Chapter 17

Oligopoly

17.1 Introduction

In this chapter we continue our study of market structure by looking at **oligopoly** or an industry with few producers. We will continue to assume that firms in the industry wish to maximize profits. This would lead firms to **collusion**. That is, firms would like to agree to restrict output to monopoly levels and then split the profits. This could give all the firms in the industry higher profits that they would obtain if they competed. The interesting question, then, is to determine why firms do not collude and if they do not collude how do they compete. Therefore, we will see that industry equilibrium is characterized by the proper balance between cooperation and competition.

In order to study oligopolistic market structure, we first need to invest a little bit in tools. The key characteristic of oligopolistic industries is the presence of **strategic interactions** among firms. This arises from the fact that with few firms in the industry the elasticity of market demand depends on the output of each firm. Therefore, one firm has a direct impact on the price that the other firms in the industry can charge. In monopoly and perfect competition this ingredient was missing. Therefore, we will introduce the concept of a **Nash Equilibrium** to study the pure non-cooperative outcome. We will study two ways in which firms compete. We will look at **Cournot competition** which means that firms compete in quantities and at **Bertrand competition** which means firms compete in prices. We will also see that as the number of firms in the industry increases

–equivalently, as market share falls– the importance of strategic interactions diminishes and we approximate the competitive solution. Similarly, as the number of firms approaches 1, we will see that we approach the monopoly solution (take with a grain of salt since threat also matters).

17.2 The Perfectly Competitive Benchmark

Let us compute the perfectly competitive benchamrk solution to the the oligopoly equilibrium first.

17.2.1 Firms

Suppose that there are fixed number (few) of firms in the industry –two firms, in this case– and that they each have one plant. We assume that marginal costs are constant. Therefore, each firms cost function is linear in output and equal to:

$$C(y_i) = c_i y_i,$$

where y_i, for $i = 1, 2$, denotes each firm's output. Each firm's profit maximization problem is given by:

$$\max_y py_i - C(y_i).$$

Suppose that the firms believe that they can not influence market price through their quantity choice and that collusion is impossible or extremely costly . Then, the firms operate in a perfectly competitive market and the first order conditions tell us that each firm will produce where price equals marginal cost:

$$[y_i] : p = c_i$$

so each firm supplies output inelastically. Each firms profit is exactly equal to 0.

17.2.2 Equilibrium

Let us suppose for simplicity that firms are identical and the market demand function is given by the following linear demand curve:

$$Y(p) = \frac{a - p}{b}.$$

Then, equilibrium in the goods market requires that the industry supply any quantity at marginal cost. Market demand tells us that, when $p = c$, market demand is equal to:

$$Y(c) = \frac{a-c}{b}.$$

Notice that $a > c$ is a necessary condition for a well posed problem. Otherwise, market demand would be insufficient to induce firms to supply any output at marginal cost. We will assume that each firm produces one-half of the industry output:

$$y_i^{PC} = \frac{a-c}{2b}.$$

Since firms are supplying at marginal cost, firm and industry profits are equal to zero.

17.2.3 Contestability

Under what conditions would we expect an outcome similar to competition despite the existence of few firms in the industry?

To be completed.

17.3 Collusion

Suppose, now, that collusion is possible. This requirement is stronger than it would seem at first glance. We wish to compute the profits of the cartel and see whether or not it would be in each firms best interest to collude.

To be completed.

Under a collusive agreement, we treat the coalition of firms –the **cartel**– as a firm with multiple plants (instead of thinking of multiple firms each with one plant). The cartel's problem is to minimize the cost of producing a given level of output –say, y– by allocating output to each of its plants (firms in the non-collusive problem). Then, united, the firms face the goods market as a monopoly. The final step is to distribute profits in such a way as to make every firm better off in collusion than in competition.

