Chapter 10

General Equilibrium of a Production Economy

10.1 Introduction

In this chapter we study the structure of a general equilibrium economy in which there is production. We consider, initially, a very simple one factor, one consumer, one period economy.

10.2 Structure of the Economy

Consider the following assumptions that explain the world that surrounds us.

To be completed.

10.2.1 Consumers

Consider an economy with a single consumer with T units of time. The consumer obtains utility from consumption and leisure, but his consumption possibilities are bounded by his labor and financial income. The consumer must therefore allocate his time endowment between leisure which gives him direct utility and labor, which allows him to expand his consumption possibilities. The consumer solves the following utility maximization problem, which we label $[C]$:

$$\begin{array}{lll} \max\limits_{C,R} & u(C,R) & \\ s.t. & L+R & = T \\ & pC & = wL+\pi \end{array}$$

Notice that we can convert this problem into a standard utility maximization problem by combining both constraints into just one constraint. By doing this we obtain:

$$pC + wR = wT + \pi.$$

Notice that if we define income as $S = wT + \pi$ we have a standard budget constraint. We must, however, be careful when we interpret income. In this case income is not the amount of money that we earn. Earned income is equal to:

$$wL + \pi$$

which is the sum of asset income (firm profits) and the labor income (notice that we work L hours and receive an hourly wage rate of w). Full income, on the other hand, is the value of our endowment plus asset income. Notice that our time endowment is worth wT dollars, so full income equals:

$$wT + \pi.$$

Notice that we give the consumer the profits of the firm since we are assuming that the only consumer in the world owns the firm. One way of thinking of full income is to assume that the consumer sells his entire endowment in the labor and obtains wT as labor income. Then he becomes a buyer in the time market and purchases a certain amount of time back. This amount of time is devoted to leisure activities and is represented by R.

The consumer's problem is given by:

$$\begin{array}{lll} \max\limits_{C,R} & u(C,R) & \\ s.t. & pC+wR & = wT+\pi \end{array}$$

where we see that the consumer spends full income on consumption and buying back part of his endowment (this is precisely the time he allocates to leisure). We say that consumers supply labor elastically since leisure gives direct utility labor supply will depend on the wage rate. Consumers supply labor inelastically when they do not get direct utility from leisure.

In this case it is consumer's self interest to supply the entire endowment of time regardless of the wage rate.

In order to solve the consumer's problem we use the method of Lagrange multipliers and obtain the following first order conditions:

$$\begin{aligned}
[C]: \quad u_C(C^*, R^*) &= \eta^* p \\
[R]: \quad u_R(C^*, R^*) &= \eta^* w \\
[\eta]: \quad pC + wR &= wT + \pi
\end{aligned}$$

As usual $[\eta]$ is the feasibility condition and the ratio of $[C]$ and $[R]$ gives us the optimality condition. Notice that the solution to the consumer's utility maximization problem is a set of Marshallian demand functions that depend on relative prices and full income, S:

$$\begin{aligned}
C^* &= C^*(p, w, S) \\
R^* &= R^*(p, w, S) \\
\eta^* &= \eta^*(p, w, S)
\end{aligned}$$

Since this is a general equilibrium problem firm profits will be endogenous and we must solve for them as part of the equilibrium.

Interpret η^*.

10.2.2 Firms

Let firms solve the following profit maximization problem, which we label $[F]$:

$$\max_l \quad pf(l) \quad - \quad wl.$$

We have assumed that there is only one factor of production, which is labor. We will assume that the marginal product of labor, $f'(l)$, is increasing but exhibits diminishing returns. That is $f' > 0$ and $f'' < 0$. Since there is only one factor of production and technology exhibits diminishing returns technology must be decreasing returns to scale. In this case, we can show that profits must be positive and that labor demand must be a decreasing function of the real wage. These results were established in Chapter xxxx. The first order condition is given by:

$$[l]: \quad pf'(l^*) \quad = \quad w.$$

Notice that there are no fixed costs so the shut down decision of the firm is trivial. Notice that the solution to the problem is given by a labor demand schedule which is a function of the wage and the price level:

