Lecture 2:
Basic Models of Trade

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Econ 357 - **International Trade (Ph.D.)**

1 Introduction

In this class, we will see two papers that will be used as building blocks of most of this class. Dornbusch, Fischer and Samuelson (1977) develop an elegant formulation of the Ricardian theory of comparative advantages in trade with a continuum of goods (and one single factor of production). This will be used among others as the starting point of the Econometrica paper by Eaton and Kortum. Krugman (1980), simplifying Krugman (1979), introduces a radically different motive for trade: if goods are differentiated, and production takes place under increasing returns to scale, then even exactly identical countries will trade with one another, and benefit from opening up to trade.

2 Dornbusch, Fischer and Samuelson (1977)

As seen in Bob Lucas' class.

3 Krugman (1980)

In the previous models we have seen (the Ricardian model of trade), Anderson's model of trade with goods differentiated by country of origin (the Armington hypothesis), as well as in the Heckscher-Ohlin model, the motive for trade is differences between countries. Consumers in every country is willing to buy goods from abroad, because goods from abroad are different. They are different either by assumption (the Armington assumption), or they are different because they are cheaper: in the Ricardian model, or in the Heckscher-Ohlin model, different countries have different technologies, and therefore there is scope for at least some partial specialization. In the Ricardian model, countries literally have different technologies; in the Heckscher-Ohlin model, they have different factor endowments, which allows them to use different combinations of those endowments, and achieve a higher efficiency in some sectors.

However, these models are at odds with at least part of the empirical evidence on trade. First, the famous gravity equations. It seems that overall, size and distance matter much more than any difference in technology, or difference in factor endowment. Second, and that was the motivation for Krugman's paper, we observe a huge lot of intra-industry trade among rich countries. No matter how much you disaggregate the trade data, they seem to export goods in exactly the same sectors as goods that they import. Countries that seem to have a very similar technology, and a very similar structure of factor endowments, do export the same kind of stuff that they export. France and Germany buy the same type of cars from one another.

Krugman (1980) offers an entirely new approach to international trade, and to the motives for international trade. He develops a very simple (simplistic) model of trade in differentiated good with increasing returns to scale. In this model, unlike the Ricardian model, or the Heckscher-Ohlin model, even exactly identical countries would trade with one another, and would gain from trade.

One thing to note. The original Krugman paper is very clear about all the simplifying assumptions that he makes: CES preferences, monopolistic competition, and iceberg transportation costs (or multiplicative transportation costs more generically). He is very specific about the fact that he is making extreme assumptions in order to get simple analytical solutions, and get a precise understanding of the forces he is describing. This model being so simple, it has been heavily (over-)used in the trade literature, and people may have tend to forget about those extreme simplifying assumptions.

Dixit-Stiglitz preferences, Krugman in autarky

Krugman uses the very simple model of monopolistic competition with iso-elastic preferences developed by Dixit and Stiglitz (1977). The very extreme simplifying assumptions embedded in the model allows for very neat analytical solutions.

I present here a slightly simplified/different version from the original Krugman paper. I assume there is a continuum of goods (which allows for exact solutions, and not approximate ones). The notations are modified to be consistent with the rest of the class.

Important assumptions:

- Isoelastic preferences.
- Monopolistic competition.
- Increasing returns to scale.

Let's consider first an economy in autarky. This economy is populated by a mass L of identical agents, who work, consume, and own domestic firms[1]. It is isomorphic to an economy with a single representative consumer, which supplies L units of labor on a competitive labor market, and independent profit maximizing producers. The representative consumer derives utility from consuming a continuum of differentiated goods, which are paid for using his labor income. If she has access to all varieties ω in the set Ω (to be determined endogenously in equilibrium), she maximizes,

$$\max_{q(\omega)} U \equiv \left(\int_{\Omega} q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}} \tag{1}$$

$$\text{s.t.} \quad \int_{\Omega} p(\omega)\, q(\omega)\, d\omega = wL$$

These isoelastic preferences give the simple isoelastic demand structure for each commodity ω,

$$q(\omega) = \left(\frac{p(\omega)}{P} \right)^{-\sigma} \frac{wL}{P} \tag{2}$$

$$\text{with } P = \left(\int_{\Omega} p(\omega)^{1-\sigma} d\omega \right)^{\frac{1}{1-\sigma}} \tag{3}$$

