)), multi period dynamic version is possible.
Pareto frontier with single type

\[ u(c, a) = \frac{c(1 - \sigma(\theta))}{(1 - \sigma(\theta))} + \chi(\theta) \frac{(1 - a)\gamma(\theta)}{\gamma(\theta)} \]

\[ P(q = \text{high}|k, a, \theta) = p(q = \text{high}|a) f(\theta) k^\alpha, \alpha = 1/3 \]
\[ P(q = \text{low}|k, a, \theta) = 1 - P(q = \text{high}|k, a, \theta) \]

Single type with \( f(\theta) = \theta = 0.6 \)

From FOC for one branch (also two-branch) monopoly wrt to utility promise

\[ \frac{S'(\omega)}{S(\omega)} = -\frac{\mu'(\omega)}{\mu(\omega)} \]

increase in promise \( w \) decreases surplus and increase market share (if solution is interior)
Surplus and Market Share elasticity
two-branch monopoly (collusion), FI
Two Branch Monopoly (collusion):
Full Information vs Moral Hazard

locations of branches are symmetric at [1/4; 3/4]

(a) Profit

(b) Profit maximizing utility offer
Two Branch Monopoly: optimal contract schedules

Full Information vs Moral Hazard

locations of branches are symmetric at \([1/4; 3/4]\)

**Consumption**

- **First-best** vs **Moral Hazard**

**Labor (effort)**

- **First-best** vs **Moral Hazard**

**Output**

- **First-best** vs **Moral Hazard**

**Capital borrowed**

- **First-best** vs **Moral Hazard**
Distinguishing Contracts: Moral Hazard from Full Information (example)

Contract properties, expected and conditional

**Consumption**

- First-best
- Moral hazard

**Labor (effort)**

- First-best
- Moral hazard

**Output**

- First-best
- Moral hazard

**Capital borrowed**

- First-best
- Moral hazard

**Conditional consumption for moral hazard**
Market Structures

- collusion (multi-branch monopoly)
- sequential Nash equilibrium (SNE) with full commitment to location and contract (business model)
- full commitment to location and simultaneous Nash on contracts (partial commitment)
- simultaneous Nash in location and contracts (no commitment)
- typically outcomes are efficient - proofs or numerical validation
- except in the case with no commitment at some spatial costs
Sequential Equilibrium (Sequential SNE)

Let’s consider a sequential game with first bank coming to location $l_1$.

This bank offers utility $\overline{\omega_1}$ and it captures the share of the market $\mu_1(\overline{\omega_1})$ that provides a surplus $S^{\overline{\omega_1}}$ for the first bank to maximize.

The first bank anticipates the entry of the second bank and it chooses its location and offer with respect to the best possible response by the second bank.

For the second entrant we find optimal location $l_2$ and type-dependent utility offer $\overline{\omega_2}$ for any given choice of the first entrant.
Conditional on bank 1 choice of location $l_1$ and offer of utility $\omega_1$ the second bank gets a share $\mu_2(\omega_2(\omega_1, \overline{l_1}), l_2(\omega_1, \overline{l_1}), \omega_1, \overline{l_1})$ of the market with agents at all locations $l$ where the following is satisfied

$$\omega_2(\omega_1, \overline{l_1}) - L * |l - l_2(\omega_1, \overline{l_1})| \geq \hat{\omega}_0$$

$$\omega_2(\omega_1, \overline{l_1}) - L * |l - l_2(\omega_1, \overline{l_1})| > \omega_1 - L * |l - \overline{l_1}|$$

