Lecture 12:
Contracts and Vertical FDI

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Econ 357 - **International Trade (Ph.D.)**

In the models we have seen so far, firms are defined as technological (or property right) entities. In the simplest Krugman model, a firm corresponds to a variety. Only one firm has the ability to produce a given variety (because of the existence of fixed cost of developping new varieties, there will never be two firms producing the same variety). One can either think that this firm is the only one with the know-how to produce this specific variety, or that it is the only one owning the property right of this variety. In the extended Melitz model, a firm corresponds both to a variety, and a technology of production. Once again, this is merely a technological definition of a firm. A firm corresponds to the entity that is able to produce this specific variety, and it has a given technology to produce this variety, none of which are transferable between firms. In the Helpman, Melitz and Yeaple model of multinational firms that we saw, once again, a firm corresponds to a variety, and a technology to produce that one variety, with the extra possibility of splitting production between different countries. The technological boundaries of firms have been extended, but we still have nothing to say about the legal boundaries of firms. Whether a firm own foreign facilities, or whether is sells off a licence to produce its variety with its productivity is indeterminate.

In all those models, there was no strict definition of the legal boundaries of firms. If one were to open up financial markets, and allow the purchase of firms, the price of a firm would exactly equal the expected discounted sum of future profits generated by this firm, so that the "legal" ownership would be indeterminate.

Strictly speaking, this is actually not exactly correct. In a world with asymmetric random shocks hitting different economies, such as in Ghironi and Melitz, households in a country will be subject to some risk, and they may be willing to diversify away those risks by investing in foreign firms. In such a case, the ownership structure may no longer

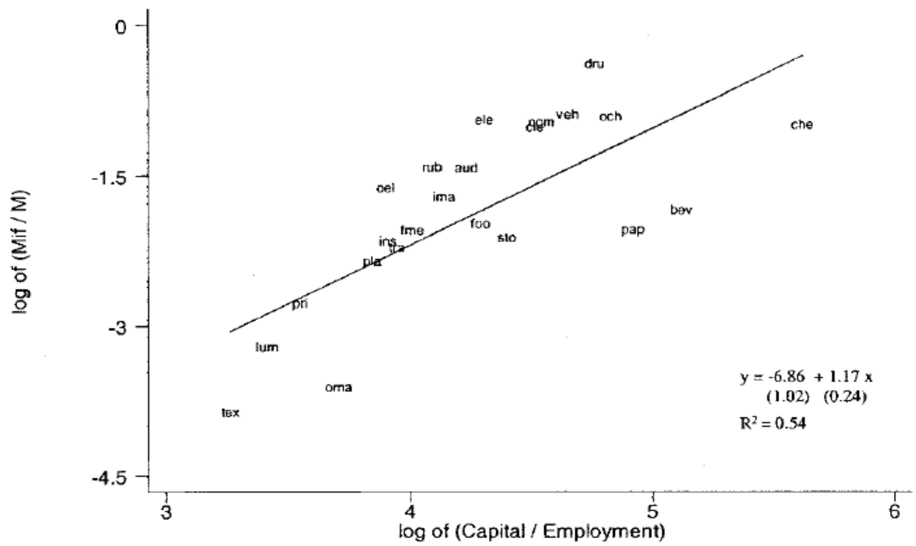

Figure 1: Share of Intrafirm Imports, in Industries with Different Capital Intensities.

be indeterminate. Actually, even in Ghironi and Melitz, there is no such motive for portfolio diversification. The authors do not solve for the optimal allocation of portfolio in a world with uncertain productivity shocks, they only solve for the transitional dynamics with no uncertainty, after the realization of an unexpected shock. However, even in such a world, because we have always assumed competitive labor markets, households may want to diversify the international portfolio of assets, but they would always own all firms in a given country. They may hold more of less of assets from different countries, but never more or less of an individual firm. So even in such a case, the legal boundaries of individual firms would be indeterminate.

