ECON 357

Lecture 4: endogenous mark-ups

Thomas Chaney

University of Chicago

Exogenous or endogenous mark-ups?

Krugman's assumptions:

- CES + many firms $\Rightarrow$ constant demand elasticity.
- Constant demand elasticity $\Rightarrow$ constant desired mark-up.
- Monopolistic competition $\Rightarrow$ desired mark-up = actual mark-up.

Relaxing Krugman's assumptions:

1. CES + few firms $\Rightarrow$ variable demand elasticity (Atkeson & Burstein *AER* 2008).
2. "Non monopolistic" competition $\Rightarrow$ desired mark-up $\neq$ actual mark-up (BEJK *AER* 2003).
3. Non CES preferences $\Rightarrow$ variable demand elasticity (Melitz & Ottaviano *REStud* 2005).

CES with few firms (e.g. Atkeson & Burstein)

- Nested CES preferences.
- Many differentiated *sectors* (i), but few differentiated *firms* (n) within each sector,

$$\begin{cases} U = \left(\int_0^1 (y_i)^{\frac{\eta-1}{\eta}} \, di\right)^{\frac{\eta}{\eta-1}} \\ y_i = \left(\sum_{n=1}^N (q_{in})^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}} \end{cases}$$

with $\rho > \eta$ (more differentiation across than within sectors).

Optimal pricing (Bertrand versus Cournot)

- Simple solution to optimal pricing,

$$p_{in}(s) = \frac{\varepsilon(s)}{\varepsilon(s) - 1} \frac{W}{\varphi_{in}}$$

$$\text{with } \begin{cases} \varepsilon_{\text{Bertrand}}(s) = s\eta + (1 - s)\rho \\ \varepsilon_{\text{Cournot}}(s) = \left(s\eta^{-1} + (1 - s)\rho^{-1}\right)^{-1} \end{cases}$$

$$\text{and } s_{in} = \frac{p_{in} q_{in}}{\sum_{m=1}^{N} p_{im} q_{im}} = \frac{(p_{in})^{1-\rho}}{\sum_{m=1}^{N} (p_{im})^{1-\rho}} = \left(\frac{p_{in}}{P_i}\right)^{1-\rho}$$

Endogenous mark-ups in Dixit-Stiglitz

- **Polar cases:** small $(s \approx 0)$ versus large $(s \approx 1)$ firms,

$$\begin{cases} p(s=0) = \frac{\rho}{\rho-1}\frac{W}{\varphi} \\ p(s=1) = \frac{\eta}{\eta-1}\frac{W}{\varphi} \end{cases} \quad \text{with } \frac{\rho}{\rho-1} < \frac{\eta}{\eta-1}$$

- **Intermediate cases:** each firm takes into account its impact on sector's price index.
  1. Big firms have a larger impact on the price index than small firms.
  2. Big firms lose fewer marker shares by raising prices than small firms.
  3. Big firms charge higher mark-ups than small firms.
- **Endogenous mark-ups:** a change in market share induces a change in mark-up.

Bernard, Eaton, Jensen and Kortum (*AER* 2003)

- N asymmetric countries.
- Fixed set of sectors $\omega \in [0, 1]$, CES preferences,

$$U_n \equiv \left(\int_0^1 q_n(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}}$$

- Simple iso-elastic demand,

$$\begin{aligned} x_n(\omega) &= \left(\frac{p_n(\omega)}{P_n} \right)^{1-\sigma} X_n \\ \text{with } P_n &= \left(\int_0^1 p_n(\omega)^{1-\sigma} d\omega \right)^{\frac{1}{1-\sigma}} \end{aligned}$$

Within sectors

- CRS technology of production.
- Many firms within sector ω differ in their labor productivity.
- Within sector ω, all firms produce the same variety.
  1. only the best survives.
  2. the best charges a price $\leq$ the second best marginal cost.
  3. Only the joint distribution of best and 2^{nd} best productivity matter.

