ECON 357

Lecture 6: estimating firm level productivity

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Estimating firm-level productivity

$$Q_{it} = F(L_{it}, M_{it}, K_{it}) \exp(\omega_{it} + u_{it}^q)$$

- L_{it}, M_{it} variable factors of production (labor and intermediates).
- K_{it} capital stock.
- ω_{it} firm level productivity (unobserved).
- u_{it} unobserved shocks.

Empirical challenges

1. **Simultaneity bias:**
   - ω_{it} affects both productivity and input choices.
   - β and β_k biased upwards.
2. **Selection bias:**
   - if low productivity realization, firms with more k_{it} more likely to survive.
   - $\Rightarrow$ sprurious negative correlation between capital and productivity.
   - β_k biased downward.

Additional challenges

$$Q_{it} = F(L_{it}, M_{it}, K_{it}) \exp(\omega_{it} + u_{it}^{q})$$

1. Unobserved prices and mark-ups.
    - Traditional method: deflate sales (r_{it}) by industry prices (p_{It}).
    - Problem: $(p_{it} - p_{It}) \neq 0$ induces bias.
2. Problem: if trade affects prices and mark-ups, impact of trade on productivity misidentified.

Note 1: we will assume Cobb-Douglas for $F(\cdot)$.
Note 2: we will assume CES demand system.

Tentative solutions

1. **Olley-Pakes:** use investment decision to control for simultaneity and selection bias.
2. **Levinsohn-Petrin:** use intermediate inputs instead.
3. **de Loecker:** use variation in firm-level exposure to trade shocks to identify mark-ups.

CES demand system

$$Q_{jt} = Q_{st} \left(\frac{P_{jt}}{P_{st}} \right)^{-\eta_s} \exp \left(\xi_{jt} + u^q_{jt} \right)$$

- Good j in segment s.
- ξ_{jt}: unobserved demand shock (may be correlated with price).
- u^q_{jt}: i.i.d. demand shock.
- **Note:** constant mark-up $\left(\frac{\eta_s}{\eta_s - 1} \right)$.

Empirical challenges: summary

$$\begin{cases} \tilde{r}_{it} = \beta_0 + \beta_l l_{it} + \beta_m m_{it} + \beta_k k_{it} + \beta_s q_{st} + \omega_{it} + \xi_{it} + \varepsilon_{it} \\ \text{with } \beta_h = \left(\frac{\eta_s}{\eta_s - 1}\right) \alpha_h, \ \ h = \{l, m, k\}, \ \beta_s = \frac{1}{\eta_s}. \end{cases}$$

- ω_{it} unobserved and correlated with input choices.
- non random exit of firms (biases estimate of β_k).
- demand shock ξ_{it} correlated with input choice.
- aggregate demand shifter correlated with unobserved demand conditions.

Multi-product dimension

$$\begin{cases} Q_{ijt} = (c_{ijt})^{\gamma} Q_{it} \\ \text{with } \sum_{j \in J(i)} c_{ijt} = 1 \text{ and } c_{ijt}^{h} = c_{ijt} \text{ for } h = \{l, m, k\}. \end{cases}$$

- Firms split up their production between different products.
- Assumption: same factor intensities for all inputs.
- γ measures returns to scale.

Multi-product production function

$$
\begin{aligned}
\tilde{r}_{it} = \beta_0 + \beta_l l_{it} + \beta_m m_{it} + \beta_k k_{it} + \\
+ \sum_{s=1}^{5} \beta_s \left(s_{is} q_{st} \right) + \ln \left(\sum_{j \in J(i)} \left(c_{ijt} \right)^{\gamma/\eta} \right) \\
+ \omega_{it} + \xi_{it} + \varepsilon_{it}
\end{aligned}
$$

- Demand shocks differ across segments (q_{st}), and firms differ in their exposure to those shocks (s_{is}).

Estimation strategy

- Investment function can be inverted:

$$\omega_{it} = h_t \left(i_{it}, k_{it}, qr_{it}, np_i \right)$$

- qr_{it} represents firm i's exposure to quota reduction, np_i number of products.

