

Chapter 11

The Planning Problem

11.1 Introduction

In the previous chapter we discussed the competitive equilibrium of a simple economy with production. The idea was to study a setting in which the exchange process was decentralized. We assumed that the consumer maximized his utility and the firm maximized its profit. These behavioral assumptions imply a set of demand and supply curves—in our case for labor and goods—which summarize a optimizing behavior in the economy. In particular, these functions relate prices and quantities. Prices—operative in a market system—coordinates firm and consumer decisions. The economy is in equilibrium when markets clear. That is, when the market sets prices of labor and consumption at which quantities supplied and demanded are equal to each other.

However, we can proceed in an alternative way. Instead of looking at a decentralized exchange process, let us assume the existence of a **benevolent social planner** who makes all the decisions in the economy. The social planner is benevolent because he only cares about the welfare of the consumer. Therefore, instead of having prices to guide the economy to equilibrium (like Smith's invisible hand), we allow the social planner to decide how the consumer's time will be allocated between leisure and labor. The only constraints the planner faces are the consumer's time endowment and the state of technology. Obviously, the social planning problem gives us the highest possible utility level for the consumer. We assume, in particular, that information is costless and readily available to the social planner. This

is a very important assumption. Hayek (1945) in an article called “The Use of Knowledge in Society” discusses these issues. In particular, he discusses how prices coordinate exchange without making substantial informational requirements. He concludes that prices are one of the reasons why the market system is superior to a centralized economic system.

In this chapter we will establish the close connection that exists between the social planning problem and the competitive equilibrium that we solved in the previous chapter. The **first welfare theorem** tells us that if an allocation and a set of prices constitute a competitive equilibrium, then the allocation will be Pareto efficient. This tells us that the competitive equilibrium is efficient; the only option we have is to redistribute endowments in order to achieve a different point on the contract curve, but there is nothing more we can do. The **second welfare theorem**, on the other hand, tells us that any social planning problem can be decentralized as a competitive equilibrium.

The welfare theorems establish a connection between the competitive equilibrium and the social planning problem. The welfare theorems also have practical implications. In particular, if we solve a planning problem we can use the welfare theorems to interpret the quantities as those arising from a competitive economy. The second welfare theorem then tells us how to obtain the associated market clearing prices. From a computational perspective, the welfare theorems will simplify much of our work but we must remember that when we solve a planning problem we are actually interested in the underlying market economy.

11.2 The Social Planner

As we discussed in the introduction, let us study the problem of a benevolent social planner who wishes to maximize a consumer’s utility. He is constrained by the consumer’s time endowment and by the current state of technology. The social planner solves:

$$\begin{aligned} \max_{c,R} \quad & u(c, R) \\ \text{s.t.} \quad & c = f(l) \\ & l + R = T \end{aligned}$$

Notice the absence of prices in the planning problem. The planning problem is an assignment or allocational problem that arises because the consumer

faces a tradeoff in the way he allocates his time. Notice that time, in the form of leisure gives direct utility, but that consumption of leisure leaves less time for labor and, therefore, reduces the agent's consumption opportunities. In other words, the opportunity cost of leisure is foregone consumption. The planner looks for the best way to allocate the consumer's time endowment by balancing labor and leisure. Notice that we use best in two ways. Best means that the planner maximizes the consumer's utility function. In this problem, best is unambiguous. However, when there are more than two agents the planner's allocation will depend on what the welfare function that we use.

Let us derive the PPF for this problem and obtain the graphical representation of the problem.

To be completed.

In order to solve the planning problem we will use the method of Lagrange multipliers. Let ϕ_t denote the Lagrange multiplier on the time constraint and ϕ_c denote the Lagrange multiplier consumption constraint. Since there are two constraints, the Lagrangean for this problem is given by:

$$L(c, R, l, \phi_c, \phi_t) = u(c, R) + \phi_c (f(l) - c) + \phi_t (T - l - R).$$

The first order conditions for this problem are given by:

$$\begin{aligned} [c] : \quad \frac{\partial u(c^*, R^*)}{\partial c} &= \phi_c^* \\ [R] : \quad \frac{\partial u(c^*, R^*)}{\partial R} &= \phi_t^* \\ [l] : \quad \phi_c^* f'(l^*) &= \phi_t^* \\ [\phi_c] : \quad c^* &= f(l^*) \\ [\phi_t] : \quad l^* + R^* &= T \end{aligned}$$

If this is not clear you should read the appropriate section in Chapter 2.

Notice that as usual, $[\phi_c]$ and $[\phi_t]$ give us the constraints. $[c]$ and $[R]$ tell us that ϕ_c^* and ϕ_t^* are the marginal utilities of consumption and leisure, respectively. Finally, $[l]$ relates both constraints. This first order condition tells us how we should assign labor time. Notice that on the left hand side of this expression we have the marginal benefit of working a little more. This is equal to the product of the marginal product of labor and marginal utility of consumption. At the optimum we must balance marginal benefit to marginal cost which is the loss in utility due to the loss of a unit of leisure.