Each firm sells a single variety ω. It is a monopolist for that variety. Production takes place under increasing returns to scale. For simplicity, this is modelled as a fixed cost of starting production, f units of labor, and a variable cost, $1/\varphi$ units of labor per unit produced[2],

$$l(q) = f + \frac{q}{\varphi} \tag{4}$$

Given the technology that the firm faces, and given the isoelastic demand for each variety, the optimal pricing strategy for each firm is to charge a constant mark-up over marginal cost,

[1] In this paper, since profits of each firm will be driven to zero, the ownership structure actually does not matter.

[2] Given this technology, we can actually assume Cournot or Bertrand competition, and in equilibrium, there will only be one producer per variety ω.

$$p(\omega) = \frac{\sigma}{\sigma - 1}\frac{w}{\varphi} \tag{5}$$

This is a very simple, and at the same time bothering result. No matter how much competition firms are facing, i.e. no matter how many varieties are available, the elasticity of substitution between any two varieties is constant, so that firm always charge the same price. This is a nice properties of the Dixit-Stiglitz model as it simplifies greatly the life of the modeler, but it is obviously in contradiction with realism. Given this pricing strategy, the firm selling $q(\omega)$ units of good ω earns net profits $\pi(\omega)$,

$$\begin{aligned} \pi(\omega) &= p(\omega)\, q(\omega) - \left(wf - \frac{wq(\omega)}{\varphi}\right) \\ &= w\left(\frac{q(\omega)}{(\sigma - 1)\,\varphi} - f\right) \end{aligned} \tag{6}$$

We assume that there is free entry. As long as there are some positive profits to be earned, new firms will be started. As more firms are started, consumers substitute at least part of their spending from existing suppliers towards those new suppliers, which depresses the profits of existing firms. This goes on until profits are driven down to zero.

$$\pi(\omega) = 0 \Rightarrow q(\omega) = (\sigma - 1)\,\varphi f \tag{FE}$$

The model is symmetric so that each firm charges the same price, sells the same quantity, and earns the same profits. To determine the total number of firms in the economy, we simply have to impose labor market clearing,

$$n\left(f + \frac{q}{\varphi}\right) = L \Rightarrow n = \frac{L}{\sigma f} \tag{LMC}$$

Among other things, we can already see that the ideal price index, which is an inverse measure of welfare, is increasing in the size of the country. Larger countries produce more varieties, and since consumers do have a preference for variety, they are better off,

$$P = \frac{\sigma w}{(\sigma - 1)\,\varphi}\left(\frac{L}{\sigma f}\right)^{\frac{1}{1-\sigma}} \tag{7}$$

Trade equilibrium

Main assumption of the trade model:

- Multiplicative transportation costs.

We can now open up trade between any two similar economies. Let's assume that each country has access to the same technology, that consumers in each country share the same preferences. The only difference between the two countries is their size, L and L^*. These two countries may trade with one another, at some cost. Precisely, transportation costs are assumed to be of the iceberg type. If one unit of good is shipped abroad, only a fraction $1/\tau$ ($\tau > 1$) arrives.

The constant elasticity preferences, and the assumption of multiplicative transportation costs give us a very nice property, even though rather unrealistic: in this integrated world, there will be exactly as many firms as in autarky, and each firm will be producing the exact same quantity as in autarky. The only difference will be in the consumption pattern: consumers now consume some foreign goods, and are therefore better off.