Productivity distributions

- Best productivity draws are Fréchet distributed,

$$\begin{aligned} F_i(z_1) &= \Pr\left[z_i^1(\omega) \leq z_1\right] \\ &= \exp\left(-T_i z_1^{-\theta}\right) \end{aligned}$$

- Best and 2nd best draws are jointly Fréchet distributed,

$$\begin{aligned} F_i(z_1, z_2) &= \Pr\left[z_i^1(\omega) \leq z_1; z_i^2(\omega) \leq z_2 \mid 0 \leq z_2 \leq z_1\right] \\ &= \left[1 + T_i\left(z_2^{-\theta} - z_1^{-\theta}\right)\right]\exp\left(-T_i z_2^{-\theta}\right) \end{aligned}$$

1. T_i indexes overall productivity in country i.
2. θ (same across countries) indexes (inverse of) dispersion of productivities.

Why the Fréchet?

- Take T independent draws from a Pareto distribution,

$$\Pr\left[z\left(\omega\right) \leq z\right] = z^{-\theta}$$

- The best and second best draws from T draws are jointly distributed Fréchet.
  1. Note: non Pareto fat tailed distributions converge *asympotically* to Fréchet.
  2. Note: the best of many Fréchet's is Fréchet.

Trade costs

- Iceberg transportation costs, $\tau_{in} \geq 1$, $\tau_{in} \leq \tau_{ik}\tau_{kn}$.
  1. Perfectly substitutable goods within sectors $\Rightarrow$ not all firms survive in a market.
  2. Positive trade costs $\Rightarrow$ some firms survive at home but not abroad.

Entry into markets

- Best producer in sector ω from i can deliver goods in n at a cost,

$$c_{in}^{1}(\omega) = \frac{\tau_{in} w_i}{z_i^{1}(\omega)}$$

- Consumers in n buy goods from the best supplier,

$$c_{n}^{1}(\omega) = \min_{i} \left\{ c_{in}^{1}(\omega) \right\}$$

Distribution of marginal costs

- Distribution of lowest marginal costs in country n is Fréchet,

$$
\begin{aligned}
G_n^1(c_1) &= \Pr\left[c_n^1(\omega) \leq c_1\right] \\
&= \Pr\left[\min_i \left\{c_{in}^1(\omega)\right\} \leq c_1\right] \\
&= 1 - \Pr\left[\min_i \left\{c_{in}^1(\omega)\right\} > c_1\right] \\
&= 1 - \prod_i \left[1 - G_{in}^1(c_1)\right] \\
&= 1 - \exp\left(-\Phi_n c_1^\theta\right) \\
\text{with } \Phi_n &= \sum_i T_i \left(\tau_{in} w_i\right)^{-\theta}
\end{aligned}
$$

Pricing

- Bertrand competition with homogenous goods $\Rightarrow$ limit pricing.
- Bertrand competition with differentiated goods $\Rightarrow$ constant Dixit-Stiglitz pricing.
  1. Only the best firm in ω survives.
  2. Limit pricing: it charges a price not lower than 2^{nd} lowest marginal cost.
  3. Dixit-Stiglitz: it charges a mark-up not higher than Dixit-Stiglitz mark-up.

$$
\begin{aligned}
p_n(\omega) &= \mu(\omega) \times c_n^1(\omega) = \min\left\{ c_n^2(\omega) ; \bar{\mu} c_n^1(\omega) \right\} \\
\text{with } \bar{\mu} &= \begin{cases} \frac{\sigma}{\sigma-1} & , \sigma > 1 \\ \infty & , \sigma \leq 1 \end{cases}
\end{aligned}
$$

Lowest and second lowest costs

- Either too firms from the same country are best and 2nd best, or the best is from one country, the 2nd best from another.

$$c_n^2(\omega) = \min_i \left\{ \min_{j \neq i} \left\{ c_{jn}^1(\omega) \right\}, c_{in}^2(\omega) \right\}$$

- Joint distribution of the lowest and the 2nd lowest costs from country i in country n.

$$\begin{aligned} G_{in}^c(c_1, c_2) &= \Pr\left[c_{in}^1(\omega) \geq c_1, c_{in}^2(\omega) \geq c_2\right] \\ &= \Pr\left[z_{in}^1(\omega) \leq \frac{\tau_{in} w_i}{c_1}, z_{in}^2(\omega) \leq \frac{\tau_{in} w_i}{c_2}\right] \\ &= \left[1 + T_i (\tau_{in} w_i)^{-\theta} \left(c_2^\theta - c_1^\theta\right)\right] \exp\left(-T_i (\tau_{in} w_i)^{-\theta} c_2^\theta\right) \end{aligned}$$

