

THE UNIVERSITY OF CHICAGO
DEPARTMENT OF ECONOMICS
Elements of Economic Analysis IV
Economics 203
FALL 2001
PROBLEM SET 4

This problem set is due at the **beginning** of the TA session on Friday.

Problem 1: Allowances, deductions and the income tax.

Consider an economy with 10 households who have the following utility function:

$$u(c, l^s) = 4\sqrt{c} - l^s.$$

The household's only income is derived from wages. Suppose there are 40 firms in the economy. They all have the same production function given by:

$$f(l^d) = l^d - \frac{1}{2}l^2.$$

- Define an equilibrium for this economy.
- Compute the equilibrium for this economy under the assumption that there is no government intervention. What is the equilibrium wage rate? What is equilibrium labor supply?

Suppose the government levies a uniform $\tau = 25\%$ income tax.

- What is the new equilibrium wage rate? How much labor does the household supply?
- How much revenue does the government obtain?

Suppose an alternative policy. Now, the government raises the tax rate to $\tau = 50\%$, but gives the households a lump sum transfer t .

- What is the household's new budget constraint and maximization problem?

- What is the new equilibrium, as function of t ? How do wages and equilibrium labor supply and consumption compare under the different tax schemes? Be precise and provide all the economic intuition necessary to support your answer. (Hint: You may wish to run your own simulation by plugging in different values for τ and t and ranking the policies by which gives higher utility to the consumer. You could also use excel and do a really nice analysis.)

Problem 2: Capital Taxation

Suppose that a household lives for two periods and has the following utility function:

$$U(c_1, c_2) = u(c_1) + \beta u(c_2).$$

Suppose the household earns w_1 in the first period and w_2 in the second period. There is a perfect capital market in which the household can borrow and lend all it wants at a fixed interest rate r . (If the household lend s , then it receives $(1 + r)s$ back.)

- a. Write down the constraint that the household faces in each period.
- b. Write down the present value budget constraint.
- c. Write down the household's maximization problem.
- d. Express the first order conditions as "MRS=price ratio." What is the economic interpretation of the condition?

Suppose that the government levies a consumption tax, τ_c , in both periods.

- e. What is the new optimization problem the household faces?
- f. Is the optimality condition the same or different? Explain the economics behind your answer clearly.

Suppose that now there is no consumption tax, but there is a proportional tax, τ_I , on all income (including interest income in the second period.

- g. Write down the new maximization problem.
- h. Does the optimality condition differ? Explain the economic intuition of this result.

Suppose that we give preferential treatment to savings. Assume that savings are tax exempt in the first period, but are taxed along with the principal in period 2.

- i. What is the new maximization problem?
- j. Is the optimality conditions the same? What are the economics of this answer.
- k. Show that this scheme is equivalent to a consumption tax.

Now assume that $u(c_i) = \log(c_i)$, $\beta = 0.8$, $w_1 = w_2 = 1$, and $r = 0.25$. Assume the government must raise revenue $g = 0.5$ in each period. Compute the necessary taxes that guarantee this level of government expenditure (these may differ across schemes and within a particular scheme, across time periods). How would the consumer rank the tax schemes? Explain the economics of the problem.