1 Antras (2003)

$$U = \left(\int_0^{n_Y} y(i)^\alpha \, di \right)^{\frac{\mu}{\alpha}} \left(\int_0^{n_Z} z(i)^\alpha \, di \right)^{\frac{1-\mu}{\alpha}}, \ \mu, \alpha \in (0,1)$$

$$y(i) = A_Y p_Y(i)^{-1/(1-\alpha)}$$
$$z(i) = A_Z p_Z(i)^{-1/(1-\alpha)}$$

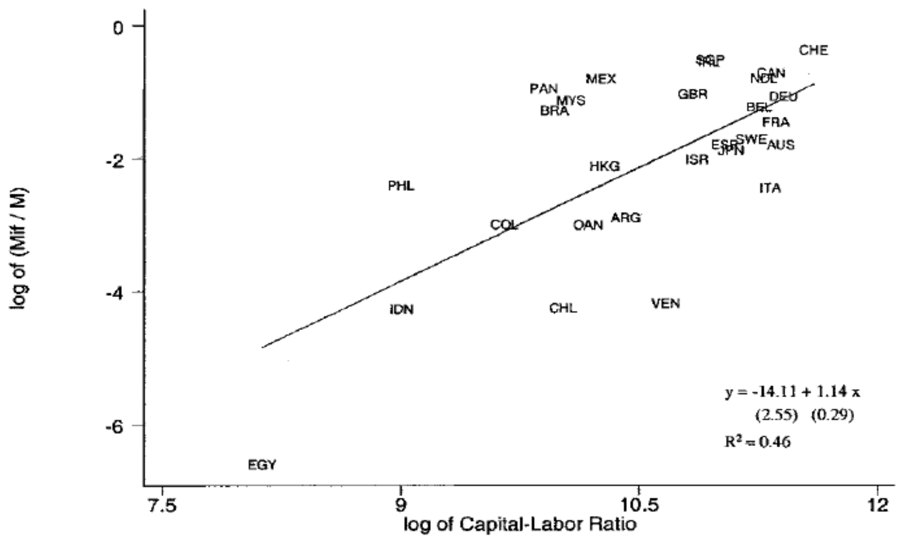

Figure 2: Share of Intrafirm Imports from Countries with Different Capital/Labor Ratios.

$$R_Y(i) = p_Y(i)\, y(i) = A_Y^{1-\alpha} y(i)^{\alpha}$$
$$R_Z(i) = p_Z(i)\, y(i) = A_Z^{1-\alpha} y(i)^{\alpha}$$

$$y(i) = x_Y(i)$$
$$z(i) = x_Z(i)$$

$$x_k(i) = \left(\frac{K_x(i)}{\beta_k}\right)^{\beta_k} \left(\frac{L_x(i)}{1-\beta_k}\right)^{1-\beta_k}$$
$$k \in \{Y, Z\}$$
$$1 > \beta_Y > \beta_Z > 0$$
$$f r^{\beta_k} w^{1-\beta_k},\ k \in \{Y, Z\}$$

final good producer: F

supplier: S

lump sum transfer $T_k(i)$ from S to F

$$\bar{\phi} = \delta^{\alpha} + \phi(1-\delta^{\alpha}) > \phi$$

$$\max_{\tilde{\phi} \in \{\phi, \bar{\phi}\}} R_k\left(K_x\left(\tilde{\phi}\right), L_x\left(\tilde{\phi}\right)\right) - rK_x\left(\tilde{\phi}\right) - wL_x\left(\tilde{\phi}\right) - f r^{\beta_k} w^{1-\beta_k}$$
$$s.t. \begin{cases} K_x\left(\tilde{\phi}\right) = \arg\max_{K_x} \tilde{\phi} R_k\left(K_x, L_x\left(\tilde{\phi}\right)\right) - rK_x \\ L_x\left(\tilde{\phi}\right) = \arg\max_{L_x} \left(1-\tilde{\phi}\right) R_k\left(K_x\left(\tilde{\phi}\right), L_x\right) - wL_x \end{cases}$$

