THE UNIVERSITY OF CHICAGO
DEPARTMENT OF ECONOMICS
Elements of Economic Analysis IV
Econ 203
SPRING 2001
PROBLEM SET 1

This problem set is due at the **beginning** of Lecture 5.

Problem 1: The competitive equilibrium of an endowment economy.

Consider a two good, two consumer endowment economy. Suppose that agent 1's endowment is given by:

$$\omega^1 = \left(\omega_x^1, \omega_y^1\right)$$

and agent 2's, endowment is given by:

$$\omega^2 = \left(\omega_x^2, \omega_y^2\right),$$

where we assume that $\omega_x^1 > \omega_x^2$ and $\omega_y^2 > \omega_y^1$. Also assume that agents preferences are identical and given by:

$$U(x, y) = \log(x) + \log(y).$$

Finally, let the prices of good x and good y be given by p_x and p_y.

a. Write down each agent's maximization problem.

b. Solve each agent's maximization problem. You should obtain x_i^*, y_i^*, and η_i^* for $i = 1, 2$ where η_i^* denotes the Lagrange multiplier on agent $i's$ budget consraint.

c. Show that the budget constraint for each agent holds with equality. That is, show that the agents spend their entire income.

c. Find X^* and Y^*, the aggregate demands for the two goods.

d. What are the equilibrium conditions for this economy?

e. Find the market clearing price for the $x - market$.

f. Use your answers to part c and part e to show that at the relative price you found in part c the $y - market$ must clear. That is, determine each agent's consumption of good y. This is Walras' law.

g. Now use the market clearing condition in the $y - market$ to find the equilibrium price. How much of good y does each agent consume?

h. Determine the values of η_i^*.

i. Graph the solution. Label your graphs clearly.

j. Would the answer be the same if I assumed that $p_x = 1$? Explain your answer clearly.

k. Who trades what with whom?

Problem 2: The planner's problem.

Assume the same economy as in problem 1, but consider the following welfare function:

$$\lambda U_1 (x_1, y_1) + (1 - \lambda) U_2 (x_2, y_2)$$

We can interpret this as a central planner that wishes to allocate the economy's endowment between the two consumers. λ is descriptive of how important agent 1 is to the central planner. Therefore, a large value of λ indicates to us that the planner gives a lot of weigh to agent 1, while the reverse is true if λ is small. The planner's problem is to maximize society's welfare by allocating consumption among the two consumers. The solution will be a Pareto optimum. We can obtain all possible Pareto optima by varying the λ weights of the problem.

a. Write down the planners problem. (Hint: there should be two constraints because aggregate consumption of each good can not exceed the economy's endowment). Label the multipliers ϕ_1 and ϕ_2.

b. What are the first order conditions of the planners problem (Hint: There should be 6)

c. Find the solution to the planner's problem and label the quantities x_i^p and y_i^p for $i = 1, 2$.

If we compare the first order conditions of the planners problem, to the first order conditions of the competitive equilibrium in problem 1, we can obtain a relationship bewtween ϕ_1, ϕ_2, λ, p_x, and p_y so that the CE and the planner's problem give us the same solution. This called **decentralizing** the planner's problem.

d. Decentralize the planners problem (That is, find the equilibrium prices and the value of λ so that the two problems have the same solution.)

e. What does this tell you about the economy? (Hint: Think about the welfare theorems)