THE UNIVERSITY OF CHICAGO
DEPARTMENT OF ECONOMICS

Elements of Economic Analysis I

Econon 200 (05)

SPRING 2001
PROBLEM SET 3

This problem set is due at the **beginning** of the TA session on Friday.

Problem 1: Further practice with utility maximization.

Suppose that Lisa has the following utility function:

$$U(x_1, x_2) = \beta\sqrt{x} + y$$

where x and y represent her consumption of goods 1 and 2, respectively. Assume that she has total income $m = 25$, and that the prices of the goods $p_x = 1$ and $p_y = 2$.

a. Solve for Lisa's optimal choices as a function of β for $\beta \in [0, 10]$.

b. Graph these choices as a function of β and provide intuition for the general shape.

c. Graph the budget constraint and the indifference curve associated with Lisa's optimal choice when $\beta = 10$.

Problem 2: Some Engel and demand curves.

Homer's utility for consuming beer and donuts is:

$$U(b, d) = b^{0.3}d^{0.7}$$

The prices are p_b and p_d and his income is m.

(a) Derive Homer's demand functions for beer and donuts using the substitution and the Lagrange methods.

(b) Draw the Engel curve for both goods.

(c) Draw the demand curve for both goods.

(d) Is either good an inferior good or a Giffen good? Explain.

(e) Are Homer's preferences homothetic? Explain.

(f) Are the goods substitutes or complements? Explain.

Problem 3: Expenditure minimization.

This question is a continuation of last problem set. The expenditure minimization problem corresponding to that exercise is given by:

$$\begin{aligned} \min_{x,y} \quad & p_x x + p_y y \\ s.t. \quad & x^\alpha y^{1-\alpha} = U \end{aligned}$$

Let η denote the Lagrange multiplier on the utility constraint.

a. What are the parameters of the problem?

b. Find the Hicksian demand functions. Label them $x_h^*(p_x, p_y, U)$ and $y_h^*(p_x, p_y, U)$.

c. Find the expenditure function: $e(p_x, p_y, U)$. What is its interpretation?

d. Find η^*. What is its interpretation?

Problem 4: Verifying some properties of the the expenditure function and hicksian demands.

In the previous problem you solved the expenditure minimiation problem associated with the following utility function:

$$U(x, y) = x^{\alpha} y^{1-\alpha},$$

obtaining Hicksian demand curves, the expenditure function and the Lagrange multiplier, η.

a. Show that the expenditure function is homogeneous of degree 1 in p_x and p_y. What is the intuition?

b. Show that $\frac{de}{dp_x} \geq 0$ and $\frac{de}{dp_y} \geq 0$. What is the intuition?

c. Find $\frac{de}{dU}$.

d. Show that $\frac{de}{dU} = \eta^* > 0$. What is the intuition?

e. Show that the expenditure function is concave in p_x. Do this graphically and mathematically. What is the intuition?

Problem 5: Using duality.

In the last problem set you solved the following $[UMP]$:

$$\begin{aligned} \max_{x,y} \quad & x^{\alpha} y^{1-\alpha} \\ s.t. \quad & p_x x + p_y y = m \end{aligned}$$

and we obtained the Lagrange multiplier, λ^*, the Marshallian demand functions $x_m^*(p_x, p_y, m)$ and $y_m^*(p_x, p_y, m)$, and the indirect utility function $v(p_x, p_y, m)$. In the previous problem you solved the corresponding $[EMP]$:

$$\begin{aligned} \min_{x,y} \quad & p_x x + p_y y \\ s.t. \quad & x^{\alpha} y^{1-\alpha} = U \end{aligned}$$

to obtain the Lagrange multiplier η^*, the Hicksian demand functions $x_h^*(p_x, p_y, U)$ and $y_h^*(p_x, p_y, U)$, and the expenditure function $e(p_x, p_y, U)$. Using the results from these two exercises verify the following statements.

a. From your expenditure function, obtain the indirect utility function. Does it coincide with your answer from last week?

b. Explain graphically why your answer makes intuitive sense.

c. From your Hicksian demands, use the properties of duality to obtain the Marshallian demands. Do they coincide with your answer from last week?

d. Explain the economic intuition of your solution graphically.

e. Can you obtain the margina utility of income? Why is your procedure economically sensible?

Problem 6: Income and substitution effects.

Suppose that Charlie consumes apples and bananas and that he orders his preferences according to the following utility function:

$$U(a, b) = ab.$$

Also assume that $p_a = 1$ and $p_b = 2$, and that $m = 40$.

a. Write down Charlie's optimal consumption problem. (Hint: Tell me what Charlie maximizes and subject to what).

b. What is Charlie's optimal demand of apples and bananas?

c. Graph the budget line with black ink and label the optimal choice with the letter A.

Suppose that the price of bananas falls to 1 dollar.

d. If I compensate Charlie for the decline in prices with sufficient money so that he can still afford the original bundle, what would this new income level be?

e. How many apples and bananas does Charlie consume at the new prices and the new income?

f. Using red ink, draw the new budget line and label the new optimal point, point B.

g. Does the substitution effect of the fall in the price of bananas lead Charlie to consume more or less bananas? How about apples? How many more or less of each?

h. How many apples and bananas does Charlie consume at the new prices and the original income? (That is $p_a = 1$, $p_b = 1$, and $m = 40$.)

i. Draw this new budget line in blue ink and label the new optimal consumption bundle,

At this point in this problem your graph should have 3 lines and three points labelled on them.

j. Draw lines that are perpendicular to the y-axis and pass through points A, B, C. Label the income, substitution and the total effect of income on bananas. How much does initial demand change due to substitution? Income?

k. Explain the economic importance of decomposing a price change into an income and a substitution effect.