

THE UNIVERSITY OF CHICAGO  
DEPARTMENT OF ECONOMICS  
**Elements of Economic Analysis II**  
**Problem Set 1**

This problem set is due at the TA session on Friday October 12.

**Problem 1: The Cost Function and Its Properties**

Consider a firm that has a Cobb-Douglas technology. The firm wishes to minimize cost of producing  $y$  units of output and has access to perfectly competitive factor markets. The firm's cost minimization problem is given by:

$$\begin{aligned} \min_{k,l} \quad & wl + rk \\ \text{s.t.} \quad & k^\alpha l^\beta = y \end{aligned}$$

Let  $\mu$  denote the Lagrange multiplier on the output constraint.

- What are the parameters of the problem?
- Find the conditional demand functions. Label them  $l^*(w, r, y)$  and  $k^*(w, r, y)$ .
- Find the cost function:  $C(w, r, y)$ . What is its interpretation?
- Find  $\mu^*$ . What is its interpretation?
- Find  $\frac{dC}{dy}$  and show that it is equal to  $\mu^*$ .
- How does  $\frac{dC}{dy}$  vary with  $(\alpha + \beta)$ ?

For the rest of the problem assume that  $(\alpha + \beta) = 1$ .

- Show that the conditional demand functions are homogeneous of degree 0 in  $w$  and  $r$ .
- Show that the cost function is homogeneous of degree 1 in  $w$  and  $r$ . What is the intuition?
- Show that  $\frac{dC}{dw} \geq 0$  and  $\frac{dC}{dr} \geq 0$ . What is the intuition?
- Show that the expenditure function is concave in  $w$ . Do this graphically and mathematically. What is the intuition?