$$l^* = l^*(w, p).$$

As we saw in Chapter xxxx, decreasing returns to scale are essentially a short run phenomenon. Following this line of reasoning allows us to obtain a different interpretation of the production side of the economy. We can reinterpret the problem as a short-run profit maximization problem in which there are, in fact two factors of productions labor and capital, say, but we assume that capital is installed and its stock can not be varied in the short run. The firm's problem, in this case would be:

$$\max_{l} \quad pF\left(\overline{k}, l\right) - wl.$$

In the short-run problem profits are the return to the fixed factor –capital, in this model. As we have seen, the return to a factor that is fixed temporarily is called a quasi-rent (recall that capital is fixed in the short term) and it is equal precisely to the profits that we obtained in the initial formulation of the problem.

We could, in fact, assume that technology is constant returns to scale. In the single factor case this amounts to assuming a linear technology:

$$f(l) = Al.$$

In this case we can easily show that the firm receives zero profits. The competitive equilibrium would exist, but demand for labor would be perfectly elastic. The firm is willing to hire whatever labor the consumer is willing to supply at the wage $w = A$ and nothing if the wage exceeds A.

10.3 Competitive Equilibrium

10.3.1 Market Structure

In order to define a competitive equilibrium properly, we must first establish the type of market in which the firms interact. Since we are studying the properties of a competitive equilibrium we will assume that markets are competitive this requires that both households and firms take prices as given when they solve their utility and profit maximization problems. Another way of describing the assumption of competitive markets is to state that prices are parameters, or that agents take prices parametrically. Thus, we assume that factor and goods markets are competitive..

10.3.2 Definition of a Competitive Equilibrium

We label equilibrium conditions as $[E]$. In this simple model, the equilibrium or consistency conditions require market clearing:

$$\begin{aligned} C^*\left(\left(\frac{p}{w}\right)^e\right) &= c^*\left(\left(\frac{p}{w}\right)^e\right) \\ L^*\left(\left(\frac{w}{p}\right)^e\right) &= l^*\left(\left(\frac{w}{p}\right)^e\right) \end{aligned}$$

where we have used homogeneity properties of supply and demand function to write supply and demand functions as functions of relative prices. Notice that the equilibrium conditions state that the market clearing relative prices equate supply and demand in both markets. We also call equilibrium prices market clearing prices and equilibrium conditions market clearing conditions.

10.3.3 Walras' Law

Notice that implicitly we have also stated that the market clearing conditions will give us market clearing relative prices. That is we can not obtain w or p separately. Another way of saying this is that we can not obtain absolute (dollar denominated) prices. The equilibrium price is always a relative price. In this case, however, we have the price of the consumption goods in terms of leisure and the price of leisure in terms of goods. Absolute prices are also relative prices, except that they measure opportunity cost of goods in terms of money. Since there is no money in this world we can not obtain an absolute price.

Notice that Walras' law tells us that when consumers are on their budget sets, if one market clears then the other market must also clear. Walras' law, then, tells us that we only need to consider one market clearing condition because, as long as consumers are on their budget set, the other market will clear at the reciprocal price. Let's look at this in greater detail.

Insert Walras' Law argument.

10.4 Extensions

Considering that we can distinguish goods by time, location, and state of the world, the analysis of general equilibrium that we have just undertaken is fairly general. For example, consider instead a dynamic economy in which consumers have access to credit markets. This economy would include two goods –consumption today and tomorrow– and can easily be handled within this framework. Similarly, in the case of uncertainty we can maximize over contingent consumption and production plans. We can support the allocation through contingent claims traded in financial markets.

Suppose that instead of solving for relative prices, we wish to obtain absolute prices. Then, we must pin down the price level which is the value of unit of money in terms of the consumption good. We must therefore expand the economy to include money and we must motivate demand for money perhaps through the utility function or through a cash-in-advance (CIA) constraint. The absence of money allows us to have one normalization (?)

Since we have computed a CE with only one agent the institutional or property rights set-up of the model is straightforward.. One agent owns everything. However in models with public goods or government intervention the assignment of property rights and the role of institutions may be fundamental.