Joint distribution of best and second best costs

$$
\begin{aligned}
& \Pr\left[c_n^1(\omega) \geq c_1, c_n^2(\omega) \geq c_2\right] \\
= & \Pr\left[\text{or} \begin{array}{c} \text{"both the lowest and the second lowest draws are above } c_2\text{"} \\ \text{"the lowest cost is in } [c_1, c_2] \text{ and the second lowest above } c_2\text{"} \end{array}\right. \\
= & \Pr\left[\text{or} \begin{array}{c} \text{for all } i's, \text{ "}c_{in}^1(\omega) \geq c_2 \text{ and } c_{in}^2(\omega) \geq c_2\text{"} \\ \text{for some } i, \left\{ \begin{array}{c} \text{"}c_1 \leq c_{in}^1(\omega) < c_2 \text{ and } c_{in}^2(\omega) \geq c_2 \text{ in } i\text{"} \\ \text{and "}c_{kn}^1(\omega) \geq c_2 \text{ and } c_{kn}^2(\omega) \geq c_2 \text{ in } k \neq i\text{"} \end{array}\right. \end{array}\right. \\
= & \ldots \\
= & \exp\left(\Phi_n c_2^\theta\right) + \Phi_n\left(c_2^\theta - c_1^\theta\right)\exp\left(\Phi_n c_2^\theta\right)
\end{aligned}
$$

Technical steps

$$
\begin{aligned}
& \Pr\left[c_n^1(\omega) \geq c_1, c_n^2(\omega) \geq c_2\right] \\
= & \prod_i \left[1 - G_{in}(c_2, c_2)\right] \\
& + \sum_i \left\{ \left(\left[1 - G_{in}(c_1, c_2)\right] - \left[1 - G_{in}(c_2, c_2)\right]\right) \prod_{k \neq i} \left[1 - G_{kn}(c_2, c_2)\right] \right\} \\
= & \prod_i e^{-T_i(\tau_{in} w_i)^{-\theta} c_2^\theta} \\
& + \sum_i \left\{ T_i (\tau_{in} w_i)^{-\theta} \left(c_2^\theta - c_1^\theta\right) e^{-T_i(\tau_{in} w_i)^{-\theta} c_2^\theta} \prod_{k \neq i} e^{-T_k(\tau_{kn} w_k)^{-\theta} c_2^\theta} \right\} \\
= & \exp\left(\Phi_n c_2^\theta\right) + \Phi_n \left(c_2^\theta - c_1^\theta\right) \exp\left(\Phi_n c_2^\theta\right)
\end{aligned}
$$

Distribution of mark-ups

- For all $\mu's$ such that $1 \leq \mu \leq \bar{\mu}$,

$$
\begin{aligned}
\Pr\left[\mu_n(\omega) \leq \mu | c_n^2(\omega) = c_2\right] &= \Pr\left[\frac{c_2}{\mu} \leq c_n^1(\omega) \leq c_2 | c_n^2(\omega) = c_2\right] \\
&= \frac{\int_{c_2/\mu}^{c_2} \frac{\partial^2 G_n}{\partial c_1 \partial c_2}|_{c_1,c_2} dc_1}{\int_0^{c_2} \frac{\partial^2 G_n}{\partial c_1 \partial c_2}|_{c_1,c_2} dc_1} \\
&= \frac{c_2^\theta - (c_2/\mu)^\theta}{c_2^\theta} \\
&= 1 - \mu^{-\theta}
\end{aligned}
$$

- Allowing the possibility of Dixit-Stiglitz mark-ups,

$$
H(\mu) = \Pr\left[\mu_n(\omega) \leq \mu\right] = \begin{cases} 1 - \mu^{-\theta} & , \ 1 \leq \mu \leq \bar{\mu}, \\ 1 & , \ \mu > \bar{\mu} = \frac{\sigma}{\sigma - 1} \end{cases}
$$

Average versus individual mark-ups

- Distribution of mark-ups is invariant (knife-edge but interesting case).
- Mark-ups depend on θ and σ:
  1. Lower θ, more dispersed productivity, higher distance between best and 2^{nd} best, higher mark-ups.
  2. Lower σ, less elastic demand (across sectors), higher mark-ups.
- Reduction in trade barriers (lower τ):
  1. If 2 best firms remain domestic, mark-up **unchanged**.
  2. If best firm domestic, better foreign competitors, mark-up goes **down**.
  3. If best firm becomes foreign, mark-up may go **up or down**.
  4. If best firm foreign, mark-up goes **up**.
  5. If 2 best firms remain foreign, mark-up **unchanged**.
  6. With Fréchet distributed productivities, all those effects cancel out.

