

The Equity Premium Puzzle in a Markov Regime switching model with rare disasters

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Abstract

Estimating a consumption-based asset pricing model with markov regime switching for a wide set of countries, this paper finds that there are two groups: one where countries are subject to rare disasters that take the form of capital destruction, and another where countries suffer strong recessions. The United States belong to the second group. To evaluate the "Peso Problem" hypothesis, a conditional estimation is performed to find that no specification of the disaster state is able to generate enough risk to match the high return on equity and low return on bills observed in data. Test of model specification points in the same direction. The results of this paper provide that the Equity Premium puzzle is still not explained.

1 Introduction

This paper develops and estimates a consumption-based model of asset pricing that allows for rare disasters as in Barro (2006). In the model, the mean and variance of the growth rate of the underlying asset are state dependent and are not observed by the econometrician and thus have to be inferred from data. Using smoothed inference, see Hamilton (1989), estimates of the primitives governing growth for a panel of countries are computed.

The estimation of the model for each country in sample provides that there are two distinct groups of countries. A first group where a rare disaster may occur in the form of capital destruction, and a second group where a recession may occur, where a recession is defined as a subsequent stream of bad realizations of GDP growth but that are part of the usual cycle of the economy. The United States belong to the second group.

The low frequency of a disaster can provide that the estimation strategy fails to capture the disaster state as an event of this type never occurred, as

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suggested by the "Peso Problem". Assuming the existence of a disaster state in the underlying asset, a conditional estimation is used to backout point estimates for the remaining parameters. By computing asset returns, I show that there is no specification of the disaster state able to generate enough risk to match the high equity premium observed in data. Tests of specification point in a similar direction.

The results of this paper points towards a single conclusion: there is not enough evidence supporting the existence of a disaster state for the United States. This implies that the Equity Premium puzzle remains unsolved.

Literature Review.

Since Mehra and Prescott (1985), the equity premium puzzle have been central part of empirical finance research. Using Lucas' (1978) asset pricing model as a benchmark, the equity premium puzzle accounts for the unreasonably high values of risk aversion needed to generate equity premium that resembles the value observed in the data: Mehra and Prescott (1985) find that for the period 1889-1979 the average real excess of returns of stocks relative to bills was about 6 percent while the real return on bills was only 0.8 percent.

Rietz (1988) suggested that the possibility of market crashes in a markov switching model framework might explain both puzzles, provided the market crashes were large enough and the unconditional probabilities were not too low. Mehra and Prescott (1988) replied rejecting the proposed solution by arguing that there was not enough evidence in the United States GDP and real interest rates time series to support the idea. Mehra and Prescott's response caused researchers to move away from Rietz line of thought.

Later, Barro (2006) argued that Rietz idea was correct. Using Rietz's original framework he calibrates the model for the U.S. economy. Assuming that every country shares the same ex-ante probability and degree of a crisis, he obtains estimates for the unconditional yearly probability of a crisis of around 1.7%. Further, the size of the disaster was estimated to be of around 45% current GDP taking into account and adjustment for risk aversion. Using historical data for the United States to calibrate the remaining parameters of the model, Barro finds that the existence of a low probability disaster state is able to generate a decrease in the risk-free rate and an increase in the equity premium.

Overview.

Section 2 builds up a simple consumption-based asset pricing model in a markov regime switching environment. The technology for production is defined, the consumer's problem is analyzed and close-form solutions for asset prices are obtained.

Section 3 describes the data used in the different sections.

To test the validity of the hypothesis econometric' instrumental is needed. Following the estimation strategy developed in Hamilton (1989) (see appendix A), Section 4 presents point estimates for the different moments of the growth equation for a set of countries for a markov model with two states. Using the estimation' results, I show that there are two kinds of countries, only one of them suitable to specifications with rare disasters. Further, the evidence supports the idea that the United States does not belong to this group. Finally, the degree

of the equity premium puzzle is analyzed using the approach in Mehra-Prescott (1985).

Similar to the "Peso Problem" in Mexico, a probable disaster, even though ex-post not occurring, generates incentives towards savings in less risky assets and thus might explain the puzzle. Section 5 explores this possibility. Assuming that a disaster state is part of the underlying process for income growth, a conditional estimation is pursued in order to test the validity of the model. Models of rare disasters are shown not to be able to correct the asset pricing puzzles, where a disaster is defined as a low conditional mean (actually, negative) in a particular state of the economy.

Section 6 asks whether countries are ex-ante identical. In order to test this assumption, Section 6 re-estimates the growth equation but by assuming that countries in sample share the same primitives. No evidence of a disaster state emerges.

Section 7 explores whether the United States suffered rare disasters in the last century. Once the existence of a regime shift post-WWII is taken into account (in contrast with Romer (1986) where the difference is only credited to measurement error), there is not enough evidence in favor of believing on rare disasters happening in the United States. The idea is that even though the market crash of the early 30's and the aftermath of WWII are rare, the degree of both events per unit of time is smaller compared, for example, to WWII in Germany.

Income data for the United States shows a sharp decline in its average value after 1950. Barro (2006), follows Romer (1986) where the observed regime shift is considered merely as measurement error. Further, Barro uses income series constructed by Romer where the measurement error is corrected. This paper sticks to the original dataset. Similar to Barro's argument, given that the proposed disasters for the United States occurred before 1950, Section 8 extends the estimation for a model with three states, where one state will capture the regime shift. Here, the United States look similar to Germany post-WWII and similar to Argentina pre-WWII but still the data does not support the idea of a disaster state being present for the United States.

Section 9 presents a Montecarlo analysis in order to check the robustness of the estimation process. Three economies were considered. One with rare disasters, another with a regular switching between expansions and recessions and a third with almost similar conditional moments across states. For the first it is shown here that the estimation strategy is able to detect the disaster state, even though some of the point estimates end up being correlated and thus not correctly identified. For the second one it is shown that the estimation strategy provides estimates with low mean square error for every coefficient, so the quality of the estimation is considered to be high. Finally, for the third one it is shown that the estimation strategy has problem distinguishing across states and so some of the estimates are far off the true values.

Section 10 concludes.

2 The Model

2.1 Production

Consider an economy that produces an homogeneous non-storable consumption good, denoted by Y , with conditional growth given by

$$\ln \frac{Y_{t+1}}{Y_t} | S_{t+1} \sim N \left(\mu_{S_{t+1}}, \sigma_{S_{t+1}}^2 \right) \quad (1)$$

where S denotes the state of the economy, $\mu_{S_{t+1}}$ is the conditional mean and $\sigma_{S_{t+1}}^2$ is the conditional variance. Further, $\pi(S'|S)$ represents the transition density between state S and state S' .

2.2 Consumers

This economy is populated by a large mass of identical consumers/firm owners, each of whom has utility

$$U(\{C_i\}_{i=t}^{\infty}) = \sum_{j=0}^{\infty} \beta^j E_t [U(C_{t+j}) | Y_t, S_t] \quad (2)$$

where $\beta \in (0, 1)$, $E_t[\cdot]$ is the expectation operator given the realization of S and current income, and C_{t+1} denotes consumption.¹

Given that the production good is non-storable, $C_t = Y_t$ and specializing $U(C_t) = \frac{C_t^{1-\alpha}}{1-\alpha}$,

$$V(Y_t, S_t) = U(\{Y_i\}_{i=t}^{\infty}, S_t) = \sum_j \beta^j E_t \left[\frac{Y_{t+j}^{1-\alpha}}{1-\alpha} | Y_t, S_t \right] \quad (3)$$

2.3 Assets

Stocks. Let $P_t(S_t, Y_t)$ be the price at time t of a tree² with current fruit equal to Y_t and state S_t , that pays dividends equal to Y_{t+j} in period $t+j$ for all $j > 0$ (given the timing of the model only information up to t is available when deciding to buy the tree). The price of this stock is

$$P_t(S_t, Y_t) = E_t \left[\sum_{j=1}^{\infty} \beta^j \left(\frac{Y_{t+j}}{Y_t} \right)^{-\alpha} Y_{t+j} | Y_t, S_t \right] \quad (4)$$

where $\beta^j \left(\frac{Y_{t+j}}{Y_t} \right)^{-\alpha}$ is the stochastic discount factor. The pricing equation can be formulated in the following recursive way:

$$P_t(S_t, Y_t) = E_t \left[\beta \left(\frac{Y_{t+1}}{Y_t} \right)^{-\alpha} (P_{t+1}(S_{t+1}, Y_{t+1}) + Y_{t+1}) | Y_t, S_t \right] \quad (5)$$

¹Given the assumptions made this far, this economy resembles one populated by a single representative consumer.