17.3.1 The Cartel's Cost Minimization Problem

Suppose that the colluded firms wish to determine the cheapest way to allocate production between the two plants. Then, the colluded firms wish to allocate production among the two plants in order to minimize total cost of producing y units of output. Formally, the cartel's cost minimization problem is:

$$\begin{array}{lccc} \min\limits_{y_1,y_2} & c_1 y_1 & + & c_2 y_2 \\ s.t. & y_1 + y_2 & = & y \end{array}$$

It is clear that if $c_1 > c_2$, then it is cost minimizing to produce the entire output at firm 2's plant, while if $c_2 > c_1$, then it is cost minimizing to produce everything at firm 1's plant. If $c_1 = c_2$, then the cartel is indifferent as to how to allocate production. Since the constraint is linear and the objective function is linear too, we must worry about corner solutions, as we can see in Figure Oligo1.

Insert Figure Oligo1.

The cartel's cost function is given by:

$$C(y) = \min\{c_1, c_2\} y.$$

Therefore, if both firms have the same technology, and therefore the same cost function, $(c_1 = c_2 = c)$, then output is indeterminate. We will assume in this case, that the cartel splits production evenly between both plants, so $y_1^C = y_2^C = \frac{1}{2}y$. The cartel's cost function is given by $C(y) = cy$, where c denotes the common (to the firms) marginal cost.

17.3.2 The Cartel's Profit Maximization Problem

Now, that the cartel has determined how to allocate production across palnts to minimize production costs, it faces the goods market as a monopoly and chooses output to maximize profit. To keep the analysis simple we will assume that the inverse market demand function is given by:

$$p(y) = a - by.$$

The cartel's profit maximization problem is given by:

$$\max_y (a - by) y - cy$$

and the first order condition is:

$$[y]: \quad a - 2by^C \quad = \quad c$$

where y^C denotes the cartel's output.

Solving the first order condition we obtain the cartel's quantity:

$$Y^C = \frac{a-c}{2b}$$

and the price it sells at:

$$p\left(Y^C\right) = \frac{a+c}{2}.$$

Finally, cartel profits are equal to:

$$\pi^C = \frac{(a-c)^2}{4b}.$$

If we assume that firms are treated symmetrically in the cartel, then each produces $\frac{1}{2}$ of industry output and each firm's profit is equal to $\frac{1}{2}$ of industry profit:

$$\pi_i^C = \frac{(a-c)^2}{8b}.$$

17.3.3 Stability of the Cartel Solution

We will see later on that the collusive outcome maximizes industry profits and, provided there is a way for firms to impose a system of sidepayments, it also maximizes each individual firm's profits. However, an important question is whether the cartel is stable. That is, do firms find it in their self interest to deviate from the agreement? We have computed the industry equilibrium assuming that the cartel is enforceable, however, many times it is profitable for firms to cheat –either by altering the quantity they sell at the going price or by changing the price at which they sell their stock. The fact that the cartel agreement is not **self-enforcing** may partially explain why, despite the fact that collusion is profit maximizing, we do not find monopolies in every industry. This suggests that if collusion itself is costly –perhaps because we have to police the behavior of the members– then firms may find it preferable to compete.

Following the analysis of the previous section, assume that the cartel is operating under the conditions outlined above. That is, each firm produces exactly one half of industry output:

$$y_i^C = \frac{a-c}{4b}$$

and sells all units at the cartel price:

$$p^C = \frac{a+c}{2}.$$

Each firm receives exactly one half of the cartel's profit:

$$\pi_i^C = \frac{(a-c)}{16b}.$$

Firm 1 Cheats

Suppose that firm 1 decides to cheat. It believes that firm 2 will not change its output and wants to see if it can profit from reneging on its agreement to collude. If firm 2, in fact, does not alter its quantity, then firm 1 would like to maximize its profits, given by:

$$\max_{y_1} \left(a - b\left(y_1 + y_2^C\right)\right) y_1 - cy_1.$$

The first order condition is given by:

$$[y] : a - 2by_1^R - \frac{a-c}{4} = c,$$

where I have used the fact that $y^C = \frac{a-c}{4b}$. Firm 1's optimal quantity if it reneges is equal to:

$$y_1^R = \frac{3(a-c)}{8b}.$$

This exceeds the quantity that firm 1 was producing in the collusive agreement:

$$\frac{a-c}{4b} = y_1^C < y_1^R = \frac{3(a-c)}{8b}.$$

Since firm 2 is assumed to not vary its quantity total industry supply will increase to:

$$Y^R = \frac{5(a-c)}{8b}$$

and the sale price will fall to:

$$p\left(Y^R\right) = \frac{3a + 5c}{8}.$$

Notice that firm 2 is selling the same number of units, but at a lower price so its profits must fall:

$$\pi_2\left(y_1^R, y_2^C\right) = \frac{3\left(a - c\right)^2}{32b}.$$

Firm 1, on the other hand sells more units at a lower, but it can not be worse off since the collusive outcome is still feasible. In fact, it is better off:

$$\pi_1\left(y_1^R, y_2^R\right) = \frac{9\left(a - c\right)^2}{64b}.$$

Therefore, cheating is profitable for firm 1 if firm 2 does nothing. Notice that consumers are happy: more output at a lower price, so the inefficiency of monopoly is weakened.

Firm 2 Reacts

Firm 2, however, realizes that if firm 1 cheats it would be worse off. Therefore, firm 2 realizes that if it was in firm 1's interest to cheat it would not choose y_1^C, but it would choose quantity y_1^R. Therefore, if firm 2 maximizes its profits it would not select y_2^C either. It would maximize its profits assuming that firm 1 chooses y_1^R:

$$\max_{y_2} \left(a - b\left(y_2 + y_1^R\right)\right) y_2 - cy_2.$$

The first order condition for this problem is:

$$[y_2] : a - 2by_2^R - by_1^R = c$$

which is equal to:

$$y_2^R = \frac{5\left(a - c\right)}{16b},$$

using the fact that $y_1^R = \frac{3(a-c)}{8b}$.

When firm 2 maximizes its profits by assuming that firm 1 cheats, he receives:

$$\pi_2\left(y_1^R, y_2^R\right) = \frac{25\left(a - c\right)}{256b}$$

and firm 1's profits are:

$$\pi_1\left(y_1^R, y_2^R\right) = \frac{15\,(a-c)}{128b}.$$

Notice that when both firms cheat, they are both worse off relative to collusion:

$$\pi_1\left(y_1^R, y_2^R\right) < \pi_1\left(y_1^C, y_2^C\right)$$

and

$$\pi_2\left(y_1^R, y_2^R\right) < \pi_2\left(y_1^C, y_2^C\right).$$

Collusion, however, is not a stable equilibrium because both firms would have to make an agreement which they would be willing to break unilaterally! If only one firm cheats –like firm 1 in our example– its profits increase, but firm 2's profit fall. If both firms cheat, then firm 2 is able to make up for the loss in profits that it would have if it did not cheat too. The outcome is that both firms cheat and the agreement breaks down. The result is a higher quantity and lower price than in the collusive outcome, so the inefficiency of the monopolistic outcome is weakened.

When Does the Cheating End?

Since retaliation by firm 2 increases its profits and decreases firm 1's profits, if firm 1 wants to maximize its profits it should not assume that firm 2 will maintain its output at the cartel level, y_2^C, but it would assume it would select the retaliatin output, y_2^R. Therefore, firm 1 should solve:

$$\max_{y_1}\left(a - b\left(y_2^R + y_1\right)\right) y_1 - cy_1.$$

which gives firm 1 higher profits. Then, firm 2 would like to change its maximization problem to account for firm 1's new quantity, and so on so forth. The question is, where will this cheating process stop? We know that the outcome is not collusion since it is not an enforceable agreement. It seems unreasonable to believe that we would go all the way to the competitive outcome, since with only two firms in the market they are likely to have some degree of market power. In order to answer this question, we should explicitly consider the strategic interaction between two the firms in the industry. We are looking for a set of quantities $\left\{y_1^S, y_2^S\right\}$ such that at those quantities neither firm as an incentive to deviate. Clearly the quantities the cartel suggests do not satisfy this property and neither do the perfectly competitive quantities. In the next section we will look at the concept of a **Nash Equilibrium** which is a tool that will help us to describe the resting point of this competitive process precisely. First, however, let us discuss briefly the importance of expectations and conditions under which we could expect the collusive agreement to hold.