Second bank profit to maximize is

$$P_2(\omega_2(\omega_1, \overline{l_1}), l_2(\omega_1, \overline{l_1}), \omega_1, \overline{l_1}) :=$$

$$\text{maximize } S^{\omega_2} \mu_2(\omega_2, l_2, \omega_1, \overline{l_1})$$
Sequential Equilibrium (Sequential SNE)

first entrant problem

After second entrant makes the offer the first entrant gets market share \( \mu_1(\omega_2(\omega_1, l_1), l_2(\omega_1, l_1), \omega_1, l_1) \) with agents at all locations \( l \) where the following is satisfied

\[
\omega_1 - \bar{L} \star |l - l_1| \geq \hat{\omega}_0
\]

\[
\omega_1 - \bar{L} \star |l - l_1| > \omega_2(\omega_1, l_1) - \bar{L} \star |l - l_2(\omega_1, l_1)|
\]

First bank profit to maximize is

\[
P_1(\omega_2(\overline{\omega_1}, \overline{l_1}), l_2(\overline{\omega_1}, \overline{l_1}), \overline{\omega_1}, \overline{l_1}) := \maximize_{\{\overline{\omega_1}, \overline{l_1}\}} S^{\overline{\omega_1}} \mu_1(\omega_2(\overline{\omega_1}, l_1), l_2(\overline{\omega_1}, l_1), \overline{\omega_1}, \overline{l_1})
\]
Nash equilibrium in case of two market entrants would be defined by \( \{\omega_1^*, l_1^*, \omega_2^*, l_2^*\} \) that satisfy

\[
S^{\omega_2^*} \mu_2(\omega_2^*, l_2^*, \omega_1^*, l_1^*) \geq S^{\omega_2} \mu_2(\omega_2, l_2, \omega_1^*, l_1^*)
\]
\[
S^{\omega_1^*} \mu_1(\omega_2^*, l_2^*, \omega_1^*, l_1^*) \geq S^{\omega_1} \mu_1(\omega_2^*, l_2^*, \omega_1, l_1)
\]

\( \forall \{\omega_1, l_1, \omega_2, l_2\} \)

variations on the theme: utility and/or location competition
Comparing Market Structures:
Real value to households

Collusion/Two Branch Monopoly vs Sequential Nash with full commitment
(Full Information)

\[ V_{\overline{\omega_1}, \overline{\omega_2}, \overline{l_1}, \overline{l_2}}(l) = \max(\overline{\omega_1} - L \cdot |l - \overline{l_1}|, \overline{\omega_2} - L \cdot |l - \overline{l_2}|, u_0) \]
The first entrant commits to location and utility offer anticipating subsequent entrance of the second entrant with location choice and utility offer.

Bertrand competition at $0 \leq \bar{L} \leq 1$, both banks at central location with maximum utility offer possible, zero profits.

Spatial separation at $1 < \bar{L}$, the second entrant has advantage due to credible threat to take the market away unless the first entrant commits to worse location and utility offer.

Local monopoly at high spatial cost with equal share of market and equal profits.
Competition: Out of Equilibrium properties

location choice for the first entrant is fixed at optimal equilibrium value

(a) Profit wrt $\omega_1$, first entrant

(b) Optimal utility offer $\omega_2$ wrt $\omega_1$

(c) Location choice $x_2$ wrt $\omega_1$
Competition: Full Information

full commitment on location, no commitment on utility offer

(a) Utility offer choice
(b) Location choice
(c) Profit

- The first entrant commits to location, then the second entrant commits to location, all anticipate subsequent simultaneous Nash equilibrium on utility offer.
- The first entrant commits to central location, the second entrant commits to marginal niche location at low spatial cost.
- Each bank secures non-zero profit at $\bar{L} > 0$.
- The first entrants dominates the market at intermediate spatial costs by credibly forcing the second entrant to commit to worse location.
## Competition: full information

### partial commitment, no commitment equilibria (all different)

### partial commitment, full information

<table>
<thead>
<tr>
<th>Spatial Cost $L = .5$</th>
<th>bank</th>
<th>location</th>
<th>utility offer</th>
<th>market share</th>
<th>profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>bank</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.44</td>
<td>3.15</td>
<td>84%</td>
<td>0.12</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.93</td>
<td>2.99</td>
<td>16%</td>
<td>0.08</td>
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</table>

