THE UNIVERSITY OF CHICAGO
DEPARTMENT OF ECONOMICS
Elements of Economic Analysis I
Econ 200 (05)
SPRING 2001
PROBLEM SET 2

This problem set is due at the **beginning** of Lecture 7.

1. Suppose that the consumer's utility maximization problem is given by:

$$\begin{aligned} \max_{x,y} \quad & x^{\alpha} y^{1-\alpha} \\ s.t. \quad & p_x x + p_y y = m \end{aligned}$$

a. What is the numeraire?

b. Find the Marshallian demand functions, and show that they are homogeneous of degree 0 in p_x, p_y, and m.

c. Show that the consumer spends all his money.

d. Find λ^*.

c. Find the indirect utility function, $v(p_x, p_y, m)$. What is its interpretation?

d. Show that $\frac{dv}{dm} > 0$.

e.What is the interpretation of $\frac{dv}{dm}$?

f. Show that

$$\lambda^* = \frac{dv(p_x, p_y, m)}{dm}$$

g. Show that $\frac{dv}{dp_x} \leq 0$ and $\frac{dv}{dp_y} \leq 0$.

2. Suppose that Bart has the following utility function:

$$U(B, C) = B^{\alpha} C^{1-\alpha}$$

where B and C equal the number of bagels and croissants consumed, respectively, and $\alpha \in [0, 1]$. Suppose further that the prices of the two goods are p_B and p_C and that Bart's total income is equal to m.

(a) Using both the substitution and Lagrange multiplier methods, solve for the optimal Marshallian demand functions. Recall that Marshallian demand functions are functions of prices and income. Explain intuitively the effect of these four parameters on Bart's optimal choices.

(b) Plot the optimal choices as a function of p_B. Give intuition for the general shape. Now plot the share of income spent on each good as a function of p_B.

What does this suggest about a consumers expenditure patterns if they have preferences defined by a Cobb-Douglas utility function?

3. Suppose that the price of coffee, p_C, is equal to 4, and that the price of gasoline, p_G, is 6. Marge's total income is $m = 120$, and the government is considering imposing a quantity tax t on her consumption of gasoline.

(a) Graph Marge's optimal choices as a function of t if her utility function is:

$$U(C, G) = C + 2G$$

Assume that possible values for t are between 0 and 6.

(b) Do the same if preferences are now governed by the following utility function:

$$U(C, G) = \min\{C, 2G\}$$

Explain the difference between the plots for (a) and (b).

(c) Suppose finally that Marge's utility function is:

$$U(C, G) = CG + 2C$$

and that $t = 2$. How much tax revenue does the government collect from Marge?

(d) Now suppose that the government instead took this total revenue directly from the her income, rather than through a gasoline tax. Is she indifferent between these two outcomes? What does this result suggest?

(5) Suppose that Sean's utility function for burritos (B) and cheeseburgers (C) is:

$$U(B, C) = BC.$$

Assume that $p_B = 6$, $p_C = 3$, and that Sean has $m = 120$ to spend. Sean is then offered the following two options. Either (1) he can receive five free burritos and buy the rest or (2) he can enjoy a 1/3 reduction (from 6 to 4) in the price of burritos.

(a) Graph the two budget sets that are available to him.

(b) Find his most preferred bundle in both cases. Which of the two options will he prefer?

(c) Now suppose instead that Sean's utility function is:

$$U(B, C) = B^6 C$$

Referring to the two graphs, which is he more likely to prefer now? Show this by deriving his optimal bundles for both options. What is the intuition (Hint: Think about what happens to the MRS).