Conjectural Variations

Notice that we assumed that the firms cheated by varying quantity at the given price. We could have also assumed that firms would keep output constant but would reduce price. Or we could have assumed that cheating would come in the form of some combination of price-output variation.

To be completed.

Repeated Games

We solved a very simple static model. The collusive agreement is a one shot option. Since there is no repeated interaction, the cartel has no way of punishing the firms that cheat. However, suppose that the firms were to interact repeatedly. Then, the cartel could punish the deviating firms by flooding the market and reduce the price drastically. If the punishment is large enough then it is reasonable to believe that the cartel could elicit cooperation from the member firms. This is known as a **trigger strategy**.

17.4 Competition

17.4.1 Cournot

Consider the case in which firm's engage in quantity competition. This is known as **Cournot** competition. Firms act as monopolists so they consider the effect that their quantity decision has on price. Each firm, however, also realizes that the output its competitor selects will affect price. Therefore, a Cournot equilibrium is a Nash equilibrium in which quantities are the strategic objects.

Each of the two firms solve:

$$\max_{y_i} \left(a - b\left(y_i + y_j\right)\right) y_i - c_i y_i.$$

Notice that y_i is the quantity selected by firm i, while y_j denotes its competitors choice. Firm i, therefore, explicitly considers the effect that its competitor's quantity choice has on market price. This is the strategic effect.

The first order condition is given by:

$$[y_i]: \quad a - 2by_i^S - by_j \quad = \quad c_i$$

which we can solve to obtain:

$$y_i^S = \frac{a - c_i - by_j}{2b}.$$

This is known as firm i's **reaction function.** A reaction function gives firm i's optimal quantity choice as a function of firm j's output. Therefore, it tells us firm i's optimal output for any possible output that firm j may choose. Since we have two firms the reaction functions are given by:

$$y_1^S = \frac{a - c_1 - by_2}{2b}$$

and

$$y_2^S = \frac{a - c_2 - by_1}{2b}.$$

A Cournot equilibrium is a Nash equilibrium in quantities. It is a pair of optimal quantities for each firm, $\left(y_1^S, y_2^S\right)$, such that given y_1^S, y_2^S maximizes firm 2's profit and given y_2^S, y_1^S maximizes firm 1's profit. Mathematically speaking, the Cournot quantities are found by solving the reaction function simulataneously.

If both firms are identical, then Cournot quantities are given by:

$$y_i^S = \frac{a - c}{3b},$$

for $i = 1, 2$. Total quantity supplied is equal to:

$$Y^S = \frac{2\left(a - c\right)}{3b}.$$

Finally, Cournot competitors sell at price:

$$p^S = \frac{a + c}{3}$$

and so their profits are equal to:

$$\pi_i^S = \frac{\left(a - c\right)^2}{9b}$$

17.4.2 The Cartel Game

Suppose we set up the game matrix for the cartel game we are analyzing in this chapter. The payoff matrix in general form is given below. The rows of the matrix contain Firm 1's possible action while the columns contain firm 2's strategies. Both firms can either collude or renege. Notice that payoffs are jointly determined by both firms actions since the actual profit level each firm receives from depends on whether it colludes or reneges and also on whether its competitor colludes or reneges (since π_i is a function of y_1 and y_2). This is the nature of the strategic interaction. The payoff matrix below then, contains the general statement of the game.

