Life-Cycle Effects of Internal Habit Formation on Portfolio Choice

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Abstract

The presence of an internal habit, interpreted as a minimum acceptable lifestyle, has important consequences for portfolio choice of agents. The risk aversion of agents varies endogenously through the life cycle depending on evolution of the agent's habit. For the case where total wealth is capitalized, I obtain analytical solutions for the value and policy functions in a continuous time finite horizon model. There is an interesting life cycle effect which I highlight. Younger agents need to sustain their habits for a longer horizon, thereby making them more risk averse and inducing them to optimally hold more conservative portfolios, as compared to older agents who have fewer outstanding periods, hence worry less about sustaining future habits and hold more aggressive portfolios. The model is applied to study portfolio decisions of retired households, in contrast to the standard model it is able to explain the data.

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1 Introduction

The introduction of habits to preferences in the asset pricing literature has shown promise in explaining the equity premium puzzle of Mehra and Prescott. For example, Constantinides (1990) studies a model with internal habit formation. He shows that through addition of habits, the observed small variation in aggregate consumption can explain the difference between the expected returns on stocks and the risk free rate. Campbell and Cochrane (1999) posit a model with external habit formation, and are able to simultaneously match a number of empirical findings including the equity premium. The puzzles in the portfolio choice literature are essentially partial equilibrium counterparts to asset pricing puzzles. For instance, standard models of portfolio choice predict that young agents should hold their entire financial wealth in stocks, contrary to empirical observation. This is primarily a partial equilibrium manifestation of the equity premium puzzle. Since, the introduction of habits has shown promise in explaining such puzzles in a general equilibrium framework, it seems natural to introduce such preferences in a portfolio choice (partial equilibrium) framework.

In this study, I assume internal habit formation. The habit stock is a weighted average of past consumption, and agents when making their consumption decision today, take into account the effect current consumption will have on their future habits and thus on their utility tomorrow. In the external habit case, agents don't take this effect into account. Their habits are thought to be determined by some reference group (the neighbors) over which the agent has no control. The internal habit representation appears to be a more natural one, especially in a partial equilibrium setting when studying an individual agent's problem.

The interpretation of the internal habit is of a 'minimum acceptable standard of living' that the agent would like to ensure he can sustain under any circumstances. This minimum standard of living evolves over time as a weighted average of current and past consumption. It is affected in a positive manner by current consumption i.e. if current consumption increases, so does the minimum standard of living. Hence, the agent won't consume extravagantly today as he realizes that this will raise his standard of living in the future. Agents will save enough to sustain their habits (or maintain their lifestyle) in the future. Consequently, agents with such preferences will hold more conservative portfolios than their time-separable counterparts, in order to ensure they can finance their standard of living in the event that the stock market falls.

The focus of this paper is the important life-cycle effect generated in portfolio choice by introduction of habits or a minimum sustainable standard of living. In contrast to most existing studies which are numerical in nature, I obtain analytical solutions in a continuous time model for the value and policy functions to highlight the economic mechanism at work. Younger agents are concerned with sustaining their standard of living for the remainder of their outstanding lives, hence will hold more conservative portfolios compared with older agents who are less concerned about their standard of living since they have fewer outstanding periods to live. By more conservative, I mean for the same level of wealth and habits, younger agents hold a smaller fraction of their wealth