<table>
<thead>
<tr>
<th>Spatial Cost $L = 1$</th>
<th>bank</th>
<th>location</th>
<th>utility offer</th>
<th>market share</th>
<th>profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>bank</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.43</td>
<td>2.13</td>
<td>56%</td>
<td>0.42</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.94</td>
<td>2.38</td>
<td>44%</td>
<td>0.25</td>
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</table>

<table>
<thead>
<tr>
<th>Spatial Cost $L = 2$</th>
<th>bank</th>
<th>location</th>
<th>utility offer</th>
<th>market share</th>
<th>profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>bank</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.45</td>
<td>2.44</td>
<td>60%</td>
<td>0.36</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.71</td>
<td>2.21</td>
<td>36%</td>
<td>0.27</td>
<td></td>
</tr>
</tbody>
</table>

### no commitment, full information

<table>
<thead>
<tr>
<th>Spatial Cost $L = 0.5$</th>
<th>bank</th>
<th>location</th>
<th>utility offer</th>
<th>market share</th>
<th>profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>bank</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.5</td>
<td>3.15</td>
<td>50%</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.5</td>
<td>3.15</td>
<td>50%</td>
<td>0.0</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Spatial Cost $L = 2$</th>
<th>bank</th>
<th>location</th>
<th>utility offer</th>
<th>market share</th>
<th>profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>bank</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.36</td>
<td>2.33</td>
<td>58%</td>
<td>0.39</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.72</td>
<td>2.14</td>
<td>39%</td>
<td>0.32</td>
<td></td>
</tr>
</tbody>
</table>
Costly entry, multiple entry, multi-stage entry

**SNE with entry cost**

Entry cost $c_E = 0.3$, Spatial Cost $L \in [0, 1)$ (entry cost absorbed)

<table>
<thead>
<tr>
<th>bank</th>
<th>location</th>
<th>utility offer</th>
<th>market share</th>
<th>profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>center</td>
<td>max to cover entry</td>
<td>50%</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>center</td>
<td>max to cover entry</td>
<td>50%</td>
<td>0</td>
</tr>
</tbody>
</table>

Entry cost $c_E = 0.3$, Spatial Cost $L = 2$ (deterrence of entry)

<table>
<thead>
<tr>
<th>bank</th>
<th>location</th>
<th>utility offer</th>
<th>market share</th>
<th>profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>center</td>
<td>2.44 (to prevent entry)</td>
<td>83%</td>
<td>0.18</td>
</tr>
<tr>
<td>2</td>
<td>border</td>
<td>2.44 (doesn’t enter)</td>
<td>0%</td>
<td>0</td>
</tr>
</tbody>
</table>

**SNE for three entrants**

Spatial Cost $L = 2$

<table>
<thead>
<tr>
<th>bank</th>
<th>location</th>
<th>utility offer</th>
<th>market share</th>
<th>profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.6250</td>
<td>2.3746</td>
<td>43.5%</td>
<td>0.2723</td>
</tr>
<tr>
<td>2</td>
<td>0.5000</td>
<td>2.3759</td>
<td>6.5%</td>
<td>0.0406</td>
</tr>
<tr>
<td>3</td>
<td>0.3750</td>
<td>2.3759</td>
<td>44%</td>
<td>0.2750</td>
</tr>
</tbody>
</table>

**SNE with simultaneous branch opening**

Spatial cost = 2

<table>
<thead>
<tr>
<th>Utility offer</th>
<th>Bank 1 branch 1</th>
<th>Bank 1 branch 2</th>
<th>Bank 2 branch 1</th>
<th>Bank 2 branch 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Share of market</td>
<td>29.5%</td>
<td>28.5%</td>
<td>42%</td>
<td>0%</td>
</tr>
<tr>
<td>Location</td>
<td>0.8350</td>
<td>0.0913</td>
<td>0.5750</td>
<td>0.5500</td>
</tr>
<tr>
<td>Profit</td>
<td>0.1909</td>
<td>0.2220</td>
<td>0.2718</td>
<td>0</td>
</tr>
</tbody>
</table>
We introduce heterogeneity amongst the agents in ability $\theta$.

