Designing a Banking System to Eliminate the Potential for Catastrophe

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November 29, 2015

Abstract This paper identifies policies that prevent financial instability and excessive risk-taking while taking into account banks’ endogenous portfolio choice and default decision. Although direct asset restrictions or market concentration can eliminate the potential for a high default risk and risk-shifting, these policies harm social welfare. Transparent banking can deregulate these policies; however, its favorable impact is offset by deposit insurance. Instead, interest rate caps and debt-type managerial compensation eliminate the potential for a catastrophic equilibrium in economic upturns and downturns, respectively. The combination of these schemes neither sacrifices social welfare nor the coverage of deposit insurance. I also find that regulators can provide banks with incentives to comply with this policy set and save the cost of monitoring business cycles to switch between the two policies by ordering banks to report business conditions. Policy implications based on the calibrated model are also proposed.

* Address: 1126 East 59th Street, 60637, USA, e-mail: urotanke@uchicago.edu. I would like to thank my advisors Ali Hortacsu, Gregor Matvos, and Zhiguo He for their help and support. I am grateful for the comments received from Anil Kashyap, Nobuhiro Kiyotaki, Casey Mulligan, and seminar participants at the 32nd International Conference of the French Finance Association, European Financial Management Association 2015 Annual Meetings, Financial Management Association 2015 Annual Meetings, Washington University in St. Louis Graduate Conference, as well as the University of Chicago Industrial Organization Working Group.
1 Introduction

Excessive risk-taking often coincides with high default risk. In the US Savings and Loan Crisis during the 1980s, failed thrifts had disproportionately high concentrations of commercial mortgages, real estate loans, and direct equity investments \cite{Barth1990}. In the Japanese Banking Crisis during the 1990s, the Japanese banks passively took excessive risk by continuing lending to their most troubled borrowers \cite{Peek2005}. In accordance with these observations, researchers suspect that this coincidence is associated with recent crises in the US and Europe\cite{Boyd2014}.

To avoid another financial crisis, regulators are required to identify forces that affect excessive risk-taking and financial instability and examine what policies can prevent them. In order to serve their needs, this paper predicts the asset volatility and default risk of a representative bank from a bank run model that endogenizes the bank’s portfolio choice and default decision in the spirit of \cite{Leland1998}. Moreover, the proposed model provides analytical implications for the effects of various policies on the asset volatility and default risk of a bank at equilibrium. Furthermore, it allows regulators to attain quantitative implications based on the parameters estimated by \cite{Egan2014}. I demonstrate it in this paper. In my model, financial fragility can be initiated by the accidental increase in the funding cost associated with uninsured deposits in the spirit of \cite{Diamond1983}. However, recent studies on industrial organization approach to finance emphasizes the importance of a bank’s endogenous action for raising capital, because the bank’s reaction to the change in uninsured depositors’ sentiment contributes further to financial instability. This is demonstrated by the models of \cite{Egan2014} and \cite{Horta2011} that endogenize the default decision of the firm. Moreover, the theory of gambling for resurrection, such as \cite{Diamond2011} and \cite{Boyd2014}, implies that a bank’s excessive risk taking can be accelerated by the bank’s high default risk. Then, at least, the direct effect of an increase in a bank’s asset volatility could further accelerate the bank’s default risk\cite{Boyd2014}. Therefore, it is important to simultaneously endogenize a bank’s portfolio choice and default decision in order to fully understand the “catastrophe” that originates from a shock to the sentiment of uninsured depositors.

\footnote{See the survey by \cite{Gorton2003}.}
\footnote{See \cite{Boyd2014} for the US and \cite{Drechsler2013} for Europe.}
\footnote{This does not mean that an increase in a bank's asset volatility always increases the bank's default risk. For example, if the bank’s return to risky assets is high, the bank’s risk-taking increases the profitability of the bank and can decrease the bank's default risk due to the bank’s high continuation value.}

\[ \]
Under the new setting, this paper enables regulators to review the analysis of policies on the following domains addressed by previous theoretical studies: (1) direct asset restrictions\textsuperscript{4} e.g., general equilibrium literature such as Nicoló et al. (2012) and Goodhart et al. (2012); (2) deposit market competition (e.g., Matutes and Vives 2000, Allen and Gale 2004); (3) transparent banking (e.g., surveys by Landier and Thesmar 2011 and Goldstein and Sapra 2014); (4) interest rate policy (e.g., Diamond and Rajan 2012); and (5) debt-type managerial compensation (e.g., Bolton et al. 2011, Edmans and Qi 2011). Reviewing these policies, I aim to construct a policy set robust to the following issues. First, I address whether the proposed policies are effective for the entire range of mean returns to risky assets, which proxy phases of business cycles. If these policies are invariant to mean asset returns, regulators do not have to monitor business cycles or ask banks to disclose business conditions. In other words, they do not have to be concerned with the cost of monitoring or banks’ incentives for compliance with regulation. Second, I consider whether these policies ruin social welfare. Even if tight regulation eliminates the potential for excessive risk-taking and financial instability, it can reduce social welfare because it reduces the opportunities of profitable investments, for example. Third, I analyze if these policies are robust to depositor composition. In particular, I address the problem of moral hazard caused by the increasing coverage for deposit insurance\textsuperscript{5}. Lastly, I check banks’ incentives for compliance with regulation.

According to my analysis, direct asset restrictions or market concentration can eliminate the potential for a high default risk and almost prevent risk-shifting. Nevertheless, these policies harm social welfare. Transparent banking forces banks to be committed to their risk strategies, leading to the relaxation of direct asset restrictions and market concentration. However, its favorable impact is offset by deposit insurance. Interest rate caps can eliminate the potential for a high default risk and almost prevent risk-shifting, although regulators may have to set severe negative caps to achieve this. If regulators only focus on the validity of policies during economic upturns, moderate caps are enough for them to achieve this. Moreover, debt-type managerial compensation can eliminate

\textsuperscript{4}In practice, asset restrictions relate to liquidity requirements and capital requirements with risk-weighted measures of assets. Banks can satisfy capital requirements either by reducing risky assets or, if possible, committing themselves to injecting sufficient equity. However, depositors may not be able to believe in bank owners’ promise under conflicts of interest between them. Therefore, banks are more likely to satisfy capital requirements by reducing risky assets.

\textsuperscript{5}As suggested by Diamond and Dybvig (1983), deposit insurance is required to prevent bank runs that arise from liquidity preference shocks to uninsured depositors. This channel is muted in this paper as I assume that deposit insurance is already set to prevents them. Therefore, I consider whether policies are independent of the depositor composition, because regulators do not want these policies to be offset by increasing coverage for deposit insurance.
risk-shifting without harming social welfare or shrinking the coverage of deposit insurance. In addition, it can eliminate the potential for a high default risk in economic downturns, if regulators allow banks to expand exposure to safe assets by providing them with sufficient liquidity during crises. Thus, the combination of interest rate caps and debt-type managerial compensation enables regulators to achieve their goal without sacrificing social welfare or deposit insurance. I also find regulators can incentivize bank managers and owners to comply with the combination of interest rate caps and debt-type managerial compensation.

Although this policy set depends on mean return to risky loans, regulators can ask banks to report business conditions if banks have incentives to comply with both policies and hence truthfully reveal information. In addition, adjustment costs are limited because regulators need to change a policy only when mean return to risky loans changes from negative value to positive one or vice versa.

While the proposed framework is rich enough to compare the efficacy of several policies, it does not address all the issues addressed by recent studies on financial regulation. In particular, there is room for future researchers to develop the proposed model at least in the following domains: asset liquidity; depositors’ preference for liquidity; depositors’ imperfect information for economic fundamentals; and entrepreneurs’ actions. By taking into account these issues, they could find policies that eliminate the potential for catastrophic equilibria other than the combination of interest rate caps and debt-type managerial compensation.

The rest of this paper is structured as follows. Section 2 analyzes the benchmark model. Section 3 analyzes the impact of transparent banking. Section 4 analyzes interest rate caps, and Section 5 characterizes the optimal tax on the CDS spread of a bank, which induces debt-type managerial compensation. Section 6 provides quantitative implications from the calibrated model. Section 7 concludes while suggesting the potential extensions of the proposed model for future researchers.

2 Model

In this section, I describe the benchmark model, in which banks cannot be committed to their risk strategies.
2.1 Players

There are \( n \) identical banks (\( n \geq 2 \)) that compete with each other to get funded from the same pool of depositors. There are \( M^I \) insured and \( 1 - M^I \) uninsured depositors (\( 0 \leq M^I < 1 \)). There is no outside option that the depositors can avail in the absence of banks. Each bank is run by a risk-neutral manager, who is hired by shareholders under a linear incentive contract\(^6\) that depends on the bank’s stock price. In other words, the manager’s incentives are perfectly aligned with the interests of the shareholders\(^7\). The manager makes portfolio choice, while the shareholders make default decision. I denote the strategy of bank \( k \) by \( s_k \).

All depositors consider deposit rates when they choose their banks. Some depositors are not insured by a deposit insurance authority; thus, their preference is also sensitive to the default risk of each bank. On the other hand, the insured depositors do not consider banks’ default risk because any shortfall is insured. The deposit insurance authority orders each bank to pay a certain premium for funding insurance. I denote the uninsured depositors’ belief about the strategy profile of all banks by \( s^N \). I also denote the strategies of all banks except bank \( k \) by \( s_{-k} \).