Because they are facing the same elasticity at home and abroad, firms will charge a constant mark-up over marginal cost both at home and abroad. The multiplicative transportation cost means that the marginal cost of selling one unit of good abroad is just the marginal cost of producing one unit of good domestically, multiplied by the iceberg cost τ. Domestic firms charge a price p for their own market,

$$p = \frac{\sigma}{\sigma - 1}\frac{w}{\varphi}$$

and a price τp if they sell abroad. The price set by foreign firms is similarly defined by p^*. The price indices at home and abroad are given by,

$$P^{1-\sigma} = np^{1-\sigma} + n^* \left(\tau p^*\right)^{1-\sigma}$$
$$P^{*1-\sigma} = n \left(\tau p\right)^{1-\sigma} + n^* p^{*1-\sigma}$$

Given those prices, firms produce $q^D(p)$ units for their domestic market, and export $\tau q^X(\tau p)$ units abroad (remember that they have to produce τ units for each unit sold abroad). The total production of each firm is $q = q^D + \tau q^X$, and the total net profits they earn is, after rearranging, $\pi = \frac{w}{\sigma\varphi} q - wf$. Exactly as in autarky, free entry drives down profits to zero, so that,

$$\pi = 0 \Leftrightarrow q = (\sigma - 1)\varphi f$$

As in autarky, labor market clearing condition requires that total domestic production equals domestic labor, and a similar condition abroad,

$$\begin{cases} n\left(f+\frac{q}{\varphi}\right)=L \\ n^{*}\left(f+\frac{q}{\varphi}\right)=L^{*} \end{cases} \Rightarrow \begin{cases} n=\frac{L}{\sigma f} \\ n^{*}=\frac{L^{*}}{\sigma f} \end{cases} \qquad (LMC)$$

Note the very interesting (and unappealing) result that the scale of production is unaffected by the opening to trade. Both before and after, each firm produces exactly the same amount of goods. The only variable of adjustment is the number of goods consumed: once trade opens up, consumers have access to foreign goods, and they are better off. In this extreme case, only the extensive margin of trade adjusts. We can get intermediate results (both the scale of production and the number of goods adjusting) if we move away from constant elasticity preferences. Krugman (1979), in a much more complicated model, gets this intermediate result. Preferences are such that the demand that each individual firm faces becomes more elastic when more goods are available. This makes some sense: as more and more goods become available, consumers can more easily substitute between them. Additional goods in a sense are less differentiated, or more precisely, make all goods less differentiated. Once this non constant elasticity is present, as trade opens up, firms will produce larger quantities. There will therefore be fewer firms producing, but still more products being consumed (imports plus domestically produced goods).

If we define the relative wage between the two countries as $\omega=\frac{w}{w^{*}}$, we can now easily solve for the aggregate exports (nominal exports, *f.o.b.*, that is not including transportation costs), $X_{f.o.b.}=n\tau p q^{X}(\tau p)$, which gives, after rearranging,

$$X_{f.o.b.}=\lambda\times L\times L^{*}\times\left(\frac{\tau\omega}{P^{*}}\right)^{1-\sigma} \qquad (8)$$

with λ a scaling constant[3]. We can actually alternatively solve for the price index (as a function of relative wages), $P^{*1-\sigma}=\lambda\left((\tau\omega)^{1-\sigma}L+L^{*}\right)$.

Intensive versus extensive margin:

Note an important prediction of the model. When a country goes from autarky to some costly trade (τ goes from $+\infty$ to some number above 1), all the new trade comes on the extensive margin. Consumer do not consume more of the goods they used to consume when their country was in autarky, n remains unchanged. They actually consume

[3] $\lambda=\frac{1}{\sigma f}\left(\frac{\sigma}{(\sigma-1)\varphi}\right)^{1-\sigma}$.

less of each (but the same number of those domestically produced goods), and they just start consuming new goods imported from abroad (all of the n^* foreign varieties). However, when a country further liberalizes trade, in the sense that trade barriers go down (τ goes down), the set of imported goods remains exactly unchanged. n^* is constant. Consumers only consume more of each imported variety. We will see later in the class that this prediction is strongly violated in the data, and we will see how introducing some heterogeneity between firms, and some selection of firms into the export market, may explain an increase of trade on both the intensive and the extensive margins.