10.5 A Simple Cobb-Douglas Example

Consider a simple economy in which preferences are given by:

$$u(C, R) = \log C + \log R$$

and the production function is:

$$f(l) = Al^{\frac{1}{2}}.$$

10.5.1 The Consumer's Problem

After solving $[C]$ we will obtain demand for consumption and leisure, labor supply, and the marginal utility of income. We proceed by constructing the Lagrangian and solving the first order conditions. The consumer's problem is given by:

$$\begin{aligned} \max_{C,R} \quad & \log C + \log R \\ s.t. \quad & pC + wR = wT + \pi \end{aligned}$$

and has first order conditions:

$$\begin{aligned} [C]: \quad \frac{1}{C^*} &= \eta^* p \\ [R]: \quad \frac{1}{R^*} &= \eta^* w \\ [\eta]: \quad pC^* + wR^* &= wT + \pi \end{aligned}$$

We can obtain the optimal values of C, R, and η by solving the first order conditions. In order to obtain labor supply we use the time constraint. The answers are summarized below:

$$\begin{aligned} C^* &= \frac{wT+\pi}{2p} \\ R^* &= \frac{T}{2} + \frac{\pi}{2w} \\ L^* &= \frac{T}{2} - \frac{\pi}{2w} \\ \eta^* &= \frac{2}{wT+\pi} \end{aligned}$$

10.5.2 The Firm's Problem

We solve $[F]$ to obtain labor demand and supply of goods. The firm's problem, assuming a single input diminishing returns technology is given by:

$$\max_{l} \quad pAl^{\frac{1}{2}} \quad - \quad wl.$$

The first order condition is given by:

$$[l]: \quad p\frac{1}{2}A\left(l^*\right)^{-\frac{1}{2}} \quad = \quad w.$$

Notice that there are no fixed costs so the shut down decision of the firm is trivial. Notice that the solution to the problem is given by a labor demand schedule which is a function of the wage and the price level:

$$l^* \quad = \quad \left(\frac{A}{2\frac{w}{p}}\right)^2.$$

Inserting labor demand into the production function gives us supply of goods:

$$c^* \quad = \quad A\left(\frac{A}{2\frac{w}{p}}\right).$$

Finally, multiplying c^* by p and subtracting costs gives us the firm's profit function:

$$\frac{\pi^*}{p} \quad = \quad A\left(\frac{A}{2\frac{w}{p}}\right) - \frac{w}{p}\left(\frac{A}{2\frac{w}{p}}\right)^2.$$

10.6 The Competitive Equilibrium

The competitive equilibrium of this economy requires that firms and consumers interact in markets. Both firms and consumers take prices as given when they solve their optimization problems. Supply and demand schedules –the solution to firm and consumer optimization problems– relate optimal quantities for all possible prices. Since supply and demand curves are functions of price, we need some way in which to make firm and consumer decisions consistent. Markets provide a setting in which firms and consumers come together and make their decisions consistent. Prices are the coordinating or allocation mechanism behind the market. The economy is in equilibrium when decisions are consistent and both firms and consumers are satisfied with their supply and demand decisions. That is, in equilibrium labor and goods markets clear and, given equilibrium prices, the consumer maximizes utility and the firm maximizes profit.

The Labor Market

Let's consider the labor market. Equilibrium in the labor market is a wage rate that equates labor supply and labor demand:

$$\left(\frac{A}{2\frac{w}{p}}\right)^2 = \frac{T}{2} - \frac{\pi}{2w} .$$

Notice that consumer's labor supply depends on firm profits. However we know that firm profits. Therefore the labor market clearing conditions is given by:

$$\left(\frac{A}{2\left(\frac{w}{p}\right)^e}\right)^2 = \frac{T}{2} - \frac{1}{2}\left(\frac{A}{2\left(\frac{w}{p}\right)^e}\right)^2,$$

where we have simplified labor supply. The first thing to notice is that as expected the labor market clearing depends only on the real wage. As we saw before, this is a consequence of Walras' Law. Notice, also, that the market clearing condition is not a function of $\frac{w}{p}$. Rather, there is a wage rate –the equilibrium wage rate $\left(\frac{w}{p}\right)^e$– that satisfies the market clearing condition. Solving for $\left(\frac{w}{p}\right)^e$ we obtain:

$$\left(\frac{w}{p}\right)^e = \frac{A}{2\left(\frac{1}{3}T\right)^{\frac{1}{2}}}.$$