2.2 Timing

The timing of the initial period is described below.

1. For each bank, the incumbent equity holders hire a risk-neutral manager under an incentive contract that offers a fixed salary and an equity-linked bonus.

2. The deposit insurance authority sets premium.

3. Each bank simultaneously chooses its strategy. The bank also proposes deposit rates and is committed to pay them to the depositors.

4. The depositors choose their banks based on the deposit rates that each bank is committed to pay and their belief about the strategy of each bank.

5. The equity of each bank is evaluated by an efficient stock market. Each bank pays a bonus to the manager in accordance with the incentive contract.

\(^6\)For example, a manager receives a fixed salary at the beginning of the year and an equity-linked bonus at the end of the year. Linear contracts are common in practice (Bose et al., 2011).

\(^7\)Later, I will relax this assumption and consider the situation where managerial incentives are not perfectly aligned with the interests of shareholders.

\(^8\)In the benchmark model, the distinction between a manager and shareholders is not important.
6. After the return on the bank assets realizes, the shareholders make a default decision. In the case of a default, the shareholders liquidate the bank assets to pay the premium and the interests. The depositors lose $\xi$ of the principal ($0 < \xi \leq 1$) after receiving the interests. The deposit insurance authority compensates any short fall of the principal for the insured after receiving the premium. The shareholders receive nothing. Otherwise, the shareholders continue business by injecting capital to pay the premium and the interests.

When the shareholders continue business, the following period proceeds as below.

1. The manager is hired by the shareholders under the same incentive contract that was used in the previous period. The manager and the shareholders plan the same strategies that they planned in the previous period. Also, the depositors choose the same bank that they selected in the previous period.

2. Same as step 5 of the initial period.

3. Same as step 6 of the initial period.

2.3 Banks’ strategies

Each bank $k$ determines (1) portfolio choice; (2) default decision; (3) insured deposit rate; and (4) uninsured deposit rate. Because (3) and (4) can be uniquely determined by (1) and (2), I can eventually reduce the strategy space of the bank to that of (1) and (2).

For (1), I allow the manager to invest in either risky loans or riskless bonds. Let $q_k$ be the exposure to risky loans ($q_k > 0$). Then, the manager invests $1 - q_k$ of the bank assets into riskless bonds. Therefore, I can represent the return on the bank assets, $\tilde{R}_k$, where $\tilde{R}_k = (1 - q_k)\mu_0 + q_k\tilde{R}$, $\mu_0$ is the risk-free rate, and $\tilde{R}$ is the return on risky loans. I assume that the return on risky loans follows a normal distribution, $\tilde{R} \sim N[\mu, \sigma]$, where $\sigma > 0$. Without loss of generality, I can set $\mu_0 = 0$ by measuring each rate relative to the risk-free rate. Throughout the paper, I denote $\Phi(\cdot), \phi(\cdot), \lambda(\cdot)$ as the CDF, PDF, and inverse Mills Ratio of standard normal distribution, respectively.

Regarding (2), Hortaçsu et al. (2011) showed that shareholders plan the reservation rate to make a default decision. Let the reservation rate be $R_k$, such that the bank continues to operate if $\tilde{R}_k > R_k$, otherwise, it liquidates its assets. Then, the probability of default is $\Phi(\frac{R_k - q_k\mu}{q_k\sigma})$. Let the normalized reservation rate be $z_k$, where $z_k = \frac{R_k - q_k\mu}{q_k\sigma}$. I represent the default decision of the bank
by $z_k$ because it sufficiently represents the probability of default. Consequently, the bank optimizes both the overall risk $q_k$ (portfolio choice) and tail risk $z_k$ (default decision). Thus, I denote the strategy of the bank by $s_k$ where $s_k = (q_k, z_k)$.

### 2.4 Deposit demand

I model the demand for deposits in a discrete choice framework. As the insured depositors are protected by deposit insurance, they only care about deposit rates when choosing a bank for opening accounts. On the other hand, the uninsured depositors are not protected by the deposit insurance; therefore, they also consider the default risk of banks besides the deposit rate. Household $j$ derives indirect utility from holding insured and uninsured deposits at bank $k$, where

$$
\tilde{u}_{jk}^I(i_k) = \alpha i_k + \epsilon_{j,k}
$$

$$
\tilde{u}_{jk}^N(i_k, z_k) = \alpha(i_k - \xi \Phi(z_k)) + \epsilon_{j,k}.
$$

Here $i_k$ represents a deposit rate. $z_k$ represents the uninsured depositors’ belief about the tail risk of bank $k$. The parameter $\alpha$ measures depositors’ effective deposit rate sensitivity, which is the total expected return on a depositor’s claim, taking into account the default risk of a bank. For example, if the deposit rate is 10%, the probability of a default is 5%, and the recovery rate is 50%, the uninsured depositor expects to gain 10 dollars and lose 5 dollars. Therefore, the total expected return on the uninsured depositor’s claim is 5 dollars. Then, the effective deposit rate is 5%. I assume that $\alpha > 0$ and $\xi > 0$.

Further, $\epsilon_{j,k}$ is the consumer’s idiosyncratic utility shock which follows a i.i.d. Type 1 Extreme Value distribution. Assuming that there are infinitely many depositors, bank $k$ acquires market shares in insured and uninsured deposit markets as follows:

$$
m^I(i_k, s_{-k}) = \frac{\exp(\alpha i_k)}{\sum_{k'=1}^{n} \exp(\alpha i_{k'})}
$$

$$
m^N(i_k, s_{-k}, s^N) = \frac{\exp(\alpha(i_k - \xi \Phi(z_k^N)))}{\sum_{k'=1}^{n} \exp(\alpha(i_{k'} - \xi \Phi(z_k^N)))}.
$$

\[^{9}\text{Egan et al. (2014) empirically rejected that the demand for insured deposits is sensitive to the banks’ default risks, while the uninsured depositors care about them.}\]
Lastly, I assume that the deposit insurance assigns a premium $c$ for each principal of insured deposit. I assume that $c \geq 0$.

2.5 Optimal deposit rates

I characterize the optimal deposit rates chosen by the bank: $i^I(s_k, s_{-k}, s^N)$ and $i^N(s_k, s_{-k}, s^N)$ as a function of $m^I(s_k, s_{-k}, s^N) = m^I(i^I(s_k, s_{-k}, s^N), s_{-k})$ and $m^N(s_k, s_{-k}, s^N) = m^N(i^N(s_k, s_{-k}, s^N), s_{-k}, s^N)$.

At the optimum, the bank makes the expected markup equal to the inverse deposit rate elasticity of the residual demand as follows:

$$q_k[\mu + \sigma\lambda(z_k)] - c - i^I(s_k, s_{-k}, s^N) = \frac{1}{\alpha[1 - m^I(s_k, s_{-k}, s^N)]}$$  \hspace{1cm} (1)

$$q_k[\mu + \sigma\lambda(z_k)] - i^N(s_k, s_{-k}, s^N) = \frac{1}{\alpha[1 - m^N(s_k, s_{-k}, s^N)]},$$  \hspace{1cm} (2)

Lemma 1. For any $(s_k, s_{-k}, s^N)$, $i^I(s_k, s_{-k}, s^N)$ and $i^N(s_k, s_{-k}, s^N)$ satisfying (1) and (2) uniquely exist.

Proof. The LHS of (1) and (2) are strictly decreasing in deposit rate; whereas the RHS of (1) and (2) are strictly increasing in it. Moreover, the RHS of (1) and (2) approach positive infinity as the deposit rate approaches positive infinity, whereas they converge to $\frac{1}{\alpha}$ as the deposit rate approaches negative infinity. Furthermore, the LHS of (1) and (2) approach negative infinity as the deposit rate approaches positive infinity; whereas they approach positive infinity as the deposit rate approaches negative infinity. These facts suggest that, for a given $s_k$, optimal deposit rates uniquely exist.

2.6 Valuation of a bank

The expected return and profit of the bank is characterized as the weighted average of the markups extracted from the insured and uninsured depositors as follows:

$$\pi(s_k, s_{-k}, s^N) = \frac{\theta^I(s_k, s_{-k}, s^N)}{\alpha[1 - m^I(s_k, s_{-k}, s^N)]} + \frac{1 - \theta^I(s_k, s_{-k}, s^N)}{\alpha[1 - m^N(s_k, s_{-k}, s^N)]}.$$  

Here $\theta^I(s_k, s_{-k}, s^N)$ is the weight of insured deposits in the bank liability. In accordance with this, I also characterize the expected profit of the bank as follows:
\[ \Pi(s_k, s_{-k}, s^N) = \frac{M^I m^I(s_k, s_{-k}, s^N)}{\alpha[1 - m^I(s_k, s_{-k}, s^N)]} + \frac{(1 - M^I) m^N(s_k, s_{-k}, s^N)}{\alpha[1 - m^N(s_k, s_{-k}, s^N)]}. \]

Let \( r \) be the discount rate \((0 < r \leq 0.15)\). By definition, the value of equity \( V(s_k, s_{-k}, s^N) \) is the discounted value of the shareholders’ claims in the future. Therefore, it has to satisfy the following Bellman equation:

\[ V(s_k, s_{-k}, s^N) = \frac{1}{1 + r} (1 - \Phi(z_k)) \left[ \Pi(s_k, s_{-k}, s^N) + V(s_k, s_{-k}, s^N) \right]. \]

Then I can characterize the equity value as the expected profit of the bank multiplied by the survival probability discounted by the risk-adjusted rate, i.e. the sum of the normal discount rate and the default risk of the bank:

\[ V(s_k, s_{-k}, s^N) = \frac{(1 - \Phi(z_k)) \Pi(s_k, s_{-k}, s^N)}{r + \Phi(z_k)}. \]

### 2.7 Equilibrium

I focus on the symmetric equilibrium of \( \Gamma \), which represents the game described above. For obtaining the analytical closed form expressions, I impose symmetry while I only require local optimality for equilibrium strategy. This is a weaker equilibrium concept, but it is still robust to the local perturbation of each player’s strategy. I require that each belief be consistent. Finally, I do not allow banks to choose mixed strategies.