Safer types have stricter larger expected output produced at all effort levels, riskier type have larger probability of failure.

Option for banks to offer type dependent contracts.

(a) Surplus, FI

(b) Surplus elasticity, FI
Adverse selection, two unobserved types

Utilities are tied to one another, cannot be varied independently, anymore - must satisfy Adverse Selection constraints.

Utility Assignment Constraints (UAC) - type dependent offer from bank is set at utility level $\bar{\omega}(\theta)$:

$$\forall \theta \in \Theta \sum_{q,c,k,a} \pi(q, c, k, a|\theta) u(c, a|\theta) = \bar{\omega}(\theta)$$

For two types

$$\theta \in \{\theta_1, \theta_2\}$$

truth-telling constraints (TTC) can be combined with UAC (given offer of utility per type) to result in explicit bounds for TTC:

$$\bar{\omega}(\theta_2) \geq \sum_{q,c,k,a} \left[ \pi(q, c, k, a|\theta_1) \frac{P(q, |k, a, \theta_2)}{P(q, |k, a, \theta_1)} u(c, a) \right]$$

$$\bar{\omega}(\theta_1) \geq \sum_{q,c,k,a} \left[ \pi(q, c, k, a|\theta_2) \frac{P(q, |k, a, \theta_1)}{P(q, |k, a, \theta_2)} u(c, a) \right]$$
Endogenous location choice - Adverse Selection
SNE on location plus simultaneous Nash on contracts

(a) Profits

(b) Utility choice

(c) Location choice
Local monopoly at fixed locations [1/4; 3/4]

Interstate banking allowed under:
  a) full information competition on contracts;
  b) adverse selection competition on contracts;

![Graph showing welfare implications of Financial Liberalization experiment]
Relationship banking and subprime lending

- Full commitment on contracts and locations with banks operating in different information regimes
- The first entrant (a local bank or an incumbent) comes to the area and it studies his customers long enough to gain full information about their true types establishing relationships in the process
- The first entrant anticipates that there will another player in the area (a global bank or a challenger) and it commits to location and contract menu based on established relationships trying to prevent a challenger from taking over his market share
- The global bank doesn’t have information advantage that the first entrant possesses and he operates in adverse selection regime
- The global bank already knows both location and contracts offered by the local bank, so potentially it is capable to undercut the competitor if such strategy is profit optimal.
Local Information Advantage: Market Shares

(a) Incumbent (full information)  (b) Challenger (adverse selection)

- at low spatial costs the incumbent gets exactly 100% of good (safer) types
- the challenger gets exactly 100% of bad (riskier) types
- the global bank specializes in what can be called "subprime lending"
- the local bank keeps relationships established with the better clients with no subprime activity
Local Information Advantage: Profits and Locations

(a) Profit

(b) Location

- both local and global bank are left with non-zero profit even at zero spatial costs
- overall, at most spatial costs the local bank makes larger profit and he keeps close to central location
- at large spatial cost both are in local monopoly condition
parameters of preferences and technology + obstacles \( \Rightarrow \omega \Rightarrow S(\omega), S'(\omega)/S(\omega) \)

trace out the surplus function as promised utility is varied. key in market structure is elasticity, level may also convey info about obstacles. otherwise, no identification of contracts and parameters conditioned on frontier.

promised utility is not observed - we do see profits but with measurement error. we infer true underlying promised utilities with error

variation in observed spatial costs \( L \), as in moving in a cross section of provinces, cause variation in market size elasticity and this effectively traces out surplus frontier

with a parameterized error structure we back out the distribution of true profits, given observed profit

subject to measurement error in profits, and measurement error in location, data on location and profits (hence utilities) allow market structure to be inferred

heterogeneity in some factor(s) impacting preferences and/or technology \( \Rightarrow \) menu of contracts \( \Rightarrow \) back out restrictions on the nature of market structure

adverse selection is distinguished from full information heterogeneity in both underlying contracts and their observables (insurance, default) as well as market outcomes (location, market share, profits)

the most power will be gained by consideration of the joint equilibrium determination of contracts and branch locations
Explicit Dynamics and Efficiency Considerations