²as in Lucas (1978).

where $P_{t+1}(S_{t+1}, Y_{t+1}) + Y_{t+1}$ is the asset's payoff. Note that the previous equation can be rewritten as:

$$Y_t^{-\alpha} P_t(S_t, Y_t) = \beta E_t [Y_{t+1}^{-\alpha} [Y_{t+1} + P_{t+1}(S_{t+1}, Y_{t+1})] | Y_t, S_t] \quad (6)$$

This equation has a straightforward interpretation: the agent can buy a share of the stock at time t and state S_t by giving up $P_t(S_t, Y_t)$ units of consumption and thus reducing utility by $Y_t^{-\alpha} P_t(S_t, Y_t)$. In period $t + 1$ and state S_{t+1} the agent receives the dividend Y_{t+1} and has the opportunity to sell the stock for $P_{t+1}(S_{t+1}, Y_{t+1})$, therefore increasing consumption in $t + 1$ by $Y_{t+1} + P_{t+1}(S_{t+1}, Y_{t+1})$ and so the discounted increase in utility being $Y_{t+1}^{-\alpha} [Y_{t+1} + P_{t+1}(S_{t+1}, Y_{t+1})]$.

The return of this tree is defined as

$$R^e(S_t, Y_t) = E_t \left[\frac{P_{t+1}(S_{t+1}, Y_{t+1}) + Y_{t+1}}{P_t(S_t, Y_t)} | Y_t, S_t \right] \quad (7)$$

It will be proved useful to characterize the price to dividend ratio. Dividing (6) by $Y_t^{1-\alpha}$,

$$\frac{P_t(S_t, Y_t)}{Y_t} = \beta E_t \left[\lambda_{t+1}^{1-\alpha} \left[1 + \frac{P_{t+1}(S_{t+1}, Y_{t+1})}{Y_{t+1}} \right] | Y_t, S_t \right] \quad (8)$$

where $\lambda_{t+1} \equiv Y_{t+1}/Y_t$, as in Mehra and Prescott (1985), denotes consumption growth. Using the conjecture that $P_t(S_t, Y_t) = \tilde{P}_t(S_t) Y_t$, the price to dividend ratio $\Phi(S_t) \equiv \frac{P_t(S_t, Y_t)}{Y_t} = \tilde{P}_t(S_t)$ is independent of current income and thus,

$$\Phi(S_t) = \beta E_t \{ \lambda_{t+1}^{1-\alpha} | Y_t, S_t \} + \beta E_t \{ \lambda_{t+1}^{1-\alpha} \Phi(S_{t+1}) | Y_t, S_t \} \quad (9)$$

Defining $\theta(S_{t+1}) \equiv \lambda_{t+1}^{1-\alpha}$ and $u(S_t) \equiv \frac{1}{1-\alpha} E_{t+1} [\theta(S_{t+1}) | Y_t, S_t]$,

$$\Phi(S_t) = \beta (1 - \alpha) u(S_t) + \beta E_t \{ \Phi(S_{t+1}) \theta(S_{t+1}) | Y_t, S_t \}$$

This last equation for every state S_t , and allows for a matrix representation

$$\vec{\Phi} = \beta (1 - \alpha) \vec{u} + \beta \Pi \text{diag}(\vec{\theta}) \vec{\Phi}$$

where Π represents the transition matrix.

Solving for $\vec{\Phi}$,

$$\vec{\Phi} = \left[I - \beta \Pi \text{diag}(\vec{\theta}) \right]^{-1} \beta (1 - \alpha) \vec{u} \quad (10)$$

After some simple algebra, (7) can be written as,

$$R^e(S_t) = E_t \left[\lambda_{t+1} \frac{\Phi(S_{t+1}) + 1}{\Phi(S_t)} \right] \quad (11)$$

This equation presents a simple formula of the return on equity. Once the price-dividend ratio is obtained from equation (10) for every state, (11) can be used to compute the return on equity conditional on state S at period t .

Bonds. Let R^f be the gross return of a one period risk free bond. The price of any return is one so that

$$1 = E_t \left[\beta \left(\frac{Y_{t+1}}{Y_t} \right)^{-\alpha} R^f | S_t, Y_t \right]$$

Given the current state R^f is constant and thus,

$$R^f(S_t) = \frac{1}{\beta E_t [\lambda_{t+1}^{-\alpha} | Y_t, S_t]} \quad (12)$$

Equity Premium. From (11) and (12), the average equity premium in difference form is

$$E \left(\overrightarrow{R^e} - \overrightarrow{R^f} \right)$$

3 Data Description

Except for Section 7, every section uses GDP per capita series from the Maddison Historical Dataset for the 1900-2003 period. The following countries were used in the analysis:

- From the OECD: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Italy, Japan, New Zealand, Norway, Portugal, Spain, United Kingdom and The United States.
- From the Americas: Argentina, Brazil, Chile, Colombia, Mexico, Peru, Uruguay and Venezuela.
- From Asia: India and Sri Lanka.

Some countries were excluded from the analysis because had missing observations for the relevant time period.

Section 7 uses data from Shiller's website. Dividends, Stock Returns, and Interest Rates for the 1900-2003 period are used.

4 Basic Estimation

In the framework developed in Section 2, a rare disaster can be thought as a state with a low unconditional probability and negative conditional mean (or high conditional variance).

Table 1 presents point estimates (and standard errors) for the United States, Germany, and Argentina, using techniques developed in Hamilton (1989).³ Germany and others⁴ suffered major disasters with two main characteristics: huge capital destruction and little spread across time (for example, a war that lasts for 1 or 2 years). This group will be labeled DIS⁵. Table 1 also presents the mean of each coefficient across countries within the group. Opposite to Germany and the DIS group, Argentina's crisis are of smaller degree per period of time but are more spread out across time (they look more like a recession). This characteristic is shared by many other countries.⁶ All of these countries are thus included in a second group labelled REC⁷.

The pair $\{\mu_0, \sigma_0\}$ denote the estimated conditional mean and variance in state 0 (similar idea for $\{\mu_1, \sigma_1\}$). State 0 will be used here as the normal/expansion state of the economy whereas state 1 will be used for recessions and rare disasters. $\pi_{0|0}$ and $\pi_{0|1}$ are used for the transition probabilities, where $i|j$ denotes the probability of switching to state i conditional on being today in state j .

The estimated values for the transition probabilities, together with the conditional moments' estimation, provides information about the growth pattern of the United States economy: growth seems to switch in a regular basis between periods of positive growth and periods of decline or stagnation. Neither state is rare nor exhibit high volatility. This can also be observed in the second panel of Figure 1, where GDP per capita growth for the United States is plotted, from where it can be observed that (1) there is a recurrent shift between periods of high growth and periods of low or negative growth (with a different scale after WWII), and (2) something happened post WWII that implies a regime shift⁸. This idea will be explored with more detail in Section 7.

³standard deviation was computed using a Montecarlo technique instead of using asymptotics as in Hamilton (1989).

⁴Austria, France, Germany, Italy, Japan, Spain, and United Kingdom.

⁵for Disaster.

⁶Australia, Belgium, Canada, Chile, Colombia, Denmark, Finland, India, Mexico, New Zealand, Norway, Peru, Portugal, Sri Lanka, Switzerland, Uruguay, and Venezuela.

⁷for Recession.

⁸some authors believe that this is only measurement error. Romer (1986) provide a good discussion.

