The University of Chicago
Department of Economics
Elements of Economic Analysis IV
Problem Set 3

This Problem Set is due on FRIDAY NOVEMBER 9, 2001 *before* the TA session.

Please be as neat as possible since no effort will be made to understand illegible writing. When possible, put your answers in a box. Write the name of the people whom you worked with at the top of the first page.

Problem 1: Firing Costs

Suppose that there are 2 industries, cars and computers, **each with 10 firms**. Suppose, also, that there are two periods and let the production function of the car industry in period 1 be given by

$$f_1^{car} = 2\sqrt{l^d}$$

while a firm in the computer industry has the following production function in period 1:

$$f_1^{PC} = \sqrt{l^d}.$$

Therefore in period 1 the car industry is more productive. Suppose that in period 2 productivities switch. Therefore:

$$f_2^{car} = \sqrt{l^d}$$

and

$$f_2^{PC} = 2\sqrt{l^d}.$$

Notice that in each period there is a high productivity industry and a low productivity industry. The car industry suffers an adverse productivity shock, while the computer industry improves its productivity.

Suppose that there are **100 identical workers** in the economy. Their utility function is given by:

$$u(c, l^s) = 2\sqrt{c} - l^s.$$

Part 1: No Government Intervention.

a. Find labor demand in the car industry in period 1 and period 2, assuming that you normalize the price of cars to 1.

b. Find labor demand in the computer industry in period 1 and period 2, assuming that you normalize the price of computers to 1.

c. Suppose that consumers care about "consumption" which is an aggregate of cars and computers. If you normalize the price of the consumption good to 1, what is the consumer's budget constraint?

d. Write down and solve the individual's utility maximization problem. What is the consumer's labor supply?

e. Find the market clearing wage in each period. (Hint: Don't forget to aggregate.)

f. Find aggregate employment in each period. How many workers are employed in the computer industry? How many are employed in the car industry?

Part 2: The Government Intervenes.

Suppose that the government introduces a firing cost, which we denote by f. The firing operates in a similar way to a tax. When firms lay off workers they save on the wage bill. By making lay-offs costly, the government attempts to discourage lay offs. In the analysis that follows, we assume that firms transfer a fraction f of the wage bill to the government, if they lay-off workers.

g. How does the firing cost affect the consumer labor supply problem?(Hint: Do consumers ever fire workers?)

h. How does the firing cost affect the computer industry's labor demand (Hint: Do they fire workers between period 1 and period 2?)

i. How does the firing cost affect the car industry's labor demand (Hint: Do they fire workers between period 1 and period 2?)

The firing restriction affects one of the two industries. Notice that now, we can not solve that industry's profit maximization problem period by period, like we did in part 1! Now, the hiring decision in period 1 affects the fine that the industry pays in the second period. The affected industry solves:

$$\max_{\{l_1^s, l_2^s\}} 2\sqrt{l_1^d} + \sqrt{l_2^d} - w_1 l_1^d - w_2 l_2^d - f\left(w_1 l_1^d - w_2 l_2^d\right).$$

Notice that the change in the wage bill between periods 1 and 2 is exactly $w_1 l_1^d - w_2 l_2^d$. Firm profits decline by $f\left(w_1 l_1^d - w_2 l_2^d\right)$ since that amount is transferred to the government.

j. Find labor demand in each period for the affected industry.

k. Find the market clearing wage in each period. (Hint: if the firing restriction doesn't affect one industry, then its labor demand is unchanged. In part j you found the labor demand for the affected industry! Just add them to get aggregate labor demand and clear the market.) You should obtain an expression for the wage in terms of f.

Part 3: A simple numerical example.

Suppose that $f = 0.1$.

m Find the market clearing wage in each period.

n. Find aggregate employment in each period.

o. Find employment in the car industry in each period.

p. Find employment in the computer industry in each period.

q. Fill in the following chart:

f	l_1^{car}	l_2^{car}	l_1^{PC}	l_2^{PC}	AL_1	AL_2	w_1	w_2
0								
0.1								

r. Using the solution that you have tabulated above explain the effect that introducing firing costs has aggregate employment, on the market clearing wage, and on sectoral employment. Be very thorough and very clear, providing economic intuition as you go along.

Problem 2. AK Model. Consider an economy in which the production function can be represented by the following form: $Y = AK$. Y is aggregate output, A is a technology parameter and K is the quanity of aggregate capital.

a) In a similar way as we derived the "intensive form of the production function" in class for the Solow model, derive the intensive form for this technology. (i.e. represent the production funtion in per capita terms).

b) What is the marginal product of capital? What kind of returns to scale does this function exhibit?

c) Following the derivation of the Solow model (also read DLS Chapter 11), derive an analogous "law of motion for capital" in this economy.

d) Assuming that the technology grows at a constant rate μ and that population grows at the constant rate γ, what is the *gross* growth rate of capital per unit of effective labor?

e) From the previous answer, how is the growth rate related to the capital stock? (inversely related as in Solow?). What does this say about *convergence*?

f) From the "law of motion" derived previously, get an expression for the growth rate of capital (i.e. $(k_t - k_{t-1})/k_{t-1}$). Graph both components of the growth rate. What is the necessary condition for achieving permanent growth?

Problem 3. Reinterpretation of AK model using Human Capital.
In this question we will reinterpret the AK model as a model in which physical and human capital coexist. Assume that the production function of an economy is a Cobb-Douglas in the aggregate quantities of human (H) and physical (K) capital: $Y = BK^\beta H^{1-\beta}$. As always $0 < \beta < 1$ and B is a constant that reflects the state of the technology. Both factors K and H can be acumulated, at the expense of consumption. Since both factors -in their use as real assets- are perfect substitutes, in an optimum agents will use them until their marginal rate of return is the same.

a) Get the marginal rate of return to physical capital and to human capital.

b) Use the condition that both rates of return must be equal in equilibrium to compute an expression that relates H to K.

c) Substitute the expression found above for H in the production function and reinterpret that function as an AK function.

d) Based on this question, what can you say about considering the AK model as a model in which human and physical capital coexist? What is the necessary condition for this to be the case?

Problem 4. Overlapping Generations. Consider the same overlapping generations economy from class. Preferences are given by:

$$u\left(c_t\left(t\right), c_t\left(t+1\right)\right) = \log\left(c_t\left(t\right)\right) + \log\left(c_t\left(t+1\right)\right)$$

Assume that N young are born every period and the endowment is $\{y, 0\}$. That is, agents receive y units of the consumption good when they are young and 0 when they are old. There are N initial old people that have $H\left(0\right)$ untis of a fiat currency. Let $p\left(t\right)$ be the price level at time t.

a. What are the potential trades that might occur in an equilibrium? I want you to tell me who trades what and with whom and what is the mechanism by which the trades could occur. Some may occur under special circumstances. Don't worry about the circumstances, but be thorough.

b. Define an equilibrium with valued fiat currency.

c. Compute an equilibrium with valued fiat currency. Be very clear.

Now suppose that the government in this economy is concerned with savings and redistribution. That is, the government taxes a lump sum equal to τ units of the consumption good to the young, and gives it to the old that same period.

d. What is the endowment process of the agents in this economy. Assume that the tax is the same for every generation.

e. Can you find an equilibrium with valued fiat currency? What restrictions must be placed on the value of τ to ensure that this equilibrium exists?

f. Can you say something about the role of money as a "social institution"? (Compare why is money similar to the redistribution scheme that the government proposed).