Wages and market size (the seed of Economic Geography):

The relative wage will be pinned down by the equilibrium in the goods market. There are three alternative ways of specifying the goods market clearing condition: 1/ market clearing for home goods, 2/ market clearing for foreign goods, 3/ trade balance. The most informative way is the latter, to equalize imports and exports of each country, $X_{f.o.b.} = X^*_{f.o.b.}$, which gives,

$$\left(L + \left(\frac{\tau}{\omega}\right)^{1-\sigma} L^*\right)\omega^{2(1-\sigma)} = (\omega\tau)^{1-\sigma} L + L^* \qquad \text{(Trade Balance)}$$

$$\Rightarrow \begin{cases} \text{if } L = L^*,\ \omega = 1 \\ \text{if } L > L^*,\ \omega > 1 \end{cases}$$

Larger countries have higher wages. Because of the presence of trade barriers, having a larger domestic market implies having a larger relative demand from domestic products, and this drives domestic wages up. Note that this is only true for positive trade barriers ($\tau > 1$). In the presence of trade barriers, there is an incentive to save on transportation costs, and locate close to final demand, which drives the demand for workers in the larger country up, and pushes their wages up.

This finding is the starting point of later work by Paul Krugman on Economic Geography: if one allows for some mobility of factors of production, then there are endogenous agglomeration forces that drive factors towards the larger markets.

Welfare:

The welfare of the representative consumer in the home country is directly given by the price index: $W = P^{-1} = \lambda^{\frac{1}{1-\sigma}} \left((\tau\omega)^{1-\sigma} L + L^*\right)^{\frac{1}{1-\sigma}}$. Keeping the relative wage constant (sort of partial equilibrium analysis), we see that reducing trade barriers will increase welfare, since it allows consumers to have access to the foreign varieties for a cheaper price.

Increasing the size of either economies also increase welfare, since it expands the number of varieties available.

Costless trade:

In the case of costless trade, we can fully solve the model in closed form. In the case of costless trade, wages are the same in each country, as are the prices of any differentiated good. Total trade between the two countries, if we normalize wages to unity, is then,

$$X = \frac{LL^*}{L + L^*} \tag{9}$$

We can easily generalize this results for costless trade with N countries, each of size L_n (potential different). Then, trade from country i to country j is,

$$X_{ij} = \frac{L_i L_j}{L^W} \tag{10}$$

$$\text{with } L^W = \sum_{n=1}^{N} L_n$$

Equations (8), (9) and (10) are three different versions of the gravity equations. As in Anderson and van Wincoop, we see that size matters, trade barriers matter, but they matter only in their relative size compared to the relative wage (a measure of the "outward resistance to trade"), and the price index of the destination country (a measure of the "inward resistance to trade").

3.1 Home market effect (Helpman Krugman, chapter 10-4)

The second part of Krugman's (1980) paper gets predictions for the home market effect.

Home market effect: increasing returns to scale industries will tend to locate in the countries with the largest market, and export their goods to other countries.

The following simplified version of the initial model, developed in the Helpman-Krugman book, derives this property in a special case of costly trade (the original Krugman model is messier). One way to get very simple closed form solutions in the case of costly trade is to make an additional simplifying assumption. This (very ugly) trick is used extensively in trade models.

Let's assume that there are two types of goods produced: a differentiated good (same as the one we have seen up til now), and a homogenous good. Consumers spend a constant share of their income on each of these goods, μ on the differentiated good, and $(1-\mu)$ on the homogenous good. In other words, they have Cobb-Douglas preferences over each type of good, and their subutility for the differentiated good is a CES aggregate with elasticity of substitution σ.

