The University of Chicago
Department of Economics
Elements of Economic Analysis IV
Problem Set 2

This Problem Set is due on Friday Oct. 19, 2001 before the TA session.

Please be as neat as possible since no effort will be made to understand illegible writing. When possible, put your answers in a box. Write the name of the people whom you worked with at the top of the first page.

Problem 1: International Trade

Suppose that an economy produces beer (x) and pizza (y) using only labor. The production functions are given by

$$x = 10\sqrt{L_x}$$

for beer and

$$y = \alpha L_y$$

for pizza. L_i for i = beer; pizza is productive labor input and $\alpha > 0$ is a constant. Suppose that labor supply is fixed at 400, so

$$L_x + L_y = 400:$$

Also, the representative consumer's utility function is given by:

$$u(x;y) = x^{\frac{1}{2}}y^{\frac{1}{2}}:$$

a. Compute the production possibility frontier for this economy and graph it.

b. Calculate the rate at which the economy converts beer into pizza. This is the rate of product transformation.

c. Solve for the equilibrium price ratio $\left(\frac{p_x}{p_y}\right)^E$ and the equilibrium quantities x^E and y^E:

d. How do the equilibrium prices and quantities change when α increases? Explain the economic intuition clearly.

e. Suppose that the economy opens up to trade with the rest of the world and the free trade price ratio is given by:

$$\left(\frac{p_x}{p_y}\right)^{FT} = 1:$$

Solve for domestic production, x^P and y^P and the quantities consumed x^C and y^C: (Assume that $\alpha > \frac{1}{4}$:)

f. How do these quantities change as α varies? Provide precise economic intuition.

g. Assume that $\alpha = 1$: For some range of international price ratio does the economy specialize completely (i.e. produce only one good)?

Problem 2: Consider a world with two countries, Chile and Argentina. There is one representative consumer in each country. The world lasts for two periods. Each consumer has a utility function given by:

$$u(c_1; c_2) = 6c_1 - c_1^2 + \beta\left[6c_2 - c_2^2\right]:$$

This is an endowment economy in which the endowment is given by: $(y_1; y_2)$: Suppose the Chilean receives $(2; 1)$; while the Argentine receives $(1; 2)$: The consumption good can not be stored. Do not assume that $\beta(1 + r) = 1$: Assume that there is a domestic credit market.

Suppose initially that both agents are in autarky.

a. Setup the agent's problem and solve it (Hint: Solve the maximization once. Then calculate the quantitiy demanded by each agent.)

b. Compute the equilibrium. What is the interest rate? What is the consumption bundle? What are utilities?

Suppose that now the economies open to Free Trade so that now there is a world credit market.

c. What is the free trade credit market equilibrium condition?

d. Compute the free trade equilibrium. What is the interest rate? What are savings in the two countries? What are the consumption bundle in the two countries? What are the utilities?

e. Are agents better off as a result of free trade?

f. What happens to imports and exports? Be clear.

Problem 3. Consider a model with two countries, Argentina and Spain. There is one representative consumer in each country. The world lasts for two periods. Each consumer has the utility function: $U(c_1; c_2) = \log(c_1) + \beta\log(c_2)$. Nobody works in this world, but each consumer has a fixed endowment y_1 and y_2 in each period. For the Spaniards, we have $y_1 = 2$ and $y_2 = 1$, while the Argentineans have $y_1 = 1$ and $y_2 = 2$. The consumption good cannot be stored.

a) Assume that both countries live under autarky, i.e., they cannot trade with each other. Each consumer maximizes utility subject to the budget constraints:

$c_1 + s = y_1$ and $c_2 = (1 + r)s + y_2$. Solve the maximization problem for each consumer. Since there is no storage and no trade, in equilibrium we must have $s = 0$ in each country. Use this condition to compute the interest rate r, consumption, and utilities in each country, as a function of $\bar{}$.

b) Now assume the consumers can trade. The maximization problem is the same, but the equilibrium condition is di¤erent since the two countries can trade with each other. Since there is no storage, if one country borrows, the other must be willing to lend. The equilibrium condition for the credit market is $s_A + s_s = 0$. Where s_A is the savings of the Argentinean and s_S is the savings of the Spaniard. The interest r has to be equal in both countries. Use these conditions to solve for the equilibrium interest rate r, consumption, savings and utilities. Compute the current account balance for each country in each period (remember: in this model savings go to the other country, and therefore are exports).

Problem 4 (Trade, Distribution and Negotiation). From the notes, in the section entitled "Trade and Distribution", we saw that even when trade is bene…cial to a country as a whole, some groups in particular might oppose opening up the economy to free trade. This problem explores this idea further. Remember that we consider a small open economy (SOE), which takes prices in the world market as given. The country is inhabited by two people, the pizza guy and the beer guy. Both are specialized, hence they will be a¤ected by trade in a di¤erent way.

Production functions are linear. Speci…cally, $y_b = l_b$ and $y_p = l_p$. That means that in every sector output equals the labor input. The utility functions for both people are $u(c_b; c_p) = \ln(c_b) + \ln(c_p)$. Wages can be di¤erent, and will be denoted by w_b and w_p. The budget constraint for each person is $c_b + pc_p = w_i$ where $i \in \{b; p\}$. As in the notes, beer is the numeraire.

From the consumption maximization problem of each consumer we get the optimal consumptions of beer and pizza of the two individuals. These are: $c_p = w_i/2p$ and $c_b = w_i/2$ (you should check this). From the …rst order conditions of the brewery and the pizza bakery we determine the wages: $w_b = 1$ and $w_p = p$ (also check this result). Finally from the aggregate consistency conditions (aggregate demand equals aggregate supply) we determined that $w_p = 1$, and by the previous result, this implies that $p = 1$. Using the expressions for the consumption of each agent, we …nd that each person consumes 1/2 units of each good. Utility is therefore $\ln(1/2) + \ln(1/2) = -1.386$, for each agent.

Now suppose the economy opens up to free trade. Assume that the world price of pizzas in terms of beer is $p = 1/2$ (the world market price).

a) What are the wages now for each person? What is the di¤erence from the Autarky case? Why?

b) Given the wages and that $p = 1=2$, compute the consumption of beer and pizza for both agents. Is one clearly better o¤ than under autarky? Any one clearly worse o¤? Any one will oppose free trade?

c) Show that even if one individual is worse o¤, trade is better for the country as a whole than autarky.

d) Now consider some distribution policy. What is the *biggest* transfer that the beer guy will be willing to give to the pizza guy so that the later accepts free trade? (hint: the transfer means giving part of his wage to the other guy. Label this transfer $t^h > 0$ and consider his utility under no trade and under trade-with-transfer).

e) What is the *lowest* transfer that the pizza guy is willing to accept from the beer guy so that the former accepts free trade? (hint: similar to previous hint, but label the transfer t^l).

d) What do the numbers t^h and t^l tell you? Is there one transfer that will make trade supported by both guys, or are there a whole bunch of them?

e) If you found more than one transfer that makes trade possible, which one do you expect to see taking place? Why? What role does "negotiation power" has in this case?