Basic Estimation

	Germany	DIS	U.S.	Argentina	REC
μ_0	0.0371 (0.004)	0.032	0.031 (0.01)	0.039 (0.013)	0.032
μ_1	-0.183 (0.058)	-0.298	-0.005 (0.039)	-0.038 (0.03)	-0.02
σ_0	0.038 (0.003)	0.038	0.039 (0.006)	0.033 (0.008)	0.032
σ_1	0.124 (0.275)	0.25	0.054 (2.1)	0.042 (1.6)	0.049
$\pi_{0 0}$	0.936 (0.03)	0.969	0.676 (0.185)	0.65 (0.158)	0.747
$\pi_{0 1}$	0.675 (0.155)	0.765	0.62 (0.174)	0.55 (0.17)	0.531

Table 1

The fact that the standard error of σ_1 for the United States is very large presents both a problem and also an important insight. The large value of the standard deviation implies that σ_1 is not correctly estimated. Further, this implies that might be in some way problematic to construct measures that involve calculations using σ_1 (for example stock returns and risk-free rates). Still, the main purpose of this paper is to analyze and understand whether there exists a disaster state in the underlying economy of every country (and, in particular, for the United States). Is in this way, as it will be clarified in section 9, that the high value of the standard error of σ_1 provides an interesting insight. As a starter, keep in mind that standard errors are calculated using a montecarlo technique.⁹ As μ_0 and μ_1 are close and σ_0 and σ_1 are not negligible, when data is generated using the estimated coefficients it happens that it will be hard (sometimes impossible) for the posterior estimation strategy to distinguish between both states and thus the estimated value for σ_1 in each repetition will be very hard to pin down. Note that it is probably the case that the growth equation for the United States can be fitted with only one state (the variance σ^2 can account for the bussines cycle fluctuations), and thus no disaster state is also aparent in the data. This suggest that with yearly data it might be the case that the United States income growth equation has no switching. In other words, that there is only one state. Hamilton (1989) finds similar results for United States quarterly data for GDP series post-WWII. Note that Argentina share the same qualitative feature. Section 9 will extend this analysis.

The probability of each state in period t can be inferred using the estimated transition probabilities.¹⁰ The third panel of Figure 1 plots the inferred probability of being in state 1 for the United States. As suggested by the transition

⁹1- Estimate the coefficients, Υ . 2- Generate data for N fake countries using Υ as input. 3- Estimate the model for each $n \in N$. 4- Calculate the standard error of the estimated coefficients across N .

¹⁰the details are in Appendix A.

probabilities, the inferred probability rejects the idea that either state is rare¹¹.

Duration of each state

	Germany	DIS	U.S.	Argentina	REC
state 0	15.6	32.26	3.1	2.86	3.95
state 1	1.48	1.3	1.61	1.81	1.88

Table 2

The estimated coefficients for Germany (and the group's average) of μ_0 and μ_1 supports the idea of countries of steady growth that encountered events of devastating magnitude. Moreover, the high predicted value for $\pi_{0|0}$ (i.e. the probability of being in state 0 conditional on state 0 happening in the previous period) implies that periods of subsequent growth are pretty common. Furthermore, switching across states is less periodical than in the United States.

Argentina present estimates that appear to be closer to the ones for the United States, as so also does the group's average. Point estimates and transition probabilities are of the same order of magnitude.

United States: GDP per capita growth and smoothed inference

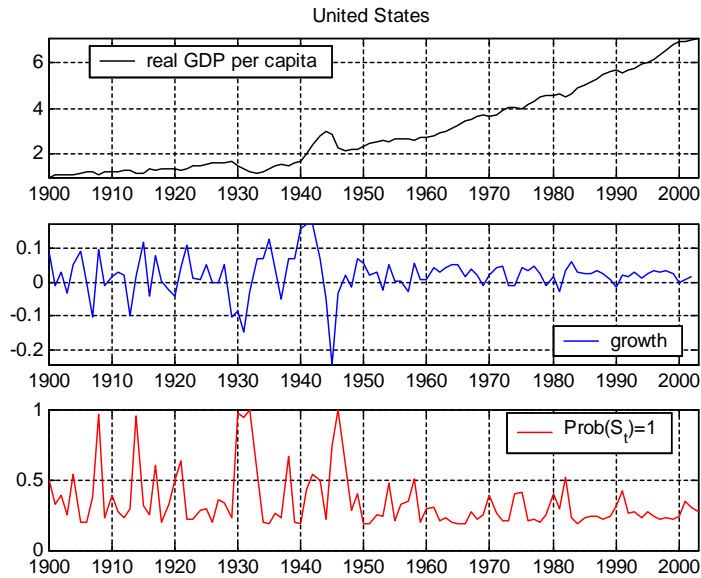


Figure 1

The duration of each state for the United States, as shown in Table 2, are closer to the ones' for Argentina and the REC group.

¹¹a rare state would have inferred probability close to 0 for almost every t and close enough to 1 for a few periods.

Germany and Argentina.: GDP per capita growth and smoothed inference

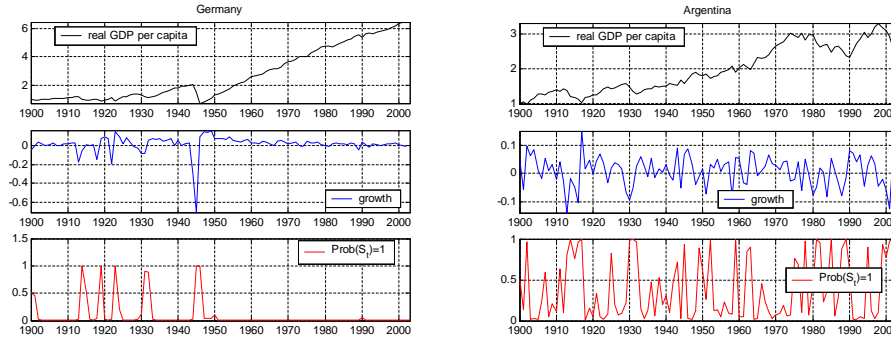


Figure 2

Figure 2 provides more information about the resemblance of the United States with the REC group. Second and third panel of each plot presents GDP per capita growth and inferred probability of state 1 for Germany and Argentina, providing the following facts: (1) switching between states is much more common in Argentina, (2) state 1 is catastrophic in Germany, while in Argentina seems to be part of the business cycle (even though with high volatility), and (3) high conditional volatility in state 1 (σ_1) is necessary to rationalize WWI and WWII in Germany (because of their disparity).

Moreover, from the third panel it is clear that state 1 is rare in Germany while common in Argentina.

The United States appears to be part of the second group (or closer to members of that group), as suggested by the comparison of the inferred probabilities for the different countries and point estimates for the conditional moments.

Disasters of the same type as in Germany are not observed in the United States data. Still, this does not imply that the U.S. is not necessary a member of the DIS group.

The existence of a disaster state, even though not present in the sampled data, might help to correct the asset pricing puzzles. The intuition is as follows: Agents in the economy know that this state exists. Furthermore, they can infer the probability of switching into that state. The chance of getting into this bad state generates incentives towards less risky positions so savings flow from equity into risk-free assets. The price of equity decreases in order to attract investors while the price of the risk-free asset increases due to the increase in demand. Thus, return on equity increases while return on the risk-free asset decreases. This behavior only requires the possibility of a future crisis, not an actual crisis occurring. Several episodes of this nature can be found in real life. The most well-known, and widely discussed, is the Peso Problem. Section 5 presents a conditional estimation as a way of testing this possibility.

4.1 The degree of the Equity Premium Puzzle

Using the point estimates conditional asset returns can be computed. With the inferred probabilities, it is possible to construct time series for both the equity premium and the risk-free rate. Third column of Table 3 was constructed following Mehra-Prescott (1985), where they look for the duple $\{\alpha, \beta\}$ necessary for rationalizing the value of the Equity Premium observed in data. The first column uses reasonable values for β and α .¹²

A subtle difference with their analysis is that here the parameters of the model (excluding α and β) were estimated from data instead of being calibrated.

