

The University of Chicago
Department of Economics
ELEMENTS OF ECONOMIC ANALYSIS I
Problem Set 4

This problem set is due on Lecture 17.

1. Suppose that Jane's utility function is given by:

$$U(x, y) = x^{\frac{1}{5}}y^{\frac{4}{5}}.$$

Assume that Jane has endowments $(\omega_x, \omega_y) = (10, 10)$ and that $p_y = 10$ throughout the problem.

a. Suppose $p_x = 10$. How much of each good will Jane consume?

b. Now assume that p_x drops to 5. What is the change in demand for good 1? Calculate the three components of this change (substitution effect, ordinary income effect, endowment income effect). Is Jane better off at this new price? Explain why or why not.

c. For which values of p_x will Jane be a net supplier of good 1?

d. For which value of p_x is Jane's utility lowest? Explain with reference to a diagram.

2. Jessica is a farmer. She grows tomatoes and squash, which she also consumes. Suppose Jessica's utility function is given by:

$$u(T, S) = T^{0.4}S^{0.6}$$

where T denotes tomatoes and S denotes squash. Assume that any quantity of tomatoes and any quantity of squash can be purchased.

a. Suppose that the price of tomatoes is 1 dollar and the price of squash is 2 dollars. Jessica's farm produces 100 pounds of tomatoes and 50 pounds of squash. On a graph, draw the budget constraint in red ink and label the initial endowment point E.

b. What are Jessica's gross demands? What are her net demands? Label her chosen (consumption) point O.

c. Suppose that before Jessica sells her endowment, the price of squash falls to 1 dollar and the price of tomatoes stays at 1 dollar. Draw the new budget line in blue ink. Does Jessica's demand for squash rise or fall? By how much? What happens to demand for tomatoes?

d. Compute the substitution effect, the ordinary income effect, and the endowment effect and plot these on your diagram labeling each chosen point clearly.

e. Suppose that the price of milk falls after Jessica has sold her endowment for money. How much squash will she consume now? How about tomatoes? What are the income and substitution effects?

f. Does the timing of the price change matter? Is Jessica better or worse off if the price change occurs before she sells her endowment, or after she sells her endowment?

3. Suppose that Nelson has an endowment of 12 hours per day, which he can spend either working (for L hours) or relaxing (for R hours, with $R+L=12$, and both R and L greater than or equal to zero). He has $M=100$ of non-labor income and the price for one unit of consumption C is simply equal to 1.

a. What is Nelson's full income? Write down and graph his budget constraint. Be sure to carefully label all axes, intercepts, etc.

b. Suppose that Nelson's utility from having C units of consumption and R hours of relaxation is simply:

$$U(R, C) = \log(R) + 3 \log(C)$$

Solve for Nelson's optimal labor supply, L , as a function of the wage rate, W . For which wage rate will Nelson choose not to work?

c. How would your answers to (b) change if his utility function were instead:

$$U(R, C) = \log(R) + \log(C).$$

Give intuition for the difference.

4. Consider a two-period model in which Marty's utility from consuming an amount c_1 in period 1 and c_2 in period 2 is simply:

$$U(c_1, c_2) = \log(c_1) + \beta \log(c_2).$$

Suppose that Marty's endowment of income in the two periods is $\omega_1 = 100$ and $\omega_2 = 100$.

a. Assume that $\beta = \frac{2}{3}$. If Marty can borrow and lend freely at an interest rate of $r=5\%$, what is his optimal consumption in each period? Does he borrow or lend in the first period?

b. Continue to assume that $\beta = \frac{2}{3}$. At what interest rate will Marty choose simply to consume his endowment in each period?

c. Now suppose that Marty's savings earn 5% but he pays a 20% interest rate to borrow. Draw his budget constraint in this case. For which values of β will Marty choose to save (hint: consider budget constraint with $r=5\%$ for this

part) and for which will he choose to borrow (now $r=20\%$). For which values of b will he locate at the kink of his budget constraint (i.e. simply consume his endowment in each period)?

5. In an isolated mountain village, the only crop is corn. Good harvests alternate with bad harvests. This year the harvest will be 1,000 bushels. Next year it will be 150 bushels. There is no trade with the outside world. Corn can be stored from one year to the next, but rats will eat 25% of what we store. Suppose that preferences between consumption today and consumption tomorrow are given by:

$$U(c_1, c_2) = \log(c_1) + \beta \log(c_2).$$

a. In red ink, draw the budget constraint showing consumption possibilities for the village. Plot a along the x-axis. Label the graph clearly.

b. What is the village's optimal consumption? How much do the rats get? Let $b=.95$

c. Suppose that a road to the village is built so that now it can trade with the rest of the world. The villagers can now buy and sell corn at the world price: 1 dollar per bushel. They can borrow and lend money at a rate of 10%. Graph the new budget line for the village. What is the new optimal consumption path for the village?

d. What happens if the situation is as in part C, but there is a transportation cost of 10 cents?