Let the strategy space of the bank be \( s = Q \times \mathbb{R} \), where \( Q \) is the closed interval \([q, \bar{q}]\), where \( \bar{q} > q > 0 \). I define the equilibrium of \( \Gamma \) as follows:\(^{11}\)

**Definition 1.** An assessment \( \{s_k\}_{k=1}^n, s^N \in \Sigma^n \times \Sigma^n \) is the equilibrium of \( \Gamma \) if \( s_k = s, \forall k = 1, ..., n \) and \( s^N = \{s_k\}_{k=1}^n \), and either

(i) there exist open rectangles \( W_k \subset \Sigma, \forall k = 1, ..., n \), such that

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\(^{10}\)I assume that \( r \) is reasonably low to have meaningful implications.

\(^{11}\)The definition above only requires local optimality, which is necessary to obtain simple analytical results. However, it is still robust to local perturbations, so it is a realistic equilibrium concept if a bank’s strategy space is narrow due to large adjustment costs required for dramatically altering its strategy from the previous one, for example. Moreover, this paper focuses on the elimination of a bad equilibrium, so any policy that does this job based on the above definition prevent a bad equilibrium based on equilibrium concepts finer than this. Therefore, it is convenient for me to define equilibrium based on local optimality to get conservative policy implications.
\( s_k \in W_k, V(s_k, s_{-k}, s^N) > V(s'_k, s_{-k}, s^N), \forall s'_k \in W_k \setminus s_k, \) and \( q_k \in (\underline{q}, \overline{q}), \) or

(ii) there exist Cartesian products of right (left) half-open interval and open interval, \( W_k \subset \Sigma, \forall k = 1, \ldots, n, \) such that
\( s_k \in W_k, V(s_k, s_{-k}, s^N) > V(s'_k, s_{-k}, s^N), \forall s'_k \in W_k \setminus s_k, \) and \( q_k = \overline{q}(\underline{q}). \)

2.8 Default decision

First, I analyze the optimal default decision of a bank. In general, the marginal value of taking tail risks is
\[
\frac{\partial V(s_k, s_{-k}, s^N)}{\partial z_k} = -\frac{(M^I m^I (s_k, s_{-k}, s^N) + (1 - M^I) m^N (s_k, s_{-k}, s^N))}{r + \Phi(z_k)} \times \left\{ \phi(z_k) \left( \frac{1 + r}{r + \Phi(z_k)} \pi(s_k, s_{-k}, s^N) - q_k \sigma(\lambda(z_k) - z_k) \right) \right\}. \tag{3}
\]

At equilibrium, this is simplified to the following:
\[
\frac{1 + r}{r + \Phi(z)} \frac{n}{\alpha(n - 1)} - q \nu [\lambda(z) - z] = 0. \tag{4}
\]

Here (4) implies that the going concern value of the bank has to be equal to the shortfall needed for the bank to continue business when the reservation rate is realized. Therefore, at the threshold, the shareholders have to be indifferent between stopping and continuing bank business. Rewriting (4), I attain the first order condition as follows:
\[
\frac{n}{\alpha(n - 1)q \nu} = \frac{r + \Phi(z)}{1 + r} [\lambda(z) - z]. \tag{5}
\]

I denote the LHS of (5) by \( g(q, n), \) which decreases in the exposure to risky loans and the number of banks in the deposit market because market competition reduces the markup earned by the bank, whereas increasing the exposure to risky loans raises the standard deviation of the portfolio return. I also denote the RHS of (5) by \( h(z). \) As \( h(z) \) is not affected by any policy parameter, \( g(q, n) \) determines the tail risk at equilibrium. As \( g(q, n) \) is not affected by \( \mu, \) the default risk of each bank is insensitive to \( \mu \) as long as \( q \) is fixed. Also, I find that the locally stable reservation rate requires the second order condition as follows:

\[14\] Hortaçsu et al. [2011] showed that the optimal reservation rate is the root of the last term in (3). This is exactly the first-order condition for the optimal tail risk.
When the discount rate is modestly low\textsuperscript{14}, \( h(z) \) decreases in \( z \) for \( z < z^1 \), attains a local minimum at \( z = z^1 \), strictly increases in \( z \) for \( z^1 < z < z^2 \), attains a local maximum at \( z = z^2 \), and strictly decreases in \( z \) for \( z > z^2 \). Moreover, I define \( z = \inf\{z|g(q,n) = h(z)\} \) and \( z = \sup\{z|g(q,n) = h(z)\} \). I categorize the equilibrium based on the performance of financial stability as shown in Figure 1.

**Definition 2.** An equilibrium strategy \( s \) is high-risk if \( z \geq z^2 \), low-risk if \( z \leq z^1 \), and middle-risk if \( z^1 < z < z^2 \).

I notice that \( h'(z) \leq 0 \) if \( z \leq z^1 \) and \( z \geq z^2 \) while \( h'(z) > 0 \) if \( z^1 < z < z^2 \). Then, I claim the following.

**Lemma 2.** If \( z \leq z^1 \) or \( z \geq z^2 \), then \( z \) satisfies (6).

For reference, I document the critical values for moderately low \( r \) in the table given below. The default risk of each bank is at least more than half for a high-risk equilibrium, whereas it is, at most, 0.04 for a low-risk equilibrium.

<table>
<thead>
<tr>
<th>( r )</th>
<th>0.01</th>
<th>0.05</th>
<th>0.1</th>
<th>0.15</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Phi(z^1) )</td>
<td>0.00097</td>
<td>0.0082</td>
<td>0.023</td>
<td>0.036</td>
</tr>
<tr>
<td>( \Phi(z^2) )</td>
<td>0.76</td>
<td>0.76</td>
<td>0.73</td>
<td>0.69</td>
</tr>
</tbody>
</table>

### 2.9 Portfolio choice

Second, I analyze the optimal portfolio choice of a bank. In general, the marginal value of the exposure to risky loans is

\[
\frac{\partial V(s_k, s_{-k}, s_N)}{\partial q_k} = \frac{1 - \Phi(z_k)}{r + \Phi(z_k)} \left( M^I m^I(s_k, s_{-k}, s_N) + (1 - M^I)m^N(s_k, s_{-k}, s_N) \right) (\mu + \sigma \lambda(z_k)).
\]

(7)

At equilibrium, this is simplified to

\textsuperscript{14}I verify that this holds for \( 0 < r \leq 0.15 \).
Note that the marginal value of the overall risk is positive if \( \mu > -\sigma \lambda(z) \), neutral if \( \mu = -\sigma \lambda(z) \), and negative if \( \mu < -\sigma \lambda(z) \). Therefore, if the expected return on risky loans is greater than the risk-free rate, the marginal value is always positive. Moreover, even if the expected return is smaller than the risk-free rate, the marginal value of the exposure to risky loans can be positive because the bank manager benefits from the upside of the unprofitable gamble without incurring its downside because of limited liability.

**Lemma 3.** Suppose \( s \) is an equilibrium strategy. \( q = \bar{q} \) if \( \mu > -\sigma \lambda(z) \) and \( q = \underline{q} \) if \( \mu < -\sigma \lambda(z) \).

I categorize the equilibrium based on the performance of credit control. I assume that a society is risk-neutral because its portfolio is well-diversified. It then wants a bank to undergo the largest exposure to risky loans, as long as the expected return exceeds the risk-free rate, but the least exposure when the expected return is below the risk-free rate. Therefore, I evaluate the equilibrium based on banks’ credit control as follows.

**Definition 3.** An equilibrium strategy \( s \) is underinvesting if \( q = \underline{q} \) and \( \mu > 0 \) and risk-shifting if \( q = \bar{q} \) and \( \mu < 0 \). The strategy can be termed as optimally credit-controlling if it is neither underinvesting nor risk-shifting.

### 2.10 Results

Now, I summarize main findings from the benchmark model.

**Proposition 1.** There exists at least one equilibrium of \( \Gamma \). If and only if \( s \) satisfies (5), (6), and either \( q = \bar{q} \land \mu > -\sigma \lambda(z) \) or \( q = \underline{q} \land \mu < -\sigma \lambda(z) \), \( s \) is an equilibrium strategy of \( \Gamma \).

**Proof.** When \( \mu = -\sigma \lambda(z) \), \( s \) that satisfies (5) and (6) is a saddle point. Therefore, it cannot be an equilibrium strategy of \( \Gamma \).

If \( \mu \geq -\sigma \lambda(z) \), then \( s = (\bar{q}, \bar{z}) \) is an equilibrium because \( \bar{z} \) is on the low or high domain, which is locally stable according to Lemma 2, and \( \mu + \sigma \lambda(\bar{z}) \geq 0 \).