Degree of Puzzles for the United States

α	4	22
β	0.96	0.79
EP	1%	4.6%
r^f	10.8%	0.3%

Table 3

As in Mehra-Prescott (1985), non-sensical values for both the risk-aversion parameter and the discount factor are needed in order to rationalize a high EP and a low risk-free rate for the United States. As in their paper, the Equity Premium Puzzle is present here.

5 Conditional Estimation

As argued in Section 4, the existence of an eventual rare disaster might be enough to explain the puzzle. Even though these might not be present in the available data for the United States (Section 4 and 7 supports the idea that no rare disasters occurred in the 20th century), it is possible to check whether the data supports this hypothesis by introducing exogenously into the model a "disaster" state and see whether model selection tests reject the proposed model and, more importantly, whether the generated equity premium matches the one observed in data.

With this in mind, Table 4 provides the conditional estimation for the United States where μ_1 is arbitrarily chosen. Further, for the conditional estimation $\sigma_1 = \sigma_0$. This restriction relays in the idea that a disaster is understood as a sharp decline in income growth. If we let σ_1 to be freely estimated, it will happen that in order to match the data the estimation strategy will provide high estimated values for σ_1 provided that the artificial disaster included in the model does not fit the data. This is an undesired result if there is interest in the estimation's quality.

Also, as discussed in Section 4 the estimation quality for the United States might not be great and thus it might be hard to draw into conclusions if those results are used. Then, instead of using the coefficients obtained in Section 4 as the unconstrained estimation results, the model is estimated again assuming

¹²Mehra-Prescott (1985) provides a discussion about what is defined as reasonable.

$\sigma_0 = \sigma_1$ and $\mu_1 = -0.005$. The likelihood ratio test for this alternative specification is 0.98 (so the alternative model is not rejected) meaning that both models are pretty similar in terms of explaining the data, and thus provides some confidence in favor of this methodology.

The first column of Table 4 presents the unconstrained estimation, and the fifth column use the conditional mean used in Barro (2006)¹³. The last row presents the likelihood ratio of each model relative to the model with $\mu_1 = -0.005$ and $\sigma_1 = \sigma_0$.

United States: Estimation conditioning on μ_1

μ_1	-0.005	-0.1	-0.15	-0.3	-0.3317	-0.5
μ_0	0.036 (0.01)	0.028 (0.004)	0.026 (0.004)	0.021 (0.004)	0.021 (0.005)	0.019 (0.005)
σ_0	0.0465 (0.009)	0.043 (0.003)	0.045 (0.003)	0.051 (0.003)	0.051 (0.003)	0.057 (0.005)
σ_1	0.0465 (0.009)	0.043 (0.003)	0.045 (0.003)	0.051 (0.003)	0.051 (0.003)	0.057 (0.005)
$\pi(0 0)$	0.6395 (0.17)	0.961 (0.02)	0.97 (0.02)	0.989 (0.01)	0.99 (0.01)	1 (0.02)
$\pi(0 1)$	0.593 (0.17)	0.577 (0.19)	0.617 (0.2)	1 (0.02)	1 (0.02)	1 (0.02)
EP	1.1%	0.3%	0.01%	1.9%	2.1%	1.4%
r^F	10.8%	10.4%	9.2%	8%	7%	9%
LR Test	— — —	4.36**	20.005***	40.9***	51.6***	35.7***

Table 4

As μ_1 decreases σ increases. In order to match the observed data, σ needs to increase to offset the low values of μ_1 . Further, this state is becoming harder to match to data and thus the implied transition probabilities records low chance of getting into that state. Eventually, the disaster state has 0 probability, as shown for the model with $\mu_1 = -0.5$. The Equity Premium is not a monotonical function of μ_1 (Figure 1-A in the appendix shows this fact for a larger set of μ_1). There are two forces defining a disaster. The first one is the actual value of the disaster given by μ_1 and σ_1 . The second force is the length of the disaster and its unconditional probability. As presented in Table 5, up to some $\mu_1 > \tilde{\mu}_1 \in (-0.3, -0.15)$ it happens that the decrease in the unconditional probability (and length) of the disaster state outweighs the size of the disaster and thus the Equity Premium falls. $\mu_1 = -0.15$ and $\mu_1 = -0.3$, even though the size of the disaster increases, the duration changes in such a way that outweighs the size. Later, for $\mu_1 < \tilde{\mu}_1$ the size end up being more important and thus the Equity Premium start to increase. Eventually, for μ_1 low enough, the estimation strategy can't fit that state into the data and thus the equity premium is fixed in a lower value (because the only risk still present in this economy is σ_0).

¹³ $-0.3317 = 0.025 + \ln(0.7)$ as in Barro (2006).

Duration of each state - conditional on μ_1

μ_1	-0.005	-0.1	-0.15	-0.3	-0.3317	-0.5
state 0	3.08	25.64	33.3	90.9	100	∞
state 1	1.61	1.73	1.62	1	1	0
unc. prob. 1	0.343	0.063	0.046	0.0105	0.009	0

Table 5

Three important issues need to be understood here. First, from inspection of Table 4 it can be observed that the existence of a disaster state, even though highly destructive in terms of growth ex-ante, is far away of solving the Equity Premium puzzle for the United States. The reason for this is that, even though the existence of a rare disaster can potentially explain the equity premium puzzle¹⁴ (this will be shown when the montecarlo analysis is done in section 9), this rare disaster is not reflected in the United States data and thus the transition probabilities adjust in order to make the disaster state disappear.

Second, likelihood ratio tests rejects the models with rare disasters. In particular, Barro' (2006) characterization is rejected with more than 99% confidence.

Last, it can be checked in Figure 3 that the inference for the model with $\mu_1 = -0.3317$ implies two disasters¹⁵:the market crash in the early 30's and the aftermaths of WWII, the two events that Barro (2006) considered as rare disasters in the United States.

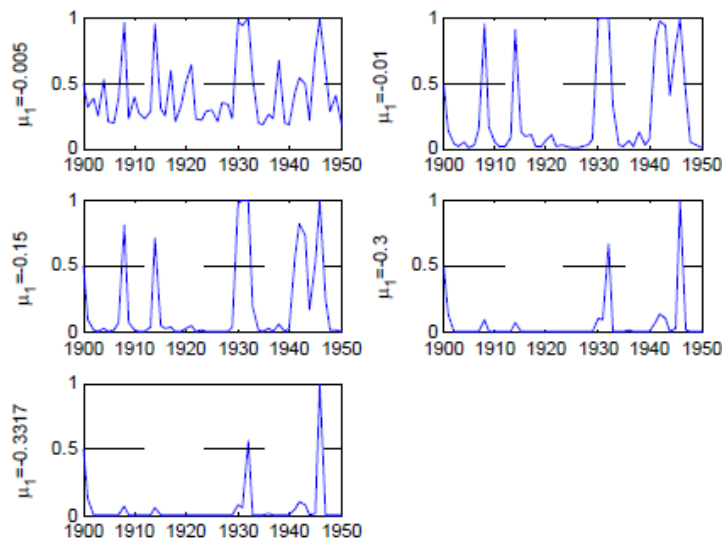
United States: smoothed inference conditioning on μ_1 1900-1950

Figure 3

¹⁴this is what Barro (2006) claims as success.

¹⁵using as a threshold that state 1 occurs if the inference is higher than 1/2.

To summarize, this section showed that models with rare disasters for the United States are not enough to solve the Equity Premium Puzzle provided that the existence of a disaster state does not generate enough risk to match the Equity Premium observed in data. Further, models of rare disasters are rejected by the data with high confidence.

6 Panel Data Estimation

Barro (2006) calibrates the disaster state as if every country in sample was ex-ante identical. In order to address this idea a panel data estimation with every country in sample is performed. As seen in Table 6, the estimation generates positive values for both conditional means and low values for the conditional variance.

