THE UNIVERSITY OF CHICAGO
DEPARTMENT OF ECONOMICS

Elements of Economic Analysis II
Problem Set 2

This problem set is due at the TA session on Friday October 19.

Problem 1: The Short Run Cost Function

In the last problem set we analyzed the cost function for a Cobb-Douglas production function of arbitrary scale. In this problem we will study the short run cost function. Consider, first, a firm that has a Cobb-Douglas technology with constant returns in the long run. The firm's technology is given by:

$$y = k^{1-\alpha} l^{\alpha}.$$

Suppose that capital is fixed in the short run, but that the firm can hire labor in perfectly competitive labor markets.

a. Derive the firm's short run demand for labor.

b. What are the firm's fixed costs?

c. What are the firm's variable costs?

d. What are is the firm's total cost function?

e. In the same coordinate axis, plot the fixed cost function in blue, the variable cost function in red, and the total cost function in black. (Hint: Since you don't have specific values for the paremeters, you must draw the curves in the proper general shape. Use first and second derivatives to characterize what the curves look like.)

f. What is marginal cost? How does it vary with α?

g. In another set of coordinate axes, plot average variable cost in red, average total cost in blue, and marginal cost in black.

Problem 2: The short run profit function.

Suppose that technology is given by:

$$y = l^{\alpha}.$$

a. What is the firm's total cost function? (Hint: You already solved a similar problem above).

b. What is the firm"s profit maximizing level of output? Denote it $y^* (\alpha, w, p)$.

c. What must be true about α so that in fact y^* maximizes the firm's profit?

Now, instead of solving for the firm's optimal output solve:

$$\max_{l} pl^{\alpha} - wl.$$

d. What is the firm's factor demand function $l(p, w)$?
e.Find the firm's profit function, $\pi\left(p, \overline{k}, w, r\right)$.
f. Show that $\frac{d\pi}{dp} = y^*(\alpha, w, p)$.
g. Show that $\frac{d\pi}{dw} = -l(p, w)$.
g. Show that y^* is homogeneous of degree 0 in prices.
h. Show that π^* is homogeneous of degree 1 in prices.