At this equilibrium wage rate supply of labor equal demand for labor and the equilibrium quantity –either L^e or l^e – is given by:

$$L^e = l^e = \frac{1}{3}T.$$

We can see this graphically in Figure xxxx.

Insert Figure xxxx.

Goods Market

Now we turn to the goods market. Notice that there is goods market clearing requires that demand for goods equal supply of goods:

$$\frac{1}{2}\frac{w}{p}T - \frac{1}{2}\frac{\pi}{p} = A\left(\frac{A}{2\frac{w}{p}}\right).$$

Once again we must substitute in for profits:

$$\frac{1}{2}\left(\frac{w}{p}\right)^e T - \frac{1}{2}A\left(\frac{A}{2\left(\frac{w}{p}\right)^e}\right) - \frac{1}{2}\left(\frac{w}{p}\right)^e\left(\frac{A}{2\left(\frac{w}{p}\right)^e}\right)^2 = A\left(\frac{A}{2\left(\frac{w}{p}\right)^e}\right).$$

Notice that once again the market clearing condition is a function only of the relative price. This condition implies that equilibrium market price of the consumption is equal to:

$$\left(\frac{p}{w}\right)^e = \frac{2\left(\frac{1}{3}T\right)^{\frac{1}{2}}}{A},$$

which is precisely the reciprocal of the real wage.

Walras' Law

Walras reasoned (find the source of this) that if all markets except for one were in equilibrium, then the final should also clear. Therefore, one equilibrium condition is redundant. Suppose that we knew that in the economy we have been studying the labor market clears. Then we have the equilibrium wage rate and the equilibrium quantity of labor. These quantities are given by:

$$\left(\frac{w}{p}\right)^e = \frac{A}{2\left(\frac{1}{3}T\right)^{\frac{1}{2}}},$$

and:

$$l^*\left(\left(\frac{w}{p}\right)^e\right) = L^*\left(\left(\frac{w}{p}\right)^e\right) = \frac{1}{3}T.$$

Walras asserts that the goods market must also be in equilibrium.

Let's verify his claim by looking at the consumer's budget constraint. We know that in equilibrium the budget constraint must hold with equality:

$$C + \left(\frac{w}{p}\right)^e R^e = \left(\frac{w}{p}\right)^e T + \left(\frac{\pi}{p}\right)^e$$

where if we recall

$$R^e + L^e = T$$

and

$$\left(\frac{\pi}{p}\right)^e = f\left(l^e\right) - \left(\frac{w}{p}\right)^e l^e.$$

If we substitute both of these constraints into the consumer's budget constraint we obtain:

$$C^e = f\left(l^e\right),$$

which states that at the equilibrium real wage, consumption demand equals its supply so the labor market must be in equilibrium.

In order to obtain the actual market clearing quantities we simply evaluate the production function at the equilibrium quantity of labor. The equilibrium relative goods market clearing price is given by the reciprocal of the real wage.

10.7 Exercises

Exercise 10.7.1 *Verify that at the posited wages the markets do in fact clear.*

Exercise 10.7.2 *Compute the comparative statics of equilibrium labor supply, the equilibrium real wage, and equilibrium consumption demand with respect to A and T in the text. in the text.*

Exercise 10.7.3 *Re-do the problem but use technology AL (CRS)*

Exercise 10.7.4 *Add money and pin down absolute prices.*

Exercise 10.7.5 *Re-do with quasi rents*

Exercise 10.7.6 *How would you set up and define the CE of an economy without production but with two goods?*

Exercise 10.7.7 *How would you handle uncertainty if insurance was fair?*

Exercise 10.7.8 *How would you do credit?*