Chapter 3

The Short-Run

3.1 Introduction

In this chapter we begin the study of the firm's **short run** factor market behavior. As in the previous chapter, we will continue to assume that factor markets are competitive. The difference between the short run and the long run is that in the long run all factors of production are assumed variable, while in the short run some factors are assumed fixed. Generally we assume that labor is variable in both the short run and the long run, whereas capital is **fixed** in the short run and variable in the long run (Note: what about land?) This affects the firm's cost minimization problem because it looses one degree of freedom in organizing its production: it must make do with the installed level of capital. Notice that regardless of the behavior of the firm in the goods market, it will always strive to minimize its cost in the factor market (question: are monopolies more or less efficient than competitive firms). In this section we will derive the **short run cost function** which is the maximum value function derived from the cost minimization problem with a fixed level of capital.

We will also study the relationship that exists between the short-run and long-run cost function as well as the relationship between marginal cost. We will see that there are fundamental differences in the marginal cost curves that lead to Marshall's laws of derived demand. The fundamental difference is that in general there are CRS in the long run and DRS in the short run.

3.2 The Length of the Run

The short run and the long run are relative terms and there is no temporal dimension to the definition. That is, the short run is not 1 month and the long run anything longer than that. In fact, the distinction between the short run and the long run depends on the problem that we are studying so that a very short period of time can be the long run and a very long period of time can be the short run. Economists draw the line between the short run and the long run by referring to the potential variability of input use. Therefore, in the short run there are some inputs that can not be varied. An extreme version of the phrase "can not be varied" takes this statement literally. However, the amount of the input may be variable but at a very high cost. Therefore, a good approximation is to treat it as fixed. In the long run, however, all factors are variable. The firm, then, has a larger choice set open to it in the long run than in the short run.

Another important aspect of the long run and short run has to do with the scale of production. As a corollary, economists often assume that in the short run the production function exhibits decreasing returns to scale, while in the long run it exhibits costant returns to scale. We will see why this is the case shortly.

To be completed.

3.3 Optimal Choice

3.3.1 The Firm's Short Run Cost Minimization Problem

As we stated in the introduction, firms seek to minimize cost subject to the constraint that they produce y units of output. That is, the firm's cost minimization problem, $[SCM]$, is given by:

$$\begin{array}{lll} \min_{l} & wl + r\overline{k} & \\ s.t. & f\left(\overline{k}, l\right) & = \quad y \\ & l \geq 0 & \end{array}$$

where $\overline{k}$ denotes the fixed level of capital with which the firm can work. We will continue to ignore the non-negatvity constraints on factor inputs by arguing that the firm is not endowed with any factors and so must be a net buyer in factor markets.

Notice that in the [SCM] there are only two parameters the wage rate, w, and the target level of output, y. If we were given values for these parameters we could obtain the actual quantities demanded. Since we are using arbitrary parameters, our solution will be a quantities of l as a functions of the wage and target production levels:

$$l^{sr}(w, y)$$

This demand functions is known as **the short run conditional or derived demand function.** Notice that conditional demand functions are functions of target output y. This means that as we vary the relative factor cost, the firm's optimal quantities will always be on the same isoquant.

3.3.2 Solving the Firm's Problem

First let us relabel the fixed capital cost as F_k. Then, the firm's problem is given by:

$$\begin{array}{lll} \min\limits_{l} & F_k + wl & \\ s.t. & f\left(\overline{k}, l\right) & = & y \end{array}$$

and we notice that it is a minimization problem in only one varaible. The solution to this problem is trivial. First, we solve the constraint for the level of labor input that, along with our installed capital, allows us to achieve output level y :

$$l^{sr} = l\left(\overline{k}, y\right).$$

The above equation simply states that short run labor demand is a function of the level of installed capital and the target output level. Notice that in this problem there is no optimality condition: the production constraint determines the intensity of factor use. If we had two variable inputs this property of the solution would not hold. In addition, notice that in this case factor prices do not enter the equation for labor demand. The reason is simple there is no tradeoff in using one input versus another since there is only one input that we can adjust meet our production constraint.