- Virtually all of the previous can be extended to include explicit dynamics, i.e. multi-period contracts for borrowers, not simply dynamic IO considerations.
- In the context of sequential competition among a local incumbent and new entrant, it seems more likely that households cannot commit to stay with the local bank especially if there is a limit in the extent to which bank can front load the contract.
- This would be true either if the new entrant comes in as a surprise move, unanticipated, or if the incumbent anticipates.
- Thee outcome would almost surely not be information constrained efficient, as the first bank cannot take full advantage of long term contracts which are otherwise beneficial.
- This is the first instance where the IO equilibrium can be inefficient and may require some regulation.
We propose the following technique to order by rank all possible strategies using the metric we call ”distance to Nash”. Both banks enter the market simultaneously and use a strategy set of 

\[ G = \{\omega_1^*, \omega_2^*, l_1^*, l_2^*\} \]

with payoff 

\[ P_1(G) = S^{\omega_1^*} \mu_1(\omega_2^*, l_2^*, \omega_1^*, l_1^*) \]

\[ P_2 = S^{\omega_2^*} \mu_2(\omega_2^*, l_2^*, \omega_1^*, l_1^*) \]

At strategy set \( G \) in Nash equilibrium there exist no deviating strategy sets \( G_1 = \{\omega_1, \omega_2^*, l_1, l_2^*\} \), \( G_2 = \{\omega_1^*, \omega_2, l_1^*, l_2\} \) that would provide higher profit for any of the deviating banks. We can define and compute the following metrics

\[
\begin{align*}
  d(G, G_1) &= \max(P_1(G_1) - P_1(G), 0), \quad P_1(G_1) = S^{\omega_1} \mu_1(\omega_2^*, l_2^*, \omega_1, l_1) \\
  d(G, G_2) &= \max(P_2(G_2) - P_2(G), 0), \quad P_2(G_2) = S^{\omega_2} \mu_2(\omega_2, l_2, \omega_1^*, l_1^*)
\end{align*}
\]
Computing Nash equilibria:
Maximization of deviation

First step: we compute $P_1(G)$ and $P_2(G)$ for a trial strategy set of $G$. Then, in the second stage we solve

$$\max_{G_1} d(G, G_1)$$

subject to $P_1(G) > 0 : \forall G_1$

$$\max_{G_2} d(G, G_2)$$

subject to $P_2(G) > 0 : \forall G_2$

We denote the solution of those maximization problems as $d(G, G_1)$ and $d(G, G_2)$. Then we compute distance to Nash as

$$d(G, G_1, G_2) = d(G, G_1) + d(G, G_2)$$
Computing Nash equilibria:
Minimization of maximum deviation

In the final stage we solve

$$\min_G d(G) : \forall\{G, G_1, G_2\}.$$  

At Nash equilibrium $G_{Nash}$ the solution of this two-step optimization problem $d(G_{Nash}) = 0$. At all other strategies this function is strictly positive and well-defined. All possible strategies can be rank-ordered by their "distance from Nash" even if there is no true Nash equilibrium possible.
Distance to Nash

(a) Full Commitment Nash, FI

(b) No Commitment Nash, FI
Computing solutions to Max problems

- The lottery formulation of contract problem for agents is solved using LP solver Gurobi (http://www.gurobi.com/) via MATLAB interface.
- SIMD processor (http://en.wikipedia.org/wiki/SIMD) optimized arrays with Kronecker tensor products are used throughout to make computations highly efficient.
- Bank profit optimization is done using a globally convergent Augmented Lagrangian Pattern Search algorithm.
- Non-linear solver KNITRO (http://www.ziena.com/threealgs.htm) with multi-start option is used for robustness checks.