Of course, this is the marginal utility of leisure which is equal to ϕ_t^* , by [R]. Since we have 5 equations and 5 unknowns we can solve the equations and obtain the optimal values of consumption, labor, leisure, and the Lagrange multipliers. Recall that the Lagrange multipliers are the shadow values of the constraints. That is, they tell us how much better off we would be if the constraint slackened slightly.

Let us now manipulate the first order conditions to obtain a suggestive solution to the planning problem. Notice that [l] implies:

$$f'(l^*) = \frac{\phi_t^*}{\phi_c^*}.$$

This tells us that the planner will equate the technical rate of substitution to the ratio of the Lagrange multipliers. Also, note that [c] and [R] together imply

$$\frac{\frac{\partial u(c^*, R^*)}{\partial R}}{\frac{\partial u(c^*, R^*)}{\partial c}} = \frac{\phi_t^*}{\phi_c^*}.$$

The quantity on the left hand side is the marginal rate of substitution of consumption for leisure. Therefore, the planner will try to equate the MRS to the ratio of the Lagrange multipliers. Obviously, we can combine the two equalities to obtain:

$$\frac{\frac{\partial u(c^*, R^*)}{\partial R}}{\frac{\partial u(c^*, R^*)}{\partial c}} = \frac{\phi_t^*}{\phi_c^*} = f'(l^*).$$

Notice that when the planner allocates time in such a way, the consumer can not be made better off. That, is the condition above characterizes the welfare maximizing bundle.

In the next section we will see how these conditions are very similar to those that we obtained in the competitive equilibrium that we analyzed in the previous chapter. We wish to determine how the welfare maximizing bundle relates to competitive equilibrium bundle. Also, we would like to know how the consumer's welfare in the competitive equilibrium relates to the maximal welfare that the planner selects. Notice that because there is a single consumer in the world we have no problem comparing utility levels. We can, however, make no statement about intensity of preference as preferences are an ordinal and not a cardinal ordering. First, however, we will analyze the planning problem associated with the competitive equilibrium we analyzed in the previous chapter.

11.3 Welfare Properties of a CE: The First Welfare Theorem

What are the welfare properties of a CE?

11.4 Decentralizing a Planning Problem: The Second Welfare Theorem

Consider the first order conditions of the planning problem.

11.5 Assumptions

An aside about the assumptions.

To be completed.

11.6 A Simple Example

Consider an economy with the same preferences, endowments, and technology as the one we studied in the previous chapter. The two objectives are, first, to explain how to use the planning problem as a computational substitute for the competitive equilibrium and, second, to explain precisely how we can interpret a planning problem as a competitive economy.

The planning problem associated with the competitive economy of the previous chapter is given by the following utility maximization problem:

$$\begin{aligned} \max_{c,R} \quad & \log c + \log R \\ \text{s.t.} \quad & c = A l^{\frac{1}{2}} \\ & l + R = T \end{aligned}$$

The first order conditions for this problem are given by:

$$\begin{aligned} [c] : \quad & \frac{1}{c^p} = \phi_c^p \\ [R] : \quad & \frac{1}{R^p} = \phi_t^p \\ [l] : \quad & \phi_c^p \frac{1}{2} (l^p)^{-\frac{1}{2}} = \phi_t^p \\ [\phi_c] : \quad & c^p = A (l^p)^{\frac{1}{2}} \\ [\phi_t] : \quad & l^p + R^p = T \end{aligned}$$

where the superscript p indicates that the quantities are the optimal quantities of the planning problem.

Solving the first order conditions we obtain the following optimal quantities:

$$\begin{aligned} l^p &= \frac{1}{3}T \\ R^p &= \frac{2}{3}T \\ c^p &= A \left(\frac{1}{3}T\right)^{\frac{1}{2}} \end{aligned}$$

We have just verified the first welfare theorem. The competitive equilibrium quantities coincide with those in the planning problem; therefore, the competitive equilibrium is efficient and maximizes the consumer's welfare.

Now, we will use the second welfare theorem to decentralize the planning problem. The second welfare theorem instructs us to use the ratio of the Lagrange multipliers as the market clearing price. The Lagrange multipliers are given by:

$$\begin{aligned} \phi_c^p &= \frac{1}{A \left(\frac{1}{3}T\right)^{\frac{1}{2}}} \\ \phi_t^p &= \frac{3}{2T} \end{aligned}$$

If we take the ratio of the Lagrange multipliers we obtain:

$$\frac{\phi_t^p}{\phi_c^p} = \frac{A}{2 \left(\frac{1}{3}T\right)^{\frac{1}{2}}}$$

which is precisely equal to the equilibrium real wage. Therefore, we have succeeded in showing that we can obtain the welfare maximizing consumption allocation of the planning problem from a competitive economy.

11.7 Exercises

Exercise 11.7.1 *I can do the planning problems for the examples in the previous chapter.*