3.4 Cost Function

If we take the conditional demand function and plug it into the objective function we obtain the firm's short run cost function. This function tells us the minimum money outlay necessary to achieve production y, given the level of installed capital and factor prices.

$$C\left(w, r, \overline{k}, y\right) = r\overline{k} + wl^{sr}\left(\overline{k}, y\right).$$

Notice that in the short run, unlike the long run, total cost is the sum of a **fixed cost** and a short run **short run variable cost.** In this case, labor costs vary with output and so are variable costs. Capital costs do not vary with output and, therefore, are fixed. We postpone a fuller discussion of the firms cost functions until the next chapter. In this section we will discuss the properties of the short run cost function.

Notice that we can look at the short run cost functio as a function of the two factor prices, the target output level, and the amount of the fixed factor. The following properties hold:

Property I: The cost function is increasing in r :

$$\frac{dC}{dr} = \overline{k} > 0$$

Property II: The cost function is increasing in $\overline{k}$:

$$\frac{dC}{d\overline{k}} = 1 + w\frac{dl^{sr}}{d\overline{k}} < 0$$

Property III: The cost function is increasing in w :

$$\frac{dC}{dw} = w\frac{dl^{sr}}{dw} + l^{sr} > 0$$

Property IV: The cost function is increasing in y :

$$\frac{dC}{dy} = w\frac{dl^{sr}}{dy} > 0$$

The intuition of the following results is clear.

To be completed.

3.5 The Short Run Cost Function and Returns to Scale

We noted in Chapter xxxx that a very important difference between consumer theory and production is that the utility function is an ordinal concept while the production function is a cardinal concept. What this means is that the output level attached to an isoquant is a meaningful quantitiy, but the utility index assigned to an indifference curve is arbitrary. Since the scale on which we measure production is meaningfully defined we can study the effect of returns to scale on the shape of the firm's cost fucntion.

To be completed.

3.6 Applications

3.6.1 Marshall's Laws of Derived Demand

Marshall came up with four laws of derived demand....

To be completed.

3.7 Exercises

Exercise 3.7.1 *Consider a firm that has a Cobb-Douglas technology. The firm wishes to minimize cost of producing* y *units of output and has access to perfectly competitive factor markets. The firm's cost minimization problem is given by:*

$$\begin{array}{lll} \min\limits_{k,l} & wl + rk & \\ s.t. & k^{\alpha} l^{1-\alpha} & = y \end{array}$$

Let μ *denote the Lagrange multiplier on the output constraint.*

a. What are the parameters of the problem?

b. Find the conditional demand functions. Label them $l^*(w,r,y)$ and $k^*(w,r,y)$.

c. Find the cost function: $C(w,r,y)$. What is its interpretation?

d. Find μ^*. What is its interpretation?

e. Find $\frac{dC}{dy}$ and show that it is equal to μ^*.

Exercise 3.7.2 *Consider a firm that produces holes with shovels and people. Assume that people and shovels are perfect complements so that technology is given by:*

$$f(k,l) = \min\{k,l\}$$

The firm wishes to minimize cost of producing y units of output and has access to perfectly competitive factor markets. Let μ denote the Lagrange multiplier on the output constraint.

a. What is the firm's cost minimization problem?

b. What is the optimality condition?

b. Find the conditional demand functions. Label them $l^*(w,r,y)$ and $k^*(w,r,y)$.

c. Find the cost function: $C(w,r,y)$. What is particular about the expression you obtained. That is discuss the economic intuition of your result.

d. Find μ^*. What is its interpretation?

e. Find $\frac{dC}{dy}$ and show that it is equal to μ^*.

f. Explain why the expression that you obtain makes intuitive sense.

Exercise 3.7.3 *Suppose that the firm's utility function is given by:*

$$f(k,l) = k^{\alpha} l^{1-\alpha}.$$

Also suppose that factor prices are given by w and r and that the target output parameter is given by y. Denote the Lagrange multiplier by μ.

a. Write down the agent's cost minimization problem. Be very precise.

b. Find the conditional demand curves, $l^*(w,r,y)$ and $r^*(w,r,y)$and show that they are homogeneous of degree 0 in w and r.

c. Find μ^*.

d. Find the cost function, $C(w,r,y)$.

e. Show that the cost function is homogeneous of degree 1 in w and r. What is the intuition?

f. Show that $\frac{dC}{dw} \geq 0$ and $\frac{dC}{dr} \geq 0$. What is the intuition?

g. Find $\frac{dC}{dy}$.

h. Show that $\frac{dC}{dy} = \mu^* > 0$. What is the intuition?

i. Show that the expenditure function is concave in w. Do this graphically and mathematically. What is the intuition?