This Problem Set is due on Friday NOV. 30 , 2001 *before* the TA session.

Please be as neat as possible since no effort will be made to understand illegible writing. When possible, put your answers in a box. Write the name of the people whom you worked with at the top of the first page.

Problem 1: Social Security

Consider a world in which there are two types of agents: young workers who earn an amount y on their working years, and old retirees who earn nothing. A generation born in period t will have preferences over consumption while young C_0^t and old C_1^t of:

$$U\left(C_0^t, C_1^t\right) = 2(C_0^t)^{1/2} + 2\beta(C_1^t)^{1/2}$$

where $0 < \beta < 1$ reflects a preference for consumption while young.

Each period t there an N_t new young workers born, each of whom produces y with certainty in their youth. The youth population N_t evolves as : $N_{t+1} = (1+n)N_t$. There is a bond market which pays a constant, riskless, real interest rate of $r > 0$, paid "overnight" on savings.

This problem concerns an "unfunded" pension system.

a) Briefly describe (briefly and to the point) the difference between funded and unfunded pension systems. Give an example of countries that have one system or the other.

The government taxes each young worker an amount τ so that it raises the following total revenue: $G = \tau N_t y$. It distributes this equally among the old. Each old agent will get G/N_{t-1} or:

$$\frac{G}{N_{t-1}} = g_{t-1} = \tau y \frac{N_t}{N_{t-1}}$$

Since population is growing at a rate n, we have:

$$g_{t-1} = \tau y(1+n)$$

Notice that, since population growth is constant, g_t does not vary with time, so we will write merely g.

b) What are the relevant budget constraints of a generation t person, as a function of t and S_t (where S_t is the private savings of the individual)?

c) Set up the maximization problem of the individual.

d) Solve the maximization problem and find the optimal value of S_t as a function of τ, y, n and r..

e) Do some comparative statics and explain what happens to private savings (S_t) when each of the relevant variables changes. In particular, is the agent better or worse off if $n < r$.

f) What happens to private savings if $n > r$? Discuss.

g)For the U.S. at the moment, is $n > r$?

h) Optional. Read "Proposals to Restructure Social Security" by Peter Diamond, in the *Journal of Economic Perspectives* (1996). (also on the course webpage). Do you think the US SS System should be privatized?