μ_0	0.0268 (0.006)	$\pi(0 0)$	0.7885 (0.14)
μ_1	0.0014 (0.04)	$\pi(0 1)$	0.6108 (0.14)
σ_0	0.033 (0.004)	EP	0.69%
σ_1	0.05 (1.5)	r^f	11.4%

Table 6

As there are more countries with no disasters (and that all of them weight equally in the estimation), the effect of rare disasters fades away and thus the estimation suggests that the underlying economy exhibits no disaster state. As in section 4, and for the same reasons, the standard error of σ_1 is too high. Further, the Equity Premium and the Risk-free rates are far away from the ones observed in data.

7 Rare disasters in the twentieth century in the United States and the regime shift post-WWII

The first panel of Figure 4 plots the United States' GDP per capita growth for 1900-2003. Two main things arise from that picture: (a) there is a regime shift post-WWII which decreased the volatility of GDP growth in an important way, and (b) the stock market crash of the early 30's and the aftermath of WWII appear to be disasters as in Barro (2006).

Concerning the first point, Romer (1986) argues that the difference in the GDP series pre- and post-WWII is just measurement error, provided that gathering information for computing the series was very hard and simple extrapolation methods were used. Further, she recomputes the whole series for GDP using newer methodologies, and no regime shift seems to be present. This is the series that Barro (2006) uses.

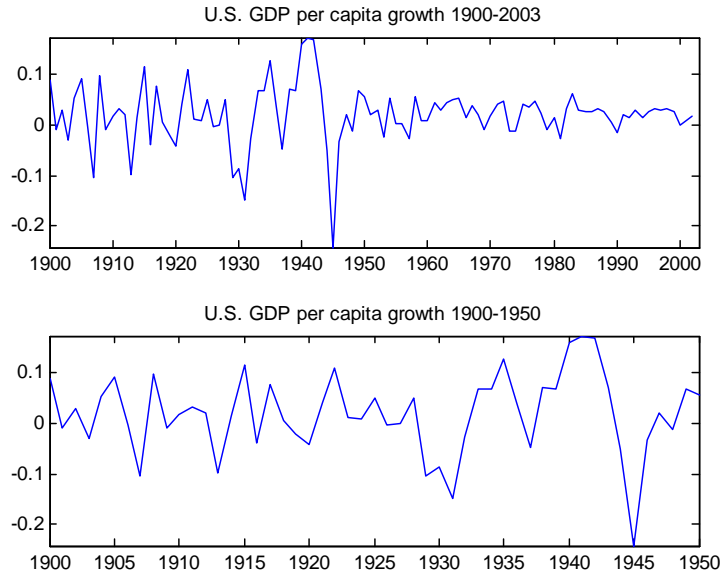


Figure 4

The concern is that other series that are easier to gather also exhibit the regime shift. Figure 5 plots dividends, stock returns, and interest rates for the United States for the 1900-2003 period. While stock returns series does not exhibit the regime-shift, both dividends and interest rates does. Further, the apparent timing of the shift is similar to the one observed for GDP per capita. Both dividends and interest rate series are easy to construct and gather and so the degree of measurement error should be not important (or at least not the sole explanation for the sharp change in both series behavior).

Given that there is enough evidence of a regime shift, the second panel of Figure 4 provide information for understanding the stock market crash and the aftermaths of WWII and their classification as rare disasters. Second panel plots GDP per capita growth but only for the sample period 1900-1950 (so the regime shift is not plotted here). In this panel, these two events blend on with the rest of the data. They seem to be of the same order. This conflicts with Barro's idea: it appears that these events are not of the same magnitude as the ones observed for Germany and Japan, for example. The first panel is confusing because there is a regime shift in 1950 that changes the scale. Up to WWII the world was highly volatile and risky, and bad times weren't rare. In the same way, great times were pretty common too.

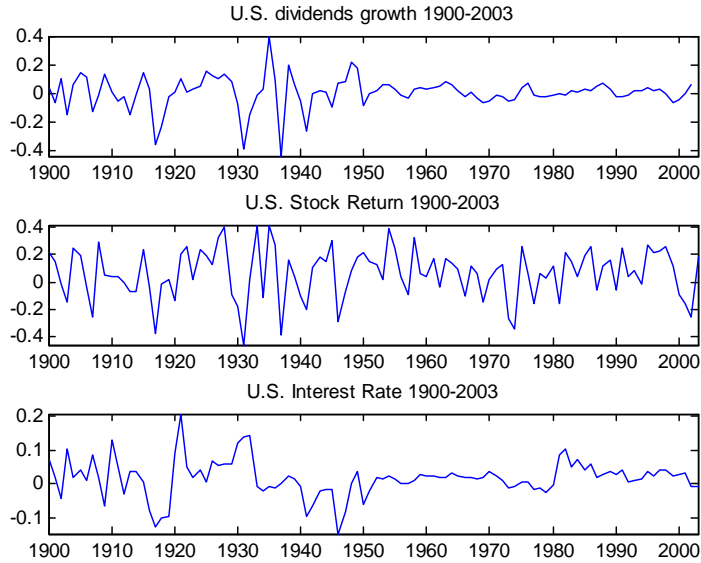


Figure 5

This facts, together with the findings of previous sections provide an important insight: the size of the disaster matters. The estimation strategy provides that the market crash in the early 30's in the United States and WWII in Germany are two very different events that affect the economy in pretty different ways.

8 Estimation of a model with three states

As discussed in the previous section there seems to be a regime shift post-WWII for income growth for the United States. Romer (1986) suggested that this "shift" in the GDP series was just measurement error. Instead of using the income series constructed by Romer that "corrects" this problem (as Barro did), this paper sticks to the original data¹⁶. The results are shown in Table 7 where the point estimates for the United States, Germany, and Argentina are presented.

¹⁶primarily because of data availability for the whole sample of countries.

Estimation of the model with three states

	Germany	United States	Argentina
μ_0	0.0199 (0.002)	0.0199 (0.003)	0.039 (0.01)
μ_1	0.08 (0.19)	0.087 (0.058)	-0.035 (0.04)
μ_2	-0.148 (0.11)	-0.036 (0.037)	-0.041 (0.03)
σ_0	0.018 (0.009)	0.023 (0.005)	0.031 (0.006)
σ_1	0.0285 (0.42)	0.046 (1.1)	0.052 (1.2)
σ_2	0.210 (0.08)	0.077 (0.02)	0.0236 (0.01)
$\pi(0 0)$	0.921 (0.04)	0.89 (0.07)	0.662 (0.19)
$\pi(1 0)$	0.005 (0.02)	0.0309 (0.04)	0.184 (0.11)
$\pi(0 1)$	0.1208 (0.33)	0.210 (0.2)	0.545 (0.33)
$\pi(1 1)$	0.808 (0.31)	0.473 (0.16)	0.298 (0.09)
$\pi(0 2)$	0.207 (0.26)	0.2 (0.2)	0.55 (0.23)
$\pi(1 2)$	0.41 (0.19)	0.297 (0.2)	0.335 (0.2)
EP	0.9%	1.1%	0.9%
R^f	1.2%	9.4%	5.8%

Table 7

From Figure 6 it can be observed that the smoothed inference of state 0 captures the regime shift (state 0 has probability of almost 1 in every period after WWII). Note that state 0 is also the most important state for Germany post-WWII, as shown in Figure 2-A. Interestingly, the coefficients for state 0 are very similar for the U.S. and Germany, suggesting that both economies start resembling each other only after WWII. Also, note that Argentina didn't experience the regime shift, as suggested by the point estimates in Table 7 or by the smoothed inference (shown in Figure 3-A).