If \( \mu < -\sigma \lambda(z) \), then \( (\underline{q}, \bar{z}) \) is an equilibrium because \( \bar{z} \) is on the low or high domain, which is locally stable according to Lemma 2.

The second statement is obvious from Proposition 1 and Lemma 3.
Proposition 2. Suppose that $q$ is an equilibrium exposure to risky loans.

If $g(q, n) > h(z^2)$, then there exists a unique low-risk equilibrium associated with $q$.

If $h(z^1) \leq g(q, n) \leq h(z^2)$, then there exist at least two equilibria associated with $q$, one of which is high-risk and the other low-risk, and there may exist a middle-risk equilibrium besides them.

If $g(q, n) < h(z^1)$, then there exists a unique high-risk equilibrium associated with $q$.

Proof. The proof is straightforward from Definition 2 and Lemma 2.

Proposition 3. An equilibrium is optimally credit-controlling if $\mu \geq 0$ or $\mu \leq -\sigma \lambda(\bar{z})$.

It is risk-shifting if $-\sigma \lambda(z) \leq \mu < 0$.

It can be both risk-shifting and optimally credit-controlling if $-\sigma \lambda(\bar{z}) < \mu < -\sigma \lambda(z)$.

There is no underinvesting equilibrium.

Proof. If $\mu \geq 0$, then $\mu + \sigma \lambda(z) > 0$ for any $z$. Therefore, $q = \bar{q}$ by Lemma 3. From Definition 3, it is not risk-shifting.

If $\mu \leq -\sigma \lambda(\bar{z})$, then $\mu + \sigma \lambda(z) < 0$ for all $z$ that satisfies $g(q, n) = h(z)$ for some $q \in [\underline{q}, \bar{q}]$ except $\bar{z}$. Therefore, $q = \underline{q}$ by Lemma 3. From Definition 3, it is also not risk-shifting.

If $-\sigma \lambda(\bar{z}) \leq \mu < 0$, then $\mu + \sigma \lambda(z) > 0$ for all $z$ that satisfies $g(q, n) = h(z)$ for some $q \in [\underline{q}, \bar{q}]$ except $\bar{z}$. Therefore, $q = \bar{q}$ by Lemma 3. From Definition 3, it is risk-shifting.

For the remaining case, $q = \bar{q}$ if $z = \bar{z}$ and $q = \underline{q}$ if $z = \bar{z}$. As neither $\bar{z}$ nor $\bar{z}$ is on the middle domain, they are both locally stable. Therefore, the banking sector attains multiple equilibria, at least one of which is optimally credit-controlling and another is risk-shifting.

These results suggest that there are often multiple equilibria one of which involves a high default risk or risk-shifting. Figure 2 and Table 1 illustrate the cases in which multiple equilibria can arise. On the one hand, the depositors may expect a low default risk and allow banks to offer low deposit rates, which increases the going concern value of banks. This lowers reservation rates. Moreover, this discourages risk-shifting because the gain from risk-shifting is proportional to the payoff of equity holders that is conditional on bank survival, which is amplified by the reservation rates of banks. On the other hand, the depositors may expect a high default risk and may require banks to offer high deposit rates, which decreases the continuation value of banks. Then, the opposite
feedback occurs. Consequently, the banking sector can take high default risk and excessive risk without regulation.

2.11 Implication for direct asset restrictions and antitrust policy

Lastly, I derive first-order policy implications from the benchmark model. I seek policies that eliminate the potential for a high default risk and achieve optimal credit control for almost entire range of $\mu$ except ones slightly below zero, for which the negative impact of risk-shifting is limited. In particular, I focus on policy parameters that affect $g(\bar{q}, n)$ and $\bar{z}$.

Proposition 4. There is a threshold such that the banking sector surely attains a unique low-risk equilibrium regardless of $\mu$ for any $\bar{q}$ strictly below the threshold.

The maximum threshold is characterized by $\eta(n)$ where $g(\eta(n), n) = h(z^2)$. Then, $\eta(n)$ is strictly decreasing in $n$.

Moreover, if $\bar{q} < \eta(n)$, the banking sector achieves optimal control if $\mu \geq 0$ or $\mu \leq -\sigma \lambda(z^1)$.

Proof. From Proposition 2, the banking sector surely attains a low-risk equilibrium if $g(\bar{q}, n) > h(z^2)$. Since $\frac{\partial g(\bar{q}, n)}{\partial \bar{q}} < 0$, $g(\bar{q}, n) > h(z^2)$ for any $\bar{q} < \eta(n)$. Notice $\eta'(n) = -\frac{\frac{\partial g(\bar{q}, n)}{\partial \bar{q}}}{\frac{\partial g(\bar{q}, n)}{\partial n}} < 0$.

For the last statement, $\bar{z} < z^1$ if $\bar{q} < \eta(n)$ from Proposition 2. Then, apply Proposition 3.

These findings suggest that direct asset restrictions or market concentration can eliminate the potential for a high-risk equilibrium by raising the going concern value of a bank. Moreover, these policies can improve credit control in the banking sector by expanding the domain of the mean returns on risky loans in which risk-shifting never occurs. Note that $\sigma \lambda(z^1)$ is negligibly small for $\sigma$ used in the calibrated model.

Although direct asset restrictions or market concentration can improve the soundness of the banking sector, these policies can harm the welfare of the society. For a risk-neutral society, it is optimal to fully invest in risky loans as long as these projects are profitable. Also, depositors are better off by market competition as they have more options when they choose their banks. Moreover, the systemic importance of an individual bank increases by market concentration. In this case, the default of one bank can impose greater costs to a society. Therefore, I propose the simple welfare criterion for judging the efficiency of financial regulation as follows.

Definition 4. The proposed policy harms social welfare if it either decreases $n$, decreases $q$ when $\mu > 0$, or increases $q$ when $\mu < 0$, where $q$ is equilibrium exposure to risky loans.
3 Transparent banking

The key friction in the previous model was lack of commitment by banks. Disclosure regulation imposes ex-post punitive cost on distorted disclosures. It then works as a commitment device. I revisit the above game in which banks are committed to their risk strategies. I find that this condition mitigates restrictions on bank assets and market concentrations that regulators require to prevent high default risk or risk-shifting because banks internalize the negative impact of the tail risk on the amount they can collect from uninsured depositors if they know that depositors know their risks when choosing banks. Although transparent banking unambiguously improves the soundness of the banking industry, its favorable effect decreases for the fraction of insured depositors because insured depositors only consider deposit rates and are insensitive to the default risk of banks. Although the counteracting force of deposit insurance can be muted by actuarially fair deposit insurance, the deposit insurance authority might be unable to implement actuarially fair deposit insurance premiums because it has an ex-post incentive not to enforce failed banks to pay high premiums associated with high default risks.

3.1 Modified game structure

Step 4 of the initial period in the previous game is altered so that depositors choose banks based on deposit rates and the strategy each bank is committed to pay and undertake. I denote the modified game by $\Gamma_1$.

Definition 5. An assessment $\{s_k\}_{k=1}^n \in \Sigma^n$ is an equilibrium of $\Gamma_1$ if $s_k = s, \forall k = 1, \ldots, n$, and either

(i) there exist open rectangles $W_k \subset \Sigma, \forall k = 1, \ldots, n$, such that

$s_k \in W_k, V(s_k, s_{-k}, \{s_k, s_{-k}\}) > V(s'_k, s_{-k}, \{s'_k, s_{-k}\}), \forall s'_k \in W_k \setminus s_k$, and $q_k \in (\underline{q}, \overline{q})$, or

(ii) there exist Cartesian products of right (left) half-open interval and open interval, $W_k \subset \Sigma, \forall k = 1, \ldots, n$, such that

$s_k \in W_k, V(s_k, s_{-k}, \{s_k, s_{-k}\}) > V(s'_k, s_{-k}, \{s'_k, s_{-k}\}), \forall s'_k \in W_k \setminus s_k$, and $q_k = \underline{q}(\overline{q})$.

3.2 Default decision

Under the modified game, beliefs by depositors do not influence equity value, so I denote $V(s_k, s_{-k}, \{s_k, s_{-k}\})$ by $V(s_k, s_{-k})$. At equilibrium, the first order condition is modified to:
\[
\frac{n}{\alpha(n-1)q^\nu} + \frac{(r + \Phi(z))(1 - \Phi(z))(1 - M^I)\xi}{(1 + r)q^\nu} = r + \Phi(z) \frac{\xi(z) - z}{1 + r}. \tag{9}
\]

In comparison to the original equation, the LHS unambiguously shifts up. I define the new term of the LHS as the cost of losing uninsured deposits \(c^N(q,z,M^I)\). \(c^N(q,z,M^I)\) strictly increases in \(z\) for \(z \leq \frac{1 - r}{2}\) and strictly decreases in \(z\) for \(z \geq \frac{1 - r}{2}\). \(c^N(q,z,M^I)\) is strictly decreasing in \(M^I\).

Furthermore, \(\lim_{z \to \infty} c^N(q,z,M^I) = 0\) and \(\lim_{z \to -\infty} c^N(q,z,M^I) = \frac{(1 - M^I)\xi}{q^\nu}\). Therefore, there exists at least one root satisfying (9) for any \(q\).