Measured productivity

- Simple measured productivity, output per worker, is simply mark-up,

$$\frac{\text{output}\,(z_1)}{\text{workers}\,(z_1)} = \mu\,(z_1)$$

- Observed distribution of productivity,

$$H_{in}\,(\mu \mid z_1) = \Pr\left[\mu_{in}\,(\omega) \leq \mu \mid z^1_{in}\,(\omega) = z^1_n\,(\omega) = z_1\right]$$
$$= \begin{cases} 1 - \exp\left[-\Phi_n\left(\mu^\theta - 1\right)\left(\tau_{in} w_i\right)^\theta z_1^{-\theta}\right] & , \; 1 \leq \mu \leq \bar{\mu}, \\ 1 & , \; \mu > \bar{\mu} = \frac{\sigma}{\sigma - 1} \end{cases}$$

1. More productive firms (higher z_1) charge higher mark-ups *on average*.
2. The easier it is to export (τ_{ij} lower or w_i lower), the higher the mark-up, *on average*.
3. The tougher the market (lower Φ_n), the lower the mark-up, *on average*.
4. The tougher the market structure (θ or σ higher), the lower the mark-up (first order stochastic dominance).

Efficiency and export status

- Not all firms export, and exporters are larger and more productive *on average*.
  1. To produce domestically, must be the best at home,
  $$z_i^1(\omega) \geq z_k^1(\omega) \frac{w_i}{\tau_{ki} w_k}, \ \forall k \neq i$$
  2. To export, must be the best abroad (after incurring the trade barrier τ_{in}),
  $$z_i^1(\omega) \geq z_k^1(\omega) \frac{\tau_{in} w_i}{\tau_{kn} w_k}, \ \forall k \neq i$$
  3. Triangular inequality, $\frac{1}{\tau_{ki}} \leq \frac{\tau_{in}}{\tau_{kn}}$, it is harder to export than to sell domestically,
  $$z_k^1(\omega) \frac{w_i}{\tau_{ki} w_k} \leq z_k^1(\omega) \frac{\tau_{in} w_i}{\tau_{kn} w_k}$$
- **Note:** since mark-ups abroad are lower than at home, firms "lose" productivity ($\frac{\text{global output}}{\text{total workers}}$) when they "become" exporters, as in Melitz.

Aggregate exports

- Aggregate exports,

$$\frac{X_{in}}{X_n} = \pi_{in} = \frac{T_i \left(\tau_{in} w_i\right)^{\theta}}{\Phi_n}$$

  1. Aggregate exports unaffected by mark-ups (Eaton-Kortum).
  2. Aggregate exports as in Krugman or Chaney.

- Welfare,

$$\begin{cases} P_n^{-1} = \gamma \times \Phi_n^{1/\theta} \\ \gamma = \left[\frac{1+\theta-\sigma+(\sigma-1)\bar{\mu}^{-\theta}}{1+\theta-\sigma} \Gamma\left(\frac{1+2\theta-\sigma}{\theta}\right)\right]^{1/(\sigma-1)} \end{cases}$$

  1. Imperfect competition distortions summarized in γ.
  2. $\gamma|_{\text{Perfect competition}} > \gamma|_{\text{Bertrand competition}} = \Gamma\left(\frac{1+2\theta-\sigma}{\theta}\right)^{1/(\sigma-1)}$.
  3. $\partial\left(\frac{\gamma_{\text{Perfect}}}{\gamma_{\text{Bertrand}}}\right)/\partial\sigma, \partial\gamma\left(\frac{\gamma_{\text{Perfect}}}{\gamma_{\text{Bertrand}}}\right)/\partial\theta > 0$.