United States: smoothed inference with three states

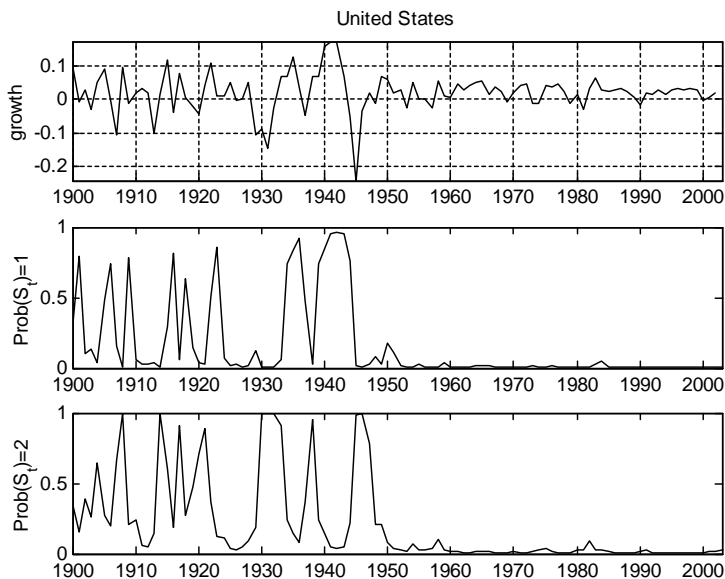


Figure 6

Still, no evidence of a disaster state can be found on Table 7 for the United States. Furthermore, The United States looks similar to Argentina for the first half of the century and after WWII they resemble Germany.

As in previous sections the standard error of σ_1 is too high. The same reasoning as in section 4 applies: with yearly data there should be only one state prior to the regime shift and also only one state after the regime shift.

As observed in Table 1-A, as μ_2 decrease (in order to artificially generate disasters) similar features to the ones observed for the case with two states emerge: (a) the Equity Premium increase and the risk free rate decrease, and (b) the models are rejected by the data. Still, the equity premium generated with the artificial disaster is far away of explaining completely de Equity Premium Puzzle.

The same idea discussed in Section 5 applies here.

9 Montecarlo Simulations

One of the central aspects of this paper and of Barro (2006 and 2007), is to understand how do low-frequency devastating events affect asset prices and, in posteriori, welfare calculations. The low frequency of these events raises some concern about the ability of the estimation strategy to capture them and to produce meaningful estimators.

With this in mind it seems natural to simulate an economy with known parameters by the econometrician and then assess the quality of the estimation.

For the purpose of this paper, the model will be specialized to have only two states. For each country in sample there are 104 years of data available. As a result, this is the number of periods to be used in the Montecarlo simulation, which consists of 20,000 repetitions.

To asses the quality of the estimation, the Mean Square Error (or MSE) of each estimator is computed. Because the MSE is affected by the scale of the underlying parameters, $MSE/(true\ value)$ is used here.

Three different economies are simulated, one with two high-frequency states, one low-frequency state, and another where the two states will be hard to disnt-guish. The first economy is such that there is a somewhat continuous switching between the possible states. This simulated economy will resemble economies with cycles but with no rare disasters. The second simulated economy presents constant growth but in rare ocassions will switch to a state were GDP growth becomes significantly negative.

Economy with cycles. As seen in Table 2-A ¹⁷ in Appendix A, the high frequency of both states implies high quality of the estimation (MSE is small for every variable).

The small values of MSE suggest that if the underlying economy is similar to the one constructed here, then the estimation strategy would provide good estimates and would allow for the construction of measures of welfare and equity returns that correspond to the underlying parametrization. What does "similar economy" means? Think about economies that have not experienced disasters but have had several periods (maybe subsequent) of economic contraction. Evidence confirming that such a group exists will be presented and discussed in later sections.

Economy with Rare Disasters. The low frequency of the catastrophic events affects the quality of the estimation. Table 3-A presents the simulation of this type of economy, where the underlying parameters were chosen to match Barro's.

In terms of coefficients' estimation, only μ_1 and σ_1 are off from the true values. As was suggested earlier, this is due to the fact that state one occurs only around twice every 100 periods¹⁸. The scarcity of data points in this state makes it impossible to produce estimates with low MSE of the conditional moments. Even though this is an important problem that decreases the quality of the estimation process, it is still true that the estimation is able to detect the periods where a disaster ocured, as shown in Figure 4-A, and thus has the ability to provide acceptable measures for the probability of the crisis. Also, although the conditional moments in state one are not estimated with precision, they are clearly implying a big crisis providing a valid qualitative analysis. The low frequency of state 1 implies that the estimation strategy is unable to distinguish between the conditional mean and conditional variance, and so a high covariance

¹⁷ $\alpha = 4$ and $\beta = 0.97$ are assumed for calculation purposes as in Barro (2005).

¹⁸ The unconditional probability of state one is $\frac{1-\pi_{0|0}}{2-\pi_{0|0}-\pi_{1|1}} = 0.017 \approx 0.02$.

between both conditional moments in the montecarlo simulation is obtained (the covariance is 0.08 while for the economy with cycles is -0.0034). This fact can also be observed in Figure 5-A.

Furthermore, the risk-free rate and expected equity premium have low MSE.

Economy with Undistinguishable States. As it can be seen in Table 4-A, the conditional moments of state 0 have a small MSE whereas the ones for state 1 don't. This fact suggests that whenever high standard errors of the conditional moments of a particular state are obtained it is the case that the underlying economy has actually only one state or undistinguishable states.

Finally, it is worthwhile pointing out that the evidence presented in Tables 2-A, 3-A and 4-A suggests that the proposed estimation strategy is powerful enough to distinguish between economies that experience rare disasters and economies that don't, even though the degree of the crisis might be somewhat inaccurate.

10 Conclusion

A consumption-based model of asset pricing was constructed in order to test the importance of rare disasters over the equity premium and risk-free rates in the United States. Further, the estimation strategy allows to check whether the data supports the proposed model.

The analysis performed in this paper indicates that there is evidence supporting the idea of the existence of two distinct groups of countries. A first group where rare disasters may occur and take the form of big capital destruction, and a second group where instead of disasters a recession may occur (where a recession is defined as a subsequent stream of bad realizations of GDP growth but that are part of the economy's cycle). Further, the evidence suggests that the United States belongs to the second group.

The usual reply to this argument is, as in the Peso Problem, that the data might not reflect a low probability event as a rare disaster is. With this in mind a conditional estimation was performed in order to understand if there is any way in which the data might support this idea. No model of conditional mean is able to generate enough risk to match the Equity Premium observed in data. Furthermore, model selection tests rejects models of rare disasters.

Dividends and GDP growth and Interest rates appear to have a regime shift after WWII (rejecting the idea of simple measurement error used by Barro (2005) and largely discussed in Romer (1986)), and so a model with three states was constructed in order to capture this shift. The same results obtained for a model with two states were obtained here.

Apparently, the existence of a disaster state is reasonable for countries such as Germany, Japan, and Spain, that experienced important capital destruction some time in the last century, but it is not suitable for the United States or countries such as Argentina or New Zealand, that have important volatility through the business cycle but no particularly bad event in the last hundred years.

Should it be concluded that rare disasters are not part of the story? This paper provides evidence in favor of the idea that the United States might not have this state, or at least that the probability of this state is smaller than in other countries. The data supports the idea of the existence of two distinct groups. A probable story that can be matched with the results of this paper is one where there are two types of countries. One where the probability of the disaster state is higher than in the other one (it might be even zero). At time zero, agents don't know which type of economy they populate and so they will extract information about the true values of the economy from the available signal: GDP growth. This uncertainty will generate that countries with the lower probability of the rare disaster might have a higher equity premium and a lower risk-free rate because agents' unconditional belief of the disaster probability is higher than the actual value. This idea calls for further research.

11 Appendix: Estimation Strategy

11.1 A brief Introduction

The next lines are from Time Series Analysis by J. Hamilton.

Let ξ_t denote a random vector whose j th element is equal to unity if $s_t = j$ and whose j th element equals zero otherwise. Thus, for a model with only two states, when $s_t = 0$ the vector ξ_t is equal to the first column of I_2 and when $s_t = 1$ the vector is equal to the second column of I_2 .