Estimation

- **First stage:** (use a polynomial approximation for $\tilde{h}$)

$$\tilde{r}_{it} = \beta_0 + \beta_l l_{it} + \beta_m m_{it} + \beta_k k_{it}$$

$$+ \sum_{s=1}^{5} \beta_s \left(s_{is} q_{st} \right) + \sum_{j \in J(i)} \delta_j D_{ij} + \sum_{g \in G(i)} \delta_g D_{ig} + \tilde{\phi}_t \left(i_{it}, k_{it}, qr_{it}, np_i \right) + \varepsilon_{it}$$

$$\tilde{\phi}_t \left(i_{it}, k_{it}, qr_{it}, np_i \right) = \beta_k k_{it} + \tilde{h}_t \left(i_{it}, k_{it}, qr_{it}, np_i \right)$$

$\Rightarrow$ get $\left(\hat{\beta}_l, \hat{\beta}_m, \hat{\beta}_s, \hat{\delta}_j, \hat{\delta}_g, \widehat{\tilde{\phi}}_{it} \right)$. Note: $\hat{\eta}_s = 1/\hat{\beta}_s$.

- **Second stage:** (use a polynomial approximation for g)

$$\tilde{r}_{it} - \hat{\beta}_l l_{it} - \hat{\beta}_m m_{it} - \sum_{s=1}^{5} \hat{\beta}_s \left(s_{is} q_{st} \right) - \sum_{j \in J(i)} \hat{\delta}_j D_{ij} - \sum_{g \in G(i)} \hat{\delta}_g D_{ig}$$

$$= \beta_0 + \beta_k k_{it+1} + g_t \left(\widehat{\tilde{\phi}}_{it} - \beta_k k_{it} \right) + \left(\tilde{\xi}_{it+1} + \zeta_{it+1} + \varepsilon_{it+1} \right)$$

$\Rightarrow$ get $\hat{\beta}_k$.

Key steps in the estimation

- Control for endogenous choice of k and endogenous selection (unbiased β's).
- Control for mark-ups (η).
- Control for simultanuous impact of demand shocks on input choices, prices, quantities and selection.
- **Method:**
  1. use firm level variation in demand induced by different trade shocks.
  2. invert investment equation to control for choice of capital.

Backing out productivity

$$\hat{\omega}_{it} = \sum_s s_{is} \left(\tilde{r}_{it} - \hat{\beta}_l l_{it} - \hat{\beta}_m m_{it} - \hat{\beta}_k k_{it} - \hat{\beta}_s \left(s_{is} q_{st} \right) \right) \left(\frac{\hat{\eta}_s}{\hat{\eta}_s - 1} \right)$$

- Key estimates to uncover ω are β's and η.
- Note that segment and sub-segment fixed effects can affect productivity.

Impact of trade on productivity

$$\hat{\omega}_{it} = \lambda_0 + \lambda_1 qr_{it} + controls + \varepsilon_{it}$$

- Different estimates of $\hat{\omega}$ will give different answers.
- If quotas affect demand, not controlling for the impact of quotas on demand will overestimate λ_1.

Impact of trade on productivity

Table 8: Impact Trade Liberalization on Productivity

Specification (# obs)	Estimated coefficient	Productivity Estimated using augmented model	OP
I (1,291)	qr	-0.0637**	-0.1068*
		(0.0366)	(0.0296)
II (1,088)	$\triangle qr$	-0.0699*	-0.1254*
		(0.0312)	(0.0327)
III (765)	$\triangle qr$	-0.0455**	-0.1347*
		(0.0272)	(0.0299)
IV (890)	*level*	0.0019*	-0.0000
		(0.0008)	(0.0008)

Std errors in parentheses, * and ** denote significant at 5 or lower and 10 percent, resp.
All regressions include quota-product classification dummies (23 categories), except for VI

Figure: Jan de Loecker (2009)