Unlike the previous game, (9) implies that the going concern value of a bank must be less than the shortfall needed for the bank to continue business when the reservation rate is realized. This implies that equity holders have \textit{ex-ante} incentive to lower the reservation rate even if it is unprofitable \textit{ex-post} because they can collect more money from the uninsured by promising that they will inject capital beyond the continuation value.

In addition, conditions required for local stability are modified to:

\[
\frac{\lambda'(z)}{n} - h'(z) > \phi(z) \frac{\xi(1 - M^I)}{(1 + r)q^\nu} \left[\alpha \xi M^I(r + \Phi(z))(1 - \Phi(z)) \frac{(n-1)^2}{n^2} - (1 - r - 2\Phi(z))\right]. \tag{10}
\]

The stability condition becomes more restrictive than before. Even if \(z\) is not in the middle domain, it may not satisfy (10). If \(M^I\) is sufficiently large, \(z\) on the low and high domains satisfies (10). If \(M^I\) is sufficiently small, \(z\) on the low domain satisfies (10) because the RHS of (10) becomes negative with sufficiently small \(M^I\) since \(1 - r - 2\Phi(z) > 0, \forall z \leq z^1\) for modestly low \(r\).

I note that a bank is better off by commitment since it is possible for a bank to implement the strategy undertaken under the game without commitment in which depositors rationally expect the strategy of each bank in this game. Indeed, the following inequality holds:

\[
\max_{s_k \in W_k} V(s_k, s_{-k}, s^N) \leq \max_{s_k \in W_k} V(s_k, s_{-k}), \forall s_{-k} \in \Sigma^{n-1}, \forall W_k \subset \Sigma, \text{ where } s^N = \{s_k\}_{k=1}^{n}. \tag{11}
\]

Thus, a bank has an incentive to comply with disclosing their businesses, and in this sense, this policy is easier to implement.

### 3.3 Results

The choice of overall risk is determined the same as before. I characterize the equilibrium of the modified game as:
Proposition 5. If and only if $s$ satisfies (9), (10), and either $q = \bar{q} \land \mu > -\sigma \lambda(z)$ or $q = \underline{q} \land \mu < -\sigma \lambda(z)$, $s$ is an equilibrium strategy of $\Gamma_1$.

Proof. Similar to the proof of Proposition 1.

Proposition 6. Suppose there is a threshold such that the banking industry surely attains a unique, low-risk equilibrium regardless of $\mu$ for any $\bar{q}$ below the threshold.

Let the maximum threshold be characterized by $\eta_1(n, M^I)$. Then, $\eta_1(n, M^I) > \eta(n)$ where $\lim_{M^I \to 1} \eta_1(n, M^I) = \eta(n)$ for all $n$.

If $\bar{q} < \eta_1(n, M^I)$, the banking industry achieves optimal control if $\mu \geq 0$ or $\mu \leq -\sigma \lambda(z^1)$.

Proof. The banking sector surely attains a low-risk equilibrium if $\tilde{g}(\bar{q}, n) + c^N(\bar{q}, z, M^I) > h(z), \forall z$, where $z > z^1$.

Define $\hat{n}_1(n, M^I)$ such that $\tilde{g}(\hat{n}_1(n, M^I), n) + c^N(\hat{n}_1(n, M^I), z^*, M^I) = h(z)$ for some $z^*$ where $z^* > z^1$ and $\tilde{g}(\hat{n}_1(n, M^I), n) + c^N(\hat{n}_1(n, M^I), z, M^I) > h(z), \forall z$ where $z > z^1 \land z \neq z^*$.

If $\hat{n}_1(n, M^I) \leq \eta(n)$, then $\tilde{g}(\hat{n}_1(n, M^I), n) > g(\eta(n), n) = h(z^2) \geq h(z), \forall z$ where $z > z^1$. Then, $\tilde{g}(\hat{n}_1(n, M^I), n) + c^N(\hat{n}_1(n, M^I), z, M^I) > h(z), \forall z$ where $z > z^1$.

This contradicts to $\tilde{g}(\hat{n}_1(n, M^I), n) + c^N(\hat{n}_1(n, M^I), z^*, M^I) = h(z^*)$ for some $z^*$ where $z^* > z^1$. Therefore, $\hat{n}_1(n, M^I) > \eta(n)$.

If $z^*$ satisfies (11), $\eta_1(n, M^I) = \hat{n}_1(n, M^I)$. If not, $\eta_1(n, M^I) > \hat{n}_1(n, M^I)$ because $\tilde{g}(\bar{q}, n) + c^N(\bar{q}, z, M^I)$ is strictly decreasing in $\bar{q}$. Therefore, $\eta_1(n, M^I) > \eta(n)$.

However, as $M^I$ approaches to 1, (9) and (10) become equivalent to (5) and (6). Therefore, $\eta_1(n, M^I)$ approaches $\eta(n)$.

The last statement is obvious from Proposition 4.

These results suggest a chance of deregulating direct asset restrictions or market concentrations to improve social welfare through transparent banking. However, the favorable effect of transparent banking decreases for the fraction of the insured depositors because they are not incentivized to monitor the default risk of each bank. Figure 3 illustrates how transparent banking can improve the soundness of the banking industry, and how its favorable effect is offset by greater coverage of deposit insurance.

Commitment to risk strategies improves the soundness of the banking industry because it reminds banks of monitoring and the threatening roles of depositors, which are essential for banking.
prudence, according to Diamond and Rajan (2001). Thus, if governments require banks to be committed to risk strategies, they should keep the coverage of deposit insurance to the minimum required to prevent bank runs.

3.4 Actuarially fair deposit insurance premium

If the deposit insurance authority can assign risk-sensitive deposit insurance premiums, it can monitor and threaten banks on behalf of insured depositors. Suppose it requires banks to pay actuarially fair premium (i.e., $\xi \Phi(z_k)$) per insured deposit instead of $c$. At equilibrium, the first and second order conditions become equivalent to those under $\Gamma_1$ when $M^I = 0$. This is because the marginal penalty of default risk imposed on a bank, which related only to uninsured deposits, is equally related to insured deposits. Therefore, depositor composition does not matter to equilibrium outcomes under $\Gamma_1$ with actuarially fair deposit insurance premiums.

**Proposition 7.** Under $\Gamma_1$ with actuarially fair deposit insurance premium, an equilibrium strategy is equivalent to the one under $\Gamma_1$ when $M^I = 0$.

Suppose there is a threshold such that the banking industry surely attains a unique, low-risk equilibrium regardless of $\mu$ for any $\bar{q}$ below the threshold.

The maximum threshold is characterized by $\eta_1(n,0)$ under $\Gamma_1$ with actuarially fair deposit insurance premiums, where $\eta_1(n,0) \geq \eta_1(n,M^I)$ for all $n$ and $M^I$ where $0 \leq M^I < 1$.

If $\bar{q} < \eta_1(n,0)$, the banking industry achieves optimal control if $\mu \geq 0$ or $\mu \leq -\sigma \lambda(z^1)$.

Despite the favorable effect of actuarially fair deposit insurance premiums on financial stability, this policy has not been undertaken seriously, at least in the United States. In 1991, the FDIC started setting risk-based premiums that differed depending on three levels of bank capitalization and three supervisory rating groups, following the financial crisis of the 1980s. However, from 1996 to 2006, more than ninety percent of all banks were categorized in the lowest risk category (Acharya et al. 2010). Why then is it difficult to implement actuarially fair deposit insurance premium? According to extant studies, governments might be unable to commit to enforcing failed banks to pay actuarially fair deposit insurance premiums because governments might find it optimal to bail out failed banks instead of ordering them to liquidate assets to pay high premiums. For example, the US government provided Wells Fargo with substantial tax credits to purchase Wachovia, though the acquiring firm should have instead owed additional premiums associated with Wachovia. Pennacchi (2009) attributes lack of commitment to political pressure to aid to the
banking industry. Moreover, Acharya and Yorulmazer (2007) addresses the commitment problem for another reason. When the number of failed banks exceeds the number of surviving banks, some failed banks would have to be purchased by investors outside the banking industry, resulting in a loss of asset value due to lack of investment and management expertise. In this case, it becomes optimal for governments to bail out these failed banks or industry insiders that purchase them on behalf of investors outside the banking industry. Thus, there is a problem of implementing actuarially fair deposit insurance premiums because governments might be unable to commit to enforcing failed banks to pay high premiums.

In the following sections, I seek an alternative approach to eliminate the potential for high default risk and nearly prevent risk-shifting, which does not require bank transparency.

4 Interest rate caps

Diamond and Rajan (2012) argue that interest rate policies can improve the soundness of the banking industry. This policy is also appealing because bank managers and owners are willing to comply with it as it does not reduce the value of equity. Following this observation, I consider the case in which government set caps on interest rates such that $i^I \leq \bar{i}^I$ and $i^N \leq \bar{i}^I + c = \bar{i}^N$.