Now, if $s_t = i$, then the j th element of ξ_{t+1} is a random variable that takes on the value unity with probability π_{ij} and zero otherwise. Such a random variable has expectation π_{ij} . Then, the conditional expectation of ξ_{t+1} given $s_t = i$ is given by

$$E[\xi_{t+1} | s_t = i] = \begin{bmatrix} \pi_{i1} \\ \pi_{i2} \end{bmatrix} \quad (13)$$

This vector is simply the i th column of the matrix Π . It follows that when $s_t = i$, the vector ξ_t corresponds to the i th column of I_2 and so the vector in (13) can be described as $\Pi\xi_t$. Hence (13) implies,

$$E[\xi_{t+1} | \xi_t] = \Pi\xi_t$$

Using the markov property,

$$E[\xi_{t+1} | \xi_t, \xi_{t-1}, \dots] = \Pi\xi_t \quad (14)$$

In this way we can write the markov chain as

$$\xi_{t+1} = \Pi\xi_t + \eta_{t+1} \quad (15)$$

with

$$\eta_{t+1} = \xi_{t+1} - E[\xi_{t+1} | \xi_t, \xi_{t-1}, \dots] \quad (16)$$

Expression (15) is a first order autoregression for ξ_t . (16) implies that the innovation η_t is a martingale difference sequence.

(15) implies

$$\xi_{t+m} = \eta_{t+m} + \Pi\eta_{t+m-1} + \Pi^2\eta_{t+m-2} + \dots + \Pi^{m-1}\eta_{t+1} + \Pi^m\xi_t$$

It follows that,

$$E[\xi_{t+m} | \xi_t, \xi_{t-1}, \dots] = \Pi^m \xi_t \quad (17)$$

Define,

$$\eta_t = \begin{bmatrix} f(\ln Y_t | s_t = 0, \ln Y_{t-1}; \vartheta) \\ f(\ln Y_t | s_t = 1, \ln Y_{t-1}; \vartheta) \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2\pi\sigma_0^2}} e^{-\frac{(\ln Y_t - \mu_0 - \ln Y_{t-1})^2}{2\sigma_0^2}} \\ \frac{1}{\sqrt{2\pi\sigma_1^2}} e^{-\frac{(\ln Y_t - \mu_1 - \ln Y_{t-1})^2}{2\sigma_1^2}} \end{bmatrix} \quad (18)$$

How do we obtain estimates of the probabilities? using (18) and (14) just iterate on

$$\hat{\xi}_{t|t} = \frac{\left(\hat{\xi}_{t|t-1} \odot \eta_t \right)}{1' \left(\hat{\xi}_{t|t-1} \odot \eta_t \right)} \quad (19)$$

and set the initial condition as

$$\xi_{1|0} = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$$

Now let $\hat{\xi}_{t|\tau}$ represent the vector whose j th element is $P(s_t = j | y_t; \vartheta)$. For $t > \tau$ this represents a forecast about the regime for some future period, whereas for $t < \tau$ it represents the smoothed inference about the regime the process was in at date t based on data obtained through some later date τ .

Smoothed inferences can be calculated using the following algorithm (Kim 1993)

$$\hat{\xi}_{t|T} = \hat{\xi}_{t|t} \odot \left(P' \left[\hat{\xi}_{t+1|T} (\div) \hat{\xi}_{t+1|t} \right] \right) \quad (20)$$

The smoothed probabilities $\hat{\xi}_{t|T}$ are found by iterating on this equation backwards for $t = T - 1, T - 2, \dots, 1$. The iteration starts at $\hat{\xi}_{T|T}$ obtained in (19).

11.2 The estimators

Given the smoothed inferences obtained we then can estimate the remaining coefficients by Maximum Likelihood and so:

$$\hat{\mu}_0 = \frac{\sum_{t=1}^T (\ln Y_t - \ln Y_{t-1}) \Pr(s_t = 0)}{\sum_{t=1}^T \Pr(s_t = 0)} \quad (21)$$

$$\hat{\mu}_1 = \frac{\sum_{t=1}^T (\ln Y_t - \ln Y_{t-1}) \Pr(s_t = 1)}{\sum_{t=1}^T \Pr(s_t = 1)} \quad (22)$$

$$\widehat{\sigma}_0^2 = \frac{\sum_{t=1}^T (\ln y_t - \hat{\mu}_0 - \ln Y_{t-1})^2 \Pr(s_t = 0)}{\sum_{t=1}^T \Pr(s_t = 0)} \quad (23)$$

$$\widehat{\sigma}_1^2 = \frac{\sum_{t=1}^T (\ln Y_t - \hat{\mu}_1 - \ln Y_{t-1})^2 \Pr(s_t = 0)}{\sum_{t=1}^T \Pr(s_t = 0)} \quad (24)$$

11.3 Algorithm

The algorithm for the estimation is going to work as follows: Assume values for ϑ (ϑ includes $\mu_0, \mu_1, \sigma_0, \sigma_1, \pi(0|0)$ and $\pi(0|1)$). Define $\vartheta_{(0)} = \vartheta$. Given an initial condition iterate (19) to obtain the probability that in period t $s_t = j$ given all the available data up to the moment. Then use (20) to compute the smoothed inferences (these are the probabilities that we are going to use). With this data we then can predict the timing of being in a bad state (given a particular value for ϑ). With this data estimate $\pi(0|0)$ and $\pi(0|1)$. Then with these information use (21), (22), (23) and (24) to estimate μ_0, μ_1, σ_0 and σ_1 . Stack all these estimates in $\vartheta_{(1)}$. If $\|\vartheta_{(1)} - \vartheta_{(0)}\| < \varepsilon$ then stop, otherwise go back to the first step.

This estimation procedure doesn't account for the standard errors of the different coefficients. A bootstrapping technique was used for this purpose.

12 Appendix A

United States: Estimation conditioning on μ_2

μ_2	-0.036	-0.2	-0.3317
μ_0	0.0199 (0.003)	0.0219 (0.004)	0.032 (0.003)
μ_1	0.087 (0.058)	0.022 (0.08)	0.003 (0.09)
σ_0	0.023 (0.005)	0.095 (0.003)	0.04 (0.003)
σ_1	0.046 (1.1)	0.043 (1.3)	0.05 (1.33)
σ_2	0.077 (0.02)	0.095 (0.009)	0.04 (0.01)
$\pi(0 0)$	0.89 (0.07)	0.352 (0.1)	0.634 (0.05)
$\pi(1 0)$	0.0309 (0.04)	0.586 (0.07)	0.365 (0.06)
$\pi(0 1)$	0.210 (0.2)	0.124 (0.26)	0.595 (0.3)
$\pi(1 1)$	0.473 (0.16)	0.869 (0.19)	0.378 (0.179)
$\pi(0 2)$	0.2 (0.2)	0.341 (0.14)	0.27 (0.2)
$\pi(1 2)$	0.297 (0.2)	0.616 (0.18)	0.729 (0.16)
EP	1.1%	1.8%	1.6%
R^F	9.4%	8.4%	8.3%
LR test	—	45.2***	107***

Table 1-A

Montecarlo Simulation - Economy with Cycles

	True value	Estimator	MSE		True value	Estimator	MSE
μ_0	0.03	0.03 (0.0017)	$6e^{-005}$	$\pi(0 0)$	0.9	0.894 (0.033)	0.0011
μ_1	-0.045	-0.0445 (0.0149)	0.237	$\pi(0 1)$	0.66	0.706 (0.136)	0.03
σ_0	0.015	0.0148 (0.0012)	$6e^{-005}$	EP	—	0.3% (0.0223)	e^{-003}
σ_1	0.04	0.04 (0.0276)	0.0175	r^F	—	10.76% (0.0287)	$7e^{-005}$

Table 2-A

Montecarlo Simulation - Economy with Rare Disasters

	True value	Estimator	MSE		True Value	Estimator	MSE
μ_0	0.025	0.0251 (0.002)	e^{-004}	$\pi(0 0)$	0.983	0.979 (0.018)	$3e^{-004}$
μ_1	-0.3317	-0.2917 (0.085)	0.02	$\pi(0 1)$	0.983	0.981 (0.052)	0.003
σ_0	0.02	0.0197 (0.001)	e^{-006}	EP	—	4% (0.09)	0.008
σ_1	0.02	0.176 (0.154)	e^{-005}	r^F	—	3.9% (0.09)	0.0045