$$\begin{array}{lll} & C & R \\ C & \pi_1\left(y_1^C, y_2^C\right), \pi_2\left(y_1^C, y_2^C\right) & \pi_1\left(y_1^C, y_2^R\right), \pi_2\left(y_1^C, y_2^R\right) \\ R & \pi_1\left(y_1^R, y_2^C\right), \pi_2\left(y_1^R, y_2^C\right) & \pi_1\left(y_1^S, y_2^S\right), \pi_2\left(y_1^S, y_2^S\right) \end{array}$$

Let us, however, replace the payoffs with the actual payoffs from the cartel with symmetric cost functions and linear demand. We obtain:

$$\begin{array}{lll} & C & R \\ C & \frac{(a-c)^2}{8b}, \frac{(a-c)^2}{16b} & \frac{3(a-c)^2}{8b}, \frac{9(a-c)^2}{64b} \\ R & \frac{9(a-c)^2}{64b}, \frac{3(a-c)^2}{32b} & \frac{(a-c)^2}{9b}, \frac{(a-c)^2}{9b} \end{array}$$

Notice that if firm 1 chooses to collude, then firm 2 finds it optimal to renege on the contract since

$$\frac{9\left(a-c\right)^2}{64b} > \frac{\left(a-c\right)^2}{8b}.$$

Similarly, if firm 1 chooses to renege, firm 2 should also renege since

$$\frac{\left(a-c\right)^2}{9b} > \frac{3\left(a-c\right)^2}{32b}.$$

The analisis is pefectly symmetric. In the absence of a mechanism that controls cheating (a self enforcing agreement) both firms find it in their self-interest to renege on the contract. The Nash equilibrium is, then, for both firms to renege on the contract even though collusion gives them higher profits!

17.4.3 A Simple Numerical Example

If we assume that $c_1 = c_2 = c$, then we can compare price, output, and firm profits under the three market structures. Consider the following Table Oligo1.

	Collusion	Cournot	Competition
y_1	$\frac{a-c}{4b}$	$\frac{(a-c)}{3b}$	$\frac{a-c}{2b}$
y_2	$\frac{a-c}{4b}$	$\frac{(a-c)}{3b}$	$\frac{a-c}{2b}$
Y	$\frac{a-c}{2b}$	$\frac{2(a-c)}{3b}$	$\frac{a-c}{b}$
p	$\frac{a+c}{2}$	$\frac{a-2c}{3}$	c
π^1	$\frac{(a-c)^2}{8b}$	$\frac{(a-c)}{9b}^2$	0
π^2	$\frac{(a-c)^2}{8b}$	$\frac{(a-c)}{9b}^2$	0
π^I	$\frac{(a-c)^2}{4b}$	$\frac{2(a-c)}{9b}^2$	0

In Table Oligo1 we have a baseline model in which to compare Collusion, which is the industry maximum against strategic competition in quantities and the competitive benchmark.

17.4.4 Bertrand

TBC

17.5 The Core Solution

The above analysis suggests that there is an important trade-off between cooperation and competition. Just like in the Prisoner's Dilemma of the Previous chapter. TBC. What does cooperative game theory contribute, then?

17.6 Market Share

It is reasonable to wonder what happens to the analysis when the number of firms in the economy varies. In particular, we are interested on the effect that varying the number of firms has on industry output, market price, and profits. The analysis that we carryout in this section presents sufficient conditions for the competitive outcome, but not necessary conditions. As we have seen, the equilibrium outcome of an imperfectly competitive market may be close to the competitive outcome even with few firms, provided there is sufficient threat of entry. Let us modify Cournot's model so that there are n identical firms in the market. We wish to determine what occurs to price, quantity and profits as we vary the number of firms inthe market between 1 and infinity. Note that since the firms are identical we will solve for a symmetric equilibrium in which all firms in the market have an equal market share.

17.7 How Should we Study Imperfect Competition?

When do we expect the collusive outcome?

TBC Telser/Stigler.

Transactions Costs

17.8 Applications

17.8.1 Cartels: OPEC and DeBeers

Repeated games and trigger strategies

17.9 Exercises