Extreme simplifying assumption (very popular in trade papers):

- The homogenous good is produced under constant returns to scale, it is costlessly traded between the two countries, and in the trade equilibrium, each country produces some of it. This assumption allows us to pin down the wage rate, and equalize it between the two countries (up to a constant, the relative labor productivity in the homogenous sector).

It can be shown that in equilibrium, all firms in the world are charging the same price, and that they are producing the same amount of goods (you should try and prove that). If each firm produces a quantity q and sells it at a price (f.o.b.) p, total output at home is,

$$nq = \frac{np^{\sigma}}{np^{1-\sigma} + n^{*}(\tau p)^{1-\sigma}} \mu w L + \frac{n(\tau p)^{\sigma}}{n(\tau p)^{1-\sigma} + n^{*}p^{1-\sigma}} \mu w L^{*} \tau$$

and a similar equation for total output abroad.

If we assume that the unit labor requirement in the homogenous sector is 1, set the price of the homogenous good to 1, then wages are equal to 1. For simplicity, we can arbitrarily set $\varphi = (\sigma - 1)/\sigma$, so that $p = 1$. We then get the following market clearing conditions,

$$\begin{cases} \frac{q}{\mu} = \frac{1}{n+n^{*}\tau^{1-\sigma}} L + \frac{\tau^{1-\sigma}}{n\tau^{1-\sigma}+n^{*}} L^{*} \\ \frac{q}{\mu} = \frac{\tau^{1-\sigma}}{n+n^{*}\tau^{1-\sigma}} L + \frac{\tau^{1-\sigma}}{n\tau^{1-\sigma}+n^{*}} L^{*} \end{cases} \qquad (LMC)$$

We can now solve for n and n^{*}. There are 3 possible cases: 1/ the home country is the only producer of differentiated good, 2/ the foreign country is the only producer of the differentiated good, 3/ each country produces some differentiated good. Whether we are in any of these 3 cases depends on the relative size of each market, on the size of trade costs.

If we define $s_n = n/(n+n^{*})$, and $s_L = L/(L+L^{*})$, the share of the home country in the global output of differentiated good, and in global labor, respectively, we get the following simple solutions for the share of differentiated output produced domestically,

$$s_n = \begin{cases} 0 & \text{for } s_L \leq \frac{\tau^{1-\sigma}}{1+\tau^{1-\sigma}} \\ \frac{(1+\tau^{1-\sigma})s_L - \tau^{1-\sigma}}{1-\tau^{1-\sigma}}, & \text{for } \frac{\tau^{1-\sigma}}{1+\tau^{1-\sigma}} \leq s_L \leq \frac{1}{1+\tau^{1-\sigma}} \\ 1 & \text{for } s_L \geq \frac{1}{1+\tau^{1-\sigma}} \end{cases}$$

If too large a fraction of the world labor force is located in either country $\left(s_L \leq \frac{\tau^{1-\sigma}}{1+\tau^{1-\sigma}} \text{ or } s_L \geq \frac{1}{1+\tau^{1-\sigma}}\right)$, then the whole output of differentiated good will be produced in the larger country. For intermediate distributions of the global labor force $\left(\frac{\tau^{1-\sigma}}{1+\tau^{1-\sigma}} \leq s_L \leq \frac{1}{1+\tau^{1-\sigma}}\right)$, a disproportionate fraction of differentiated output will be produced in the larger country, so that it will be a net exporter of that good (the slope of s_n as a function of s_L is steeper than the 45° line). One can see that when trade costs decrease,the band of diversification of the global production of differentiated goods shrinks, and it becomes more and more likely that the production of good will agglomerate in a single location (for $\tau \approx 1$, production of the differentiated product will only happen in the largest country, no matter how small the difference in the size of the two countries).