Table 3-A

Montecarlo Simulation - Economy with undistinguishable states

	True value	Estimator	MSE		True Value	Estimator	MSE
μ_0	0.02	0.0173 (0.0049)	e^{-003}	σ_1	0.045	2.68 (1.37)	196
μ_1	-0.005	0.0006 (0.19)	0.7	$\pi(0 0)$	0.76	0.9645 (0.12)	0.07
σ_0	0.04	0.0397 (0.0036)	$3e^{-004}$	$\pi(0 1)$	0.62	0.9309 (0.12)	0.17

Table 4-A

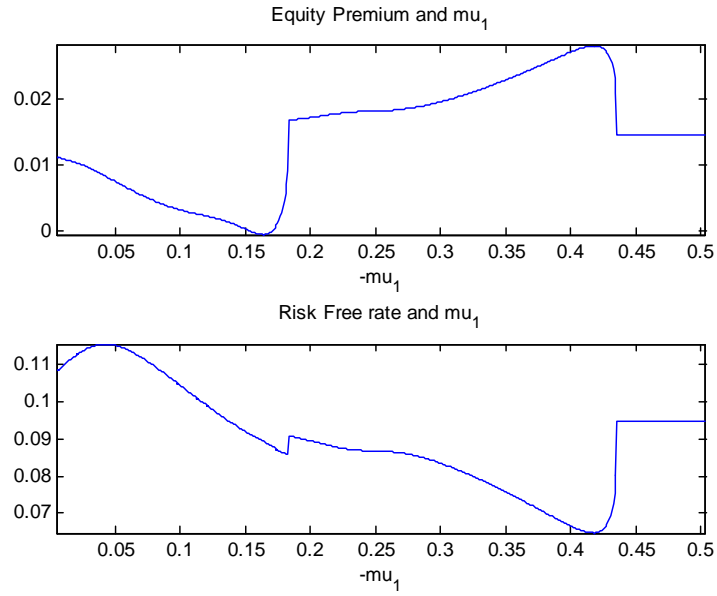


Figure 1-A

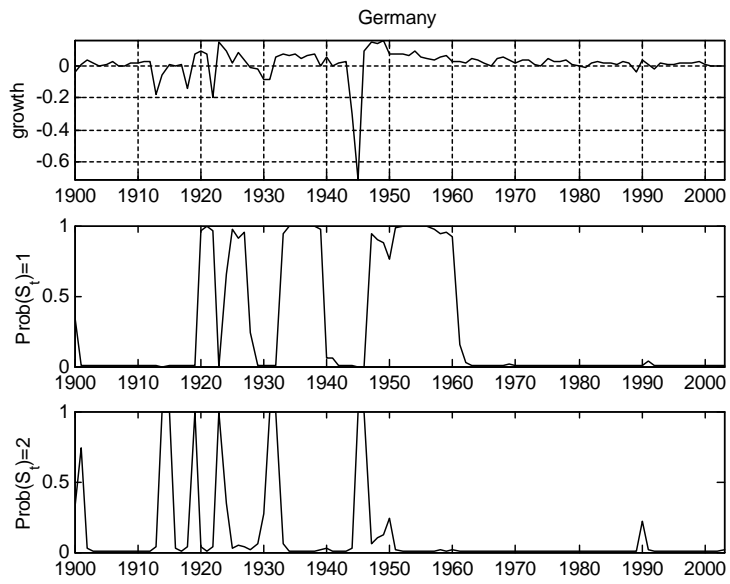


Figure 2-A

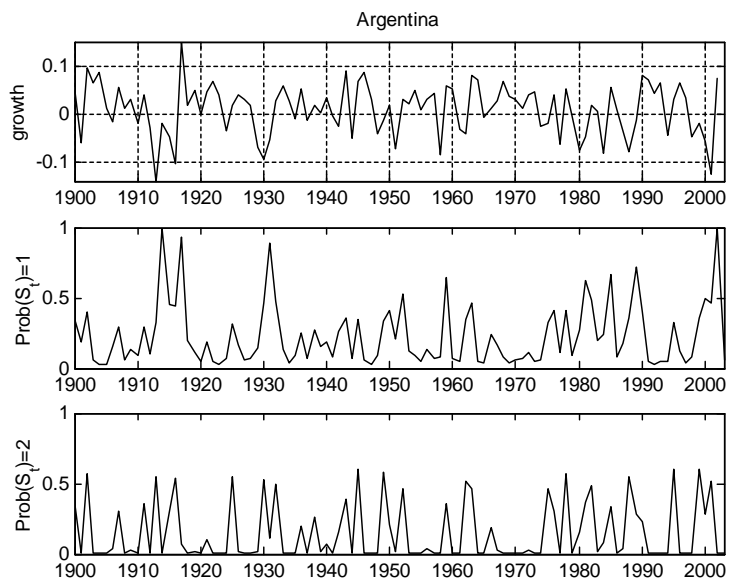


Figure 3-A

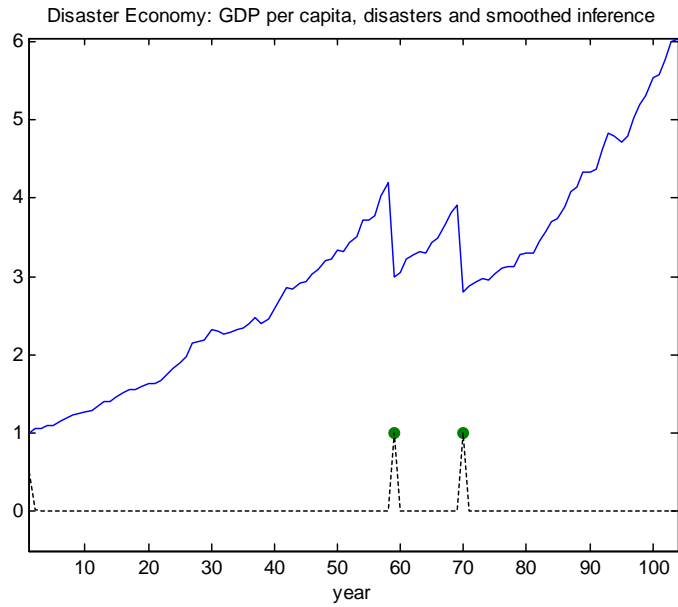


Figure 4-A

In Figure 5-A the dotted line is the smoothed inference of state 1 while the two big dots are the true moments of the crisis.

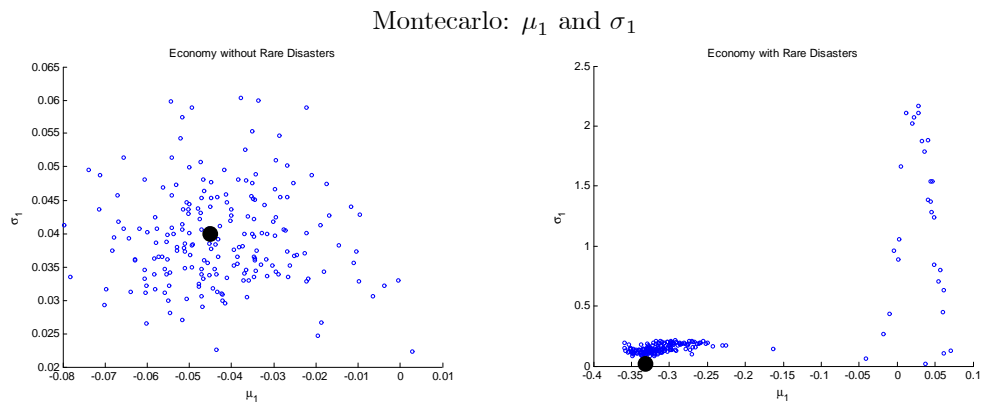


Figure 5-A

In figure 6-A the black dot is the true value while every blue dot is the pair of conditional mean - conditional variance of each simulation of the montecarlo.

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