In the presence of such caps, shareholders earn risk-adjusted returns which can exceed the one without caps at equilibrium. This occurs when $z$ is large and hence deposit rates are high. Each bank’s going concern value is higher than before when $z$ is large, which might eliminate the potential for high default risk at equilibrium. Indeed, the first order condition is modified to:

$$\frac{n}{\alpha(n-1)q\nu} + \max\left\{\frac{\mu}{\sigma} + \lambda(z) - \frac{\bar{i}^N}{q\nu} - \frac{n}{\alpha(n-1)q\nu}, 0\right\} = \frac{r + \Phi(z)}{1 + r}(\lambda(z) - z).$$

(12)

The new term of the LHS represents the additional opportunity cost of defaulting due to interest rate caps. I denote the new term $d(q, z, \mu, \bar{i}^N)$, where $d(q, z, \mu, \bar{i}^N)$ strictly increases in $z$ and $\mu$ and strictly decreases in $\bar{i}^N$, if caps are binding. $\lim_{z \rightarrow -\infty} d(q, z, \mu, \bar{i}^N) = 0$ and $\lim_{z \rightarrow \infty} d(q, z, \mu, \bar{i}^N) = \infty$. Therefore, there exists at least one root satisfying (12) for any $q$.

When $\frac{\mu}{\sigma} + \lambda(z) - \frac{\bar{i}^N}{q\nu} < g(q, n)$, interest rate caps do not influence the default decisions of shareholders because $d(q, z, \mu, \bar{i}^N) = 0$. Therefore, when $z$ or $\mu$ is low, interest rate caps are ineffective. When $\bar{i}^N$ is high, they are also ineffective.

I find that conditions required for local stability are modified to:
\[
\frac{\lambda'(z)}{n} - h'(z) > 0 \text{ if } d(q, z, \mu, \tilde{i}^N) = 0,
\]
\[
\lambda'(z) - h'(z) > 0 \text{ if } d(q, z, \mu, \tilde{i}^N) > 0.
\]

Local stability conditions are less restrictive when interest rate caps are binding.

4.1 Results

Optimum exposure to risky loans is determined the same way as before. Therefore, I can characterize the equilibrium of the modified game as:

Proposition 8. There exists at least one equilibrium of \( \Gamma \) with interest rate caps.

If and only if \( s \) satisfies (12), (13), and either \( q = \bar{q} \land \mu > -\sigma \lambda(z) \) or \( q = \bar{q} \land \mu < -\sigma \lambda(z) \), \( s \) is an equilibrium strategy of \( \Gamma \) with interest rate caps.

Proof. Similar to the proof of Proposition 1.

Interest caps work only when \( \mu \) is at least a certain threshold. Figure 4 illustrates how interest caps can improve the soundness of the banking industry, and how its favorable effect is offset by greater \( \mu \). Let the threshold be \( \underline{\mu} \). The goal is to find the caps that induce the banking industry to attain a low-risk equilibrium. I focus on least severe ones because they are least likely to be opposed by depositors.

Proposition 9. Suppose that \( g(\bar{q}, n) > h(z^2), h(z^1) < g(\bar{q}, n) < h(z^2) \), and \( \lambda'(z) - h'(z) > 0 \) for all \( z \). Let the intersection of \( g(\bar{q}, n) \) and \( h(z) \) on the middle domain be \( \bar{z}^M \).

If \( \bar{i}^N < \delta(\bar{q}, n, \underline{\mu}) \) where \( g(\bar{q}, n) + d(\bar{q}, \bar{z}^M, n, \delta(\bar{q}, n, \underline{\mu})) = h(\bar{z}^M) \), the banking industry surely attains a unique, low-risk equilibrium when \( \mu \geq \underline{\mu} \) for given \( \bar{q} \) and \( n \).

If \( \bar{i}^N < \delta(\bar{q}, n, \mu) \) with \( \underline{\mu} = -\sigma \lambda(\bar{z}) \), the banking industry achieves optimal credit control when \( \mu \geq 0 \) or \( \mu \leq -\sigma \lambda(z^1) \) and surely attains a unique, low-risk equilibrium regardless of \( \mu \).

Proof. Suppose \( h(z^1) < g(\bar{q}, n) < h(z^2) \) and \( \lambda'(z) - h'(z) > 0 \) for all \( z \), which is satisfied in a later numerical analysis with realistic parameters. The middle-risk equilibrium is locally stable when caps are binding.

Since \( d(\bar{q}, \bar{z}^M, n, \bar{i}^N) = 0 \) and \( d(\bar{q}, z, n, \bar{i}^N) > 0 \) when \( z = \bar{z}^M + \epsilon \) for any \( \epsilon > 0 \) as \( \lambda'(z) - h'(z) > 0 \) for all \( z \), \( g(\bar{q}, n) + d(\bar{q}, z, n, \bar{i}^N) > h(z) \) for all \( z \) greater than \( \bar{z}^M \).
If \( \tilde{i}^N < \delta(\tilde{q}, n, \mu) \), \( g(\tilde{q}, n) + d(\tilde{q}, \tilde{z}^M, n, \tilde{i}^N) > g(\tilde{q}, n) + d(\tilde{q}, \tilde{z}^M, n, \mu) = h(\tilde{z}^M) \). \( g(\tilde{q}, n) + d(\tilde{q}, z, n, \tilde{i}^N) > h(z) \) for all \( z \) at least as large as \( \tilde{z}^M \). Therefore, it eliminates the potential for high default risk when \( q = \tilde{q} \). \( g(q, n) + d(q, z, n, \tilde{i}^N) > g(q, n) > h(z^2) \) for all \( z \).

For \( z \) to be at equilibrium, \( z \leq \tilde{z} \). Therefore, \( \mu + \sigma\lambda(z) \leq \mu + \sigma\lambda(\tilde{z}) \) for any \( z \) at equilibrium.

Set \( \tilde{i}^N < \delta(\tilde{q}, n, \mu) \) and \( \mu = -\sigma\lambda(\tilde{z}) \). If \( \mu < \mu = -\sigma\lambda(\tilde{z}) \), \( \mu + \sigma\lambda(z) < 0 \) and \( q = \tilde{q} \) so there is no risk-shifting. Any equilibrium must be low-risk. If \( \mu \geq \mu, z \leq z^1 \) at equilibrium, and hence risk-shifting does not occur if \( \mu \leq -\sigma\lambda(z^1) \).

These results suggest that there is a chance of strictly improving the soundness of the financial sector through interest rate caps. Consequently, regulators might be able to deregulate direct asset restrictions or market concentrations fully. When \( \mu \) is sufficiently low, it is possible to eliminate risk-shifting for most of \( \mu \). In this case, it is also possible to eliminate the potential for high default risk because interest rate caps eliminate the potential for high default risk when \( \mu \geq \mu \), while there is no chance of high default risk if \( \mu < \mu \) because \( q = \tilde{q} \) at equilibrium. Thus, interest rate caps are helpful to regulators since they eliminate the potential for high default risk for every \( \mu \), and prevent risk-shifting for \( \mu \) except those slightly below zero.

However, \( \delta(\tilde{q}, n, -\sigma\lambda(\tilde{z})) \) is typically severe negative. Therefore, there remains a practical issue of imposing negative interest rate caps because depositors are unlikely to approve them. For caps to be moderate or at least non-negative, regulators cannot make \( \mu \) too low. If regulators have a good bet that \( \mu \) is unlikely to be negative, they may want to set \( \mu = 0 \) instead of \( \mu = -\sigma\lambda(\tilde{z}) \) to make a cap acceptable to depositors.

5 Debt-type managerial compensation

In the previous section, I show that interest rate caps can improve the soundness of the banking industry without sacrificing social welfare, though it may not work if \( \mu \) is low. In this section, I link managerial compensation to the tail risk of a bank as a policy that eliminates the potential for bad equilibrium when \( \mu \) is low. This makes managerial incentives align with depositors, and induces a conflict of interest between a manager and shareholders. I show that the agency problem between them rather helps a bank asymptotically achieve optimum credit control without sacrificing market competition. The proposed policy is unaffected by the fraction of insured depositors. Prevention of risk-shifting can also eliminate the potential for a high-risk equilibrium when \( \mu < 0 \) if the minimum
exposure to risky loans is low. Although regulators are reluctant to restrict *maximum* exposure to risky loans, they are justified to restrict *minimum* exposure.

5.1 Modified game structure

Even if a manager is able to choose exposure to risky loans independently, it is difficult to mandate that shareholders inject a specified amount of capital. Therefore, I assume that a manager determines only exposure to risky loans, and shareholders determine tail risk. A manager commits to undertake a portfolio choice planned. The shareholders plan the default decision after observing the portfolio choice planned by the manager. I change steps 1 and 5 of \( \Gamma \) as:

- At each bank, \( k \), the incumbent equity holders hire a manager under an incentive contract as they do in \( \Gamma \). However, the government imposes taxes on the compensation of the manager, which link to the credit default swap (CDS) spread of the bank. Therefore, the manager’s compensation is:

\[
W(s_k, s_{-k}, s^N) = W_0 + \delta E V(s_k, s_{-k}, s^N) - \tau(C(z_k)),
\]

where \( W_0 \) is a fixed wage, \( \delta E \) is the shares of equity (\( \delta E > 0 \)), and \( \tau(C(z_k)) \) is the tax associated with the CDS spread of the bank. I denote it by \( f(z_k) = \tau(C(z_k)) \).

- \( V(s_k, s_{-k}, s^N) \) and \( C(z_k) \) are determined by efficient markets. Each bank pays bonuses to the manager following the incentive contract, and the government collects taxes \( \tau(C(z_k)) \) from the manager.

I denote the modified game by \( \Gamma_2 \).