$$p_{k,V}=\frac{1}{\bar{\phi}^{\beta_k}\left(1-\bar{\phi}\right)^{1-\beta_k}}\frac{r^{\beta_k}w^{1-\beta_k}}{\alpha}>\frac{r^{\beta_k}w^{1-\beta_k}}{\alpha}=p_k^*$$
$$p_{k,O}=\frac{1}{\phi^{\beta_k}\left(1-\phi\right)^{1-\beta_k}}\frac{r^{\beta_k}w^{1-\beta_k}}{\alpha}>\frac{r^{\beta_k}w^{1-\beta_k}}{\alpha}=p_k^*$$

$$\Theta\left(\beta_k\right)=\frac{\pi_{k,V}^F+fr^{\beta_k}w^{1-\beta_k}}{\pi_{k,O}^F+fr^{\beta_k}w^{1-\beta_k}}$$

$$\Theta\left(\beta_k\right)=\left(1+\frac{\alpha\left(1-\phi\right)\delta^{\alpha}\left(1-2\beta_k\right)}{1-\alpha\left(1-\beta_k\right)+\alpha\phi\left(1-2\beta_k\right)}\right)\left(1+\frac{\delta^{\alpha}}{\phi\left(1-\delta^{\alpha}\right)}\right)^{\frac{\alpha\beta_k}{1-\alpha}}\left(1-\delta^{\alpha}\right)^{\frac{\alpha}{\alpha-1}}$$

$$\Theta\left(\hat{\beta}\right)=1$$
$$\beta_Y>\hat{\beta}>\beta_Z$$

$$E=rK+wL$$

$$\sum_{k\in\{Y,Z\}}n_k\left(K_{x,k}+K_{f,k}\right)=K$$
$$\sum_{k\in\{Y,Z\}}n_k\left(L_{x,k}+L_{f,k}\right)=L$$

$$\frac{w}{r}=\frac{\sigma_L}{1-\sigma_L}\frac{K}{L}=\frac{\mu\left(1-\tilde{\beta}_Y\right)+\left(1-\mu\right)\left(1-\tilde{\beta}_Z\right)}{\mu\tilde{\beta}_Y+\left(1-\mu\right)\tilde{\beta}_Z}\frac{K}{L}$$

$$\tilde{\beta}_Y=\beta_Y\left(1+\alpha\left(1-\beta_Y\right)\left(2\bar{\phi}-1\right)\right)$$
$$\tilde{\beta}_Y=\beta_Z\left(1+\alpha\left(1-\beta_Z\right)\left(2\phi-1\right)\right)$$

$$E=rK+wL$$

$$\sum_{k\in\{Y,Z\}}n_k\left(K_{x,k}+K_{f,k}\right)=K$$
$$\sum_{k\in\{Y,Z\}}n_k\left(L_{x,k}+L_{f,k}\right)=L$$

$$\frac{w}{r}=\frac{\sigma_L}{1-\sigma_L}\frac{K}{L}=\frac{\mu\left(1-\tilde{\beta}_Y\right)+\left(1-\mu\right)\left(1-\tilde{\beta}_Z\right)}{\mu\tilde{\beta}_Y+\left(1-\mu\right)\tilde{\beta}_Z}\frac{K}{L}$$

