The University of Chicago
Department of Economics
HONORS ECONOMETRICS
PROBLEM SET #3

This problem set explores the last three topics we have covered in class. There are three question on regression algebra, 3 questions on hypothesis testing and 4 questions on model mis-specification.

Problem R1: Scaling and units of measurement

Consider the following regression model:

$$Y_i = \beta_0 + \beta_1 X_{1i} + \epsilon_i.$$

Let $\widehat{\beta}_0$ and $\widehat{\beta}_1$ denote OLS estimators. Let ω_x and ω_y denote scaling factors for X and Y, respectively. Thus $Y^* = \omega_y Y$ and $X^* = \omega_x X$. What is the relationship between:

a. $\widehat{\beta}_0$ and $\widehat{\beta}_0^*$
b. $\widehat{\beta}_1$ and $\widehat{\beta}_1^*$
c. $var\left(\widehat{\beta}_0\right)$ and $var\left(\widehat{\beta}_0^*\right)$
d. $var\left(\widehat{\beta}_1\right)$ and $var\left(\widehat{\beta}_1^*\right)$
e. $\widehat{\sigma}^2$ and $\left(\widehat{\sigma}^*\right)^2$
f. R_{xy}^2 and $R_{x^*y^*}^2$

Problem R2: Computing Regression Coefficients using Matrices

A Cobb-Douglas production function

$$Y_i = AK_i^{\alpha} L_i^{\beta} u_i$$

was estimated by OLS using the model:

$$y_i = \alpha k_i + \beta l_i + \epsilon_i$$

where $y_i = y_i^* - \overline{y}$, $k_i = k_i^* - \overline{k}$, and $l_i = l_i^* - \overline{l}$. Let $y_i^* = \log(y_i)$, $k_i^* = \log(k_i)$, $l_i^* = \log(l_i)$, and $\epsilon_i = \log(u_i)$.

The following data is computed from the 303 available observations:

$$\sum_{i=1}^n y_i^2 = 50.4 \quad \sum_{i=1}^n k_i^2 = 15 \quad \sum_{i=1}^n k_i l_i = 10$$
$$\sum_{i=1}^n l_i^2 = 20 \quad \sum_{i=1}^n y_i k_i = 12 \quad \sum_{i=1}^n y_i l_i = 20$$

a. Show that you can transform the Cobb-Douglas production function into the estimated model.
b. Obtain the OLS estimates for α and β.
c. Obtain the OLS estimate for $\widehat{\sigma}^2$.
d. Can you find R^2? Why or why not?

Problem R3: A problem in simple regression

Suppose that you run the simple regression of Y on X and obtain $\widehat{\beta}_{yx}$. If you run the regression of X on Y then:

$$\widehat{\beta}_{xy} = \frac{1}{\widehat{\beta}_{yx}}.$$

If true explain why, if false find a (very) simple expression relating both coefficients. Explain your result.

Problem MS1: Short and Long Regression

Let the short regression model be given by:

$$y = x_1\beta_1^S + u$$

Let the long regression model be given by:

$$y = x_1\beta_1^L + x_2\beta_2 + u$$

where the data are in deviations and x_1 and x_2 are both $nx1$ vectors. The only

difference is that the long regression holds constant the effect of x_2. In this question we explore the nature of "holding a variable constant." We wish to know how $\widehat{\beta}_1^S$ and $\widehat{\beta}_1^L$ relate to each other. In this problem you are best off following the methodology from class and using the A, M, Q matrices until the very end.

a. Compute $\widehat{\beta}_1^S$

b. Compute $\widehat{\beta}_1^L$.

c. Using the expressions from parts a. and b. find an expression relating both coefficients.

d. In what sense does the long regression hold constant the effect of x_2?

Suppose that you have the following function:

$$y = f(x_1, x_2)$$

e. Totally differentiate it with respect to x_1.

f. Explain your solution to part d. in these terms.

Problem MS2: Residual regression

Consider the same set-up as in question R3. Show that there are two ways in which you can obtain the long regression estimate for β_1^L. One is by performing the long regression. The other is by regressing y on the residuals of the regression of x_2 on x_1. What happens if you run a regression of the residuals of y on x_2 on the residuals of x_1 on x_2?

Problem MS3: Piece-wise linear regression

Consider the following example. A firm pays commisions based on sales in the following manner. There is a minimum commission level β_0 and a threshold level of sales x^*. Below the threshold level sales commissions increase linearly with sales at the rate β_1. After the threshold value, commissions continue to increase at a linear rate, but the rate is higher. In other words, the commission rate changes at the threshold value x^*. Construct a regression model that allows you to estimate the commision rates sales representatives obtain if you had data on sales commisions and total sales as well as the thresshold value. You must label the equation and be very explicit about all the variables that you use. Also, graph your solution.

Problem MS4: Milton Friedman's Permanent Income Model

Friedman's theory states that consumers do not consume a fixed fraction of their current income, but use asset markets to smooth their consumption. Therefore, the consumption function is given by:

$$Y_i^* = \alpha + \beta X_i^*$$

where Y_i^* and X_i^* denote permanent income and consumption , respectively. The problem is that these quantities are unobservable. Instead we observe:

$$\begin{aligned} Y_i^* &= Y_i + u_i \\ X_i^* &= X_i + v_i \end{aligned}$$

with u_i and v_i are measurement errors. We assume that $E(u_i) = E(v_i) = 0$, $var(u_i) = \sigma_u^2$, $var(v_i) = \sigma_v^2$ and that all covariances are 0.

a. Obtain an estimable version of the consumption function.

b. Find $plim\left(\widehat{\beta}\right)$.

c. Is the estimator consistent?

d. What can you say about the nature of the bias in $\widehat{\beta}$?

Problem HT1

You run two regressions on annual time series for the years 1935-1978:

$$\begin{aligned} \widehat{y} &= 50 + 0.2x_2 + 0.5x_3 - 2x_4, \qquad R^2 = 0.8 \\ \widehat{y} &= 100 + 0.3x_2 + 0.4x_3, \qquad R^2 = 0.76 \end{aligned}$$

Determine whether the following is true or false: The standard error for $\widehat{\beta}_4$ is $\frac{1}{\sqrt{2}}$.

Problem HT2

Suppose you run the following two regressions:

$$y = x_1\beta_1 + x_2\beta_1 + x_3\beta_1 + x_4\beta_1 + x_5\beta_1 + u$$

and

$$w = z_1\delta_1 + z_2\delta_2 + v$$

where $w = y - x_4$, $z_1 = x_1$, $z_2 = x_2 + x_4$.

a. State the joint null hypothesis that is testable by comparison of the SSE of these two regressions.

b. What is the numerator degrees of freedom for that test?

Problem HT3

Consider the Cobb-Douglas production function:

$$Y = \beta_0 L^{\beta_1} K^{\beta_2}.$$

Suppose that you have data on $\frac{Y}{K}$, $\frac{L}{K}$, and K. Can you test the hypothesis that there are constant returns to scale? If so how would you do it?