

THE UNIVERSITY OF CHICAGO
DEPARTMENT OF ECONOMICS
Elements of Economic Analysis IV
Econ 203
SPRING 2001
PROBLEM SET 1

Problem 1: The competitive equilibrium of an endowment economy.

Consider a one good, one consumer production economy. Suppose that the agent has an endowment of T units of time. Also assume that the agent's preferences are given by:

$$U(x, y) = \log(c) + \log(R).$$

Finally, let the price of the consumption good be p and the wage rate be w .

a. Write down the agent's maximization problem. Be very clear about the constraints.

b. Solve each agent's maximization problem. You should obtain c^* , R^* , and η^* , where η^* denotes the Lagrange multiplier on the agent's budget constraint.

Now, suppose that there is a firm in the economy that produces according to the following production technology:

$$f(l) = l^\alpha$$

c. What is the firm's labor demand curve $l^D(w, p)$?

d. What are the firms' profits?

e. What are the equilibrium conditions for this economy?

f. Find the market clearing real wage.

g. Show that the consumer's budget constraint holds with equality.

h. Use your answers to part g and labor market clearing to show that at the equilibrium real wage, you found in part f the goods market must clear.

i. Now use the market clearing condition in the goods market to find the equilibrium price.

j. Determine the values of η^* .

k. Graph the solution. Label your graphs clearly.

l. Would the answer be the same if I assumed that $p = 1$? Explain your answer clearly.

Problem 2: The planner's problem.

Assume the same economy as in problem 1, but now consider the planning problem.

- a. What is the PPF?
- b. Write down the planners problem. (Hint: there should be two constraints because aggregate consumption of each good can not exceed the economy's endowment). Label the multipliers ϕ_c and ϕ_t .
- c. What are the first order conditions of the planners problem?
- d. Find the solution to the planner's problem and label the quantities x_i^p and y_i^p for $i = 1, 2$.

If we compare the first order conditions of the planners problem, to the first order conditions of the competitive equilibrium in problem 1, we can obtain a relationship between ϕ_c , ϕ_t , p , and w , so that the CE and the planner's problem give us the same solution. This called **decentralizing** the planner's problem.

- d. Decentralize the planners problem (That is, find the equilibrium prices so that the two problems have the same solution.)
- e. What does this tell you about the economy? (Hint: Think about the welfare theorems)
- f. Explain clearly the sense in which prices solve a coordination problem between consumption and and production.