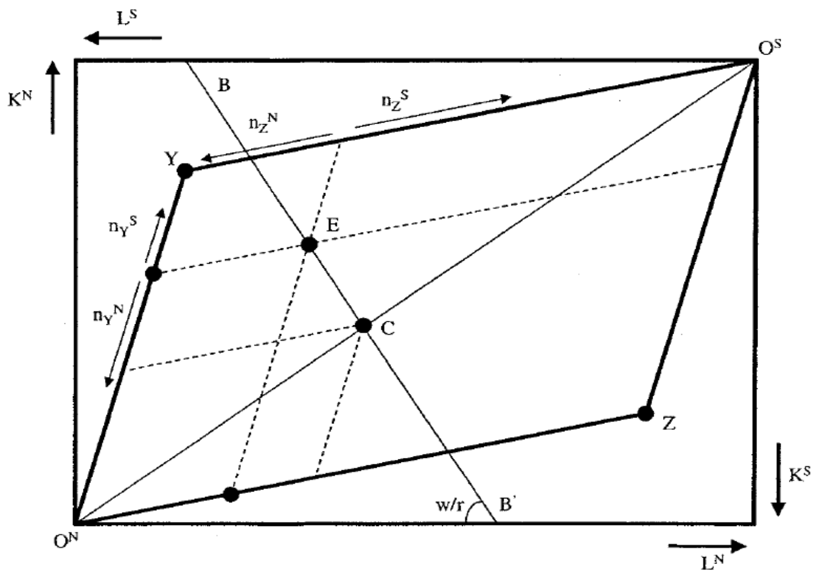

Figure 3: Patterns of Production (2 country case).

$$\tilde{\beta}_Y = \beta_Y \left(1 + \alpha \left(1 - \beta_Y\right) \left(2\bar{\phi} - 1\right)\right)$$
$$\tilde{\beta}_Z = \beta_Z \left(1 + \alpha \left(1 - \beta_z\right) \left(2\bar{\phi} - 1\right)\right)$$

$$\tilde{\beta}_Y > \tilde{\beta}_Z$$

$$\sum_{k \in \{Y,Z\}} n_k^j \left(K_{x,k}^j + K_{f,k}^j\right) = K^j, \ \forall j$$
$$\sum_{k \in \{Y,Z\}} n_k^j \left(L_{x,k}^j + L_{f,k}^j\right) = L^j, \ \forall j$$

$$M^{ij} = s^i \left(n_Y^j p_Y y + n_Z^j p_Z z\right) = s^i s^j \left(rK + wL\right)$$
$$M_{intra}^{ij} = s^i s_Y^j p_Y y$$
$$S_{intra}^{ij} = \frac{\left(1 - \tilde{\beta}_Z\right) \left(1 - \sigma_L\right) \frac{K^j}{L^j} - \tilde{\beta}_Z \sigma_L \frac{K}{L}}{\left(\tilde{\beta}_Y - \tilde{\beta}_Z\right) \left(\left(1 - \sigma_L\right) \frac{K^j}{L^j} + \sigma_L \frac{K}{L}\right)}$$

2 Antras (2005)

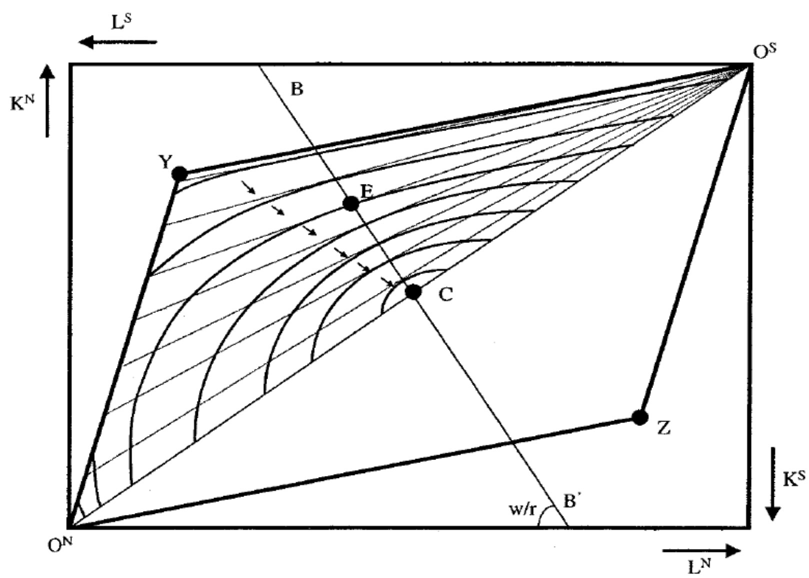

Figure 4: Volume of Intrafirm Imports.