**Definition 6.** An assessment \( \{q_k\}_{k=1}^n, \{z_k\}_{k=1}^n, s^N \in Q^n \times R^n \times \Sigma^n \) is an equilibrium of \( \Gamma_2 \) if \( q_k = q, z_k = z, \forall k = 1, ..., n \) and \( s^N = \{s_k\}_{k=1}^n \), and either

(i) there exist open intervals \( M_k \subset Q, \forall k = 1, ..., n \), such that \( q_k \in M_k, W(q_k, z_k, s_{-k}, s^N) > W(q_k', z_k, s_{-k}, s^N), \forall q_k' \in M_k \setminus q_k, \) and \( q_k \in (q, \bar{q}) \), or

(ii) there exist right (left) half-open intervals, \( M_k \subset Q, \forall k = 1, ..., n \), such that \( q_k \in M_k, W(q_k, z_k, s_{-k}, s^N) > W(q_k', z_k, s_{-k}, s^N), \forall q_k' \in M_k \setminus q_k, \) and \( q_k = q(\bar{q}) \), and there exist open intervals \( S_k \subset R, \forall k = 1, ..., n \), such that \( z_k \in S_k, V(q_k, z_k, s_{-k}, s^N) > V(q_k', z_k, s_{-k}, s^N), \forall z_k' \in S_k \setminus z_k \).
5.2 Portfolio choice

The optimality condition for the tail risk is the same as the one in \( \Gamma \), but the marginal value of the overall risk is modified to:

\[
\frac{\partial V(s_k, s_{-k}, s^N)}{\partial q_k} - \frac{f'(z_k)}{\delta E} \frac{d z_k}{d q_k} = \frac{1 - \Phi(z_k)}{r + \Phi(z_k)} \left( M^k m^k(s_k, s_{-k}) + (1 - M^k)m^N(s_k, s_{-k}) \right) (\mu + \sigma \lambda(z_k)) - \frac{f'(z_k)}{\delta E} \frac{d z_k}{d q_k}. \tag{14}
\]

Unlike the previous games, the manager considers the marginal cost of increasing the default risk determined by the shareholders because of additional exposure to risky loans. Even if a manager is unable to choose tail risk, he/she can affect it by adjusting exposure to risky loans. At equilibrium, the marginal value of the overall risk is:

\[
\frac{1 - \Phi(z)}{r + \Phi(z)} \frac{\mu + \sigma \lambda(z)}{n} - \frac{f'(z)}{\delta E} \frac{n}{\alpha(n-1)} \frac{\left[ \mu + \sigma \lambda(z) \right]}{n} = 1 - \frac{\Phi(z)}{r + \Phi(z)} \frac{n}{\alpha(n-1)} \frac{\left[ \mu + \sigma \lambda(z) \right]}{n}. \tag{15}
\]

5.3 Tax on the tail risk of a bank

If (15) is negative, \( q \) becomes an equilibrium exposure to risky loans. Suppose \( f'(z) \) is the following:

\[
\hat{f}'(z) = \delta E g^{-1}(z, n) n^2 \sigma \left( \frac{\lambda(z)}{n} - h'(z) \right) \left[ 1 - \Phi(z) \right] \frac{n}{r + \Phi(z)} \sigma \lambda(z). \tag{16}
\]

Since I am interested in the policy that is independent of \( \mu \), \( \hat{f}(z) \) is not a function of \( \mu \). \( g^{-1}(z, n) \) satisfies \( g(g^{-1}(z, n), n) = h(z) \), which is defined well and continuous. (16) is rewritten as\(^{14}\)

\[
\frac{1 - \Phi(z)}{r + \Phi(z)} \frac{\mu + \sigma \lambda(z)}{n} - \frac{\hat{f}'(z)}{\delta E} \frac{n}{\alpha(n-1)} \frac{\left[ \mu + \sigma \lambda(z) \right]}{n} = \frac{1}{n} \frac{1 - \Phi(z)}{r + \Phi(z)} \left[ \mu + O \left( \frac{1}{n} \right) \right].
\]

Therefore, as the number of banks approaches infinity, managerial interests align asymptotically with the interests of a society.

It is possible to assign any value to the marginal increase in tax with respect to tail risk for \( z \) that cannot be at equilibrium. Therefore, I set zero for \( z \) that satisfies \( \frac{\lambda(z)}{n} - h'(z) \leq 0 \) and

\(^{14}\)Notice: \( \lim_{n \to \infty} \frac{\bar{z}}{n} < \infty \) and \( \lim_{n \to \infty} \frac{n}{\alpha(n-1)} \frac{g^{-1}(z, n) \mu + \sigma \lambda(z)}{n} = \frac{1}{\alpha} \).
\( \hat{f}'(z) < 0 \). Setting the boundary condition \( f(\hat{z}) = 0 \), the optimum debt-based tax is characterized by:

\[
f(z) = \int_{\hat{z}}^{z} 1[\hat{f}'(x) \geq 0] \hat{f}'(x) dx.
\] (17)

A no arbitrary condition suggests:

\[ C(z_k) = \xi \Phi(z_k). \]

\( z \) can then be expressed as the monotonic transformation of the CDS spread, and \( f(z) \) as a function of the CDS spread. I characterize optimum compensation as:

\[
W(s_k, s_{-k}, s^N) = W_0 + \delta^E V(s_k, s_{-k}, s^N) - f \left( \Phi^{-1} \left( \frac{C(z_k)}{\xi} \right) \right),
\] (18)

where \( f(.) \) satisfies (17).

\( W_0 \) is neutral to a manager’s risk taking, meaning that the level of managerial compensation is neutral to the risk taking by a manager. Instead, the compensation structure affects the investment decisions of the manager. Thus, it is possible to prevent managers from escaping to unregulated jurisdictions by raising \( W_0 \) so they are not worse off by taxation.

5.4 Results

**Proposition 10.** As \( n \) reaches infinity, the banking industry almost surely achieves optimum credit control under \( \Gamma_2 \) with managerial compensation characterized by (18).

This result suggests that regulators can eliminate the potential of risk-shifting without direct asset restrictions or market concentrations. This approach is independent of depositor composition, and therefore it is neutral to the coverage of deposit insurance. This compensation structure can be implemented by taxes linked to the CDS spread of a bank. Prevention of risk-shifting can eliminate the potential for a high-risk equilibrium in economic downturns. If \( q \) is sufficiently small to satisfy \( g(q, n) > h(z^2) \), I can eliminate the potential for a high-risk equilibrium because a manager voluntarily reduces exposure to risky loans to the minimum level if \( \mu < 0 \).

**Proposition 11.** Suppose \( q \) satisfies \( g(q, n) > h(z^2) \). As \( n \) reaches infinity, the banking industry almost surely attains a unique, low-risk equilibrium when \( \mu < 0 \) under \( \Gamma_2 \) with managerial compensation characterized by (18).
Enhanced market competition lowers the continuation value of a bank. Therefore, regulators might need to lower $q$ to maintain the going concern value of a bank. In previous findings, restrictions on maximum exposure to risky loans were not recommended fully because they harm credit enhancement in society. However, restrictions on minimum exposure to risky loans do not harm social welfare because society is willing to lower exposure to risky loans that are unprofitable. Although depositors might be worse off by a lower $q$, the decrease in their surplus associates with a reduction on the gain from risk-shifting that depositors extract from the surplus of shareholders. Consequently, regulators are justified to increase the continuation value of a bank by providing liquidity to banks and lowering $q$ during an economic downturn, though they might reduce the welfare of depositors.

These results suggest that market competition complements avoidance of risk-shifting, whereas it was a substitute for prevention of risk-shifting in previous results, because market competition makes the correlation between asset volatility and default risk less sensitive to $\mu$, which helps regulators design a policy insulated from $\mu$. An increase to exposure to risky loans increases default risk by increasing portfolio return volatility (i.e., direct effect), but it also raises the expected return on bank assets and the continuation value of a bank (i.e., indirect effect), which depends on $\mu$. In competitive markets, the latter effect is small because market competition forces a bank to transfer a large fraction of the gain from expanding exposure to risky loans to depositors. Thus, the correlation between asset volatility and default risk becomes asymptotically independent of $\mu$.

Lastly, I note regulators are more willing to tax on managerial compensation than on equity shares. As they are reluctant to order failed banks to pay high deposit insurance premiums, they might want to bail out failed banks instead of ordering them to pay high taxes\textsuperscript{15}. Although regulators can achieve the similar effect by taxing on equity, they are unlikely to do so.

6 Calibration

I calibrate the model and attain quantitative implications to design policy packages that improve the soundness of the U.S. commercial banking sector.

\textsuperscript{15}See Subsection 3.4 for why.
6.1 Data

I obtained demand parameters from Egan et al. (2014). Since the authors report demand estimates separately for each type of depositor, I use the middle of them for calibration. From demand estimates for uninsured depositors, I can recover the recovery rate by dividing sensitivity to deposit rates by sensitivity to default risk. I find that the recovery rate is fifty percent, which accords with extant studies such as Carrizosa and Ryan (2013).

I use data from the Federal Reserve H8 to obtain exposure to risky loans. I compute cash and Treasury and agency securities as a proportion of total assets, and subtract it from one. Since Egan et al. (2014) focus on large U.S. banks, I use data for large, domestically chartered commercial banks. Since regulators are interested in whether a current policy is sufficiently robust, I estimate maximum exposure to risky loans by using data as of November 12, 2014. The expected return on risky loans is likely above the risk-free rate in 2014. Banks are then likely to invest in risky loans as much as possible. Consequently, the exposure to risky loans I estimate from data as of November 12, 2014 represents maximum exposure to risky loans to date.

The standard deviation of the return on risky loans is the remaining input for the calibration. Egan et al. (2014) report the calibrated standard deviation of the return on deposits as of March 31, 2009. Since I can estimate exposure to risky loans using the Federal Reserve data as of March 31, 2009. I can recover the standard deviation of returns to risky loans consistently. The table given below summarizes parameters used for calibration.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>40</td>
</tr>
<tr>
<td>$\xi$</td>
<td>0.5</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.2</td>
</tr>
<tr>
<td>$r$</td>
<td>0.05</td>
</tr>
<tr>
<td>$\bar{q}$</td>
<td>0.7</td>
</tr>
</tbody>
</table>

6.2 Results

I calculate the boundary of the maximum exposure to risky loans and the number of banks below which the banking industry surely attains a unique, low-risk equilibrium. I compute the threshold mean return on risky loans above which risk-shifting can occur. For each, I solve the threshold under the following conditions: (1) without bank commitment, (2) with bank commitment in
which the fraction of insured depositors is ten percent, fifty percent, and ninety percent of all the depositors, and (3) with bank commitment and actuarially fair deposit insurance premiums. Figure 5 summarizes results for the former estimates, and Table 2 shows latter estimates.

I find that the potential for a high-risk or risk-shifting equilibrium is persistent without disclosure of banks’ risk. Even if there are only two banks in the sector, in which banks take the least risk, there exists equilibrium in which the probability of default is nearly 1, and banks can undergo maximum exposure to risky assets even if risky loans underperform safe assets by more than forty percentage points per year. Therefore, the impact of eliminating the potential for catastrophic equilibrium is substantial.

If regulators attempt to keep the current level of credit enhancement, transparent banking is necessary to prevent an unfavorable equilibrium. However, this can eliminate the potential for such an equilibrium only if there are only two banks, and if regulators insure half of depositors.\textsuperscript{16} Therefore, if regulators try to maintain the current maximum exposure to risky loans, they must make the banking industry extremely concentrated even when they order the transparency of banks’ risk. If regulators instead attempt to enhance market competition, they must cap maximum exposure to risky loans. For example, suppose the maximum exposure to risky loans is reduced to fifty percent. If half of depositors must be covered by deposit insurance, then according to estimates, the banking industry can accommodate up to four banks. With actuarially fair insurance premiums, the industry can accommodate up to seven.

Figure 6 presents the threshold mean return on risky loans above which the banking industry surely attains a unique, low-risk equilibrium. According to estimates, when there are thirty banks, regulators can cap interest rates so banks cannot pay uninsured deposit rates greater than four percent, and insured ones greater than four percent less costs associated with insured deposits, to prevent high default risk and most risk-shifting when mean return on risky loans is non-negative.

Figures 7, 8, and 9 show the optimal tax schedule by number of banks. When there are thirty banks, the optimal tax schedule covers full support of tail risk that can be attained at equilibrium. If the minimum exposure to risky loans is twenty-eight percent, the banking industry can attain a low-risk equilibrium that also achieves optimum credit control when the mean return on risky loans is negative, even if there are thirty banks in the industry. The minimum exposure to risky loans can be reduced by liquidity provision to banks. Consequently, regulators can hedge against risk-shifting and eliminate the potential for a high-risk equilibrium when the mean return on risky

\textsuperscript{16}In the largest commercial banks, approximately half of deposits are uninsured [Egan et al. 2014].
loans is negative, if they accommodate thirty banks, taxes on executive compensation are based on the tail risk of banks, and they allow banks to convert sixty percent of their risky assets to liquid assets during crises.

7 Discussion

I review various policies that eliminate the potential for high default risk or risk-shifting for all $\mu$ except those slightly below zero, both analytically and quantitatively. I find that direct asset restrictions or market concentrations can achieve this, though they harm social welfare. Transparent banking relaxes restrictions on the level of risky loans and market concentrations required to prevent catastrophic consequences, while its favorable impact is offset by deposit insurance. Interest rate caps can eliminate high default risk and nearly prevent risk-shifting, but there remains a practical problem of imposing severe caps for making it effective at low $\mu$. Debt-type managerial compensation can eliminate the potential for risk-shifting with a large number of banks in a deposit market. It eliminates the potential for high default risk when combined with liquidity provision to banks during economic downturns. According to these results, there exists a tradeoff between efficiency and sound banking for policies that prevent risk-shifting and high default risk for $\mu$ except those slightly below zero. However, if regulators can deploy debt-type managerial compensation when $\mu < 0$ and set moderate interest rate caps when $\mu \geq 0$, the tradeoff vanishes. I find that regulators do not have to incur the cost of monitoring business cycles by asking banks to report business conditions because they can incentivize banks to comply with both policies and reveal truthful information. I also note that adjustment costs are limited because regulators must change a policy only when mean return to risky loans changes from negative to positive, or vice versa.

Although the proposed model allows regulators to analyze various types of policies, it does not address all issues investigated in recent literature. First, it does not consider asset liquidity. Although it considers loss of principal at default, the loss is independent of exposure to risky loans. This means asset value uniformly decreases at default because risky loans are as liquid as safe assets. However, the proposed model offers implications for liquidity constraints because liquid assets such as cash are involatile.

Second, it does not consider shocks to depositors’ preference for liquidity, instead assuming deposit insurance is preset to preclude bank runs that arise from these shocks, so the proposed model

\textsuperscript{17}Recent studies, such as Vives (2014) and Wagner (2007), cover this issue.
Mean return to risky loans $\mu < -\sigma \lambda (\bar{z})$  
$-\sigma \lambda (\bar{z}) < \mu < -\sigma \lambda (\bar{z})$  
$-\sigma \lambda (\bar{z}) < \mu$

Exposure to risky loans $q$  
$q, \bar{q}$  
$ar{q}$

Table 1: Potential for multiple equilibria (overall risk)

<table>
<thead>
<tr>
<th>Number of banks</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>No commitment</td>
<td>-0.490</td>
<td>-0.698</td>
<td>-0.797</td>
<td>-0.855</td>
<td>-0.894</td>
<td>-0.921</td>
<td>-0.942</td>
</tr>
<tr>
<td>Commitment $(M^I = 0.9)$</td>
<td>-0.475</td>
<td>-0.698</td>
<td>-0.797</td>
<td>-0.855</td>
<td>-0.894</td>
<td>-0.921</td>
<td>-0.942</td>
</tr>
<tr>
<td>Commitment $(M^I = 0.5)$</td>
<td>-0.000</td>
<td>-0.695</td>
<td>-0.796</td>
<td>-0.855</td>
<td>-0.894</td>
<td>-0.921</td>
<td>-0.942</td>
</tr>
<tr>
<td>Commitment $(M^I = 0.1)$</td>
<td>-0.000</td>
<td>-0.692</td>
<td>-0.796</td>
<td>-0.855</td>
<td>-0.894</td>
<td>-0.921</td>
<td>-0.942</td>
</tr>
<tr>
<td>Actuarially fair premium</td>
<td>-0.000</td>
<td>-0.691</td>
<td>-0.795</td>
<td>-0.855</td>
<td>-0.894</td>
<td>-0.921</td>
<td>-0.942</td>
</tr>
</tbody>
</table>

Table 2: Minimum mean return to risky loans above which risk-shifting can occur

is isolated from these shocks. As a result, transparent banking does not affect depositors’ withdrawal decisions, though it influences depositors’ withdrawal or investment decisions, as Parlatore (2014) and Dang et al. (2014) suggest.

Third, depositors have perfect information for economic fundamentals in the proposed model, making implications to transparent banking incomparable to findings from Bouvard et al. (2015) that suggest a disclosure policy can aggravate financial fragility because imperfectly informed depositors notice economic fundamentals by whether a policy is undertaken, which can increase financing costs by raising depositors’ beliefs about default risks.

Lastly, the proposed model does not endogenize borrowers’ actions. Unlike the proposed model, Boyd and Nicoló (2005) and Martinez-Miera and Repullo (2010) endogenize borrowers’ actions. Consequently, market concentrations in the banking industry do not necessarily imply financial stability because higher loan rates required by a concentrated banking industry can encourage moral hazards on the part of entrepreneurs.

Thus, there is room for future researchers to develop a model at least on these dimensions. By considering these issues, they might discover policies that eliminate the potential for catastrophic equilibria other than the combination of interest rate caps and debt-type managerial compensation.
Figure 1: Cost of capital and the classification of tail risk
The ellipses represent potential equilibria. The shaded ellipses are locally stable, and the unshaded ellipse can be locally unstable.

Figure 2: Potential for multiple equilibria (tail risk)
The ellipses represent potential equilibria. The shaded ellipses are locally stable, and the unshaded ellipse can be locally unstable.

Figure 3: Effect of transparent banking by the fraction of insured depositors
The ellipses represent potential equilibria. The shaded ellipses are locally stable, and the unshaded ellipse can be locally unstable.

Figure 4: Effect of interest rate caps by mean return to risky loans
Figure 5: Boundary of policy parameters below which the financial system attains a unique, low-risk equilibrium.
Figure 6: Boundary of policy parameters above which the financial system attains a unique, low-risk equilibrium
Figure 7: Optimal tax schedule as a function of tail risk
Figure 8: Optimal tax schedule as a function of CDS spread (I)
Figure 9: Optimal tax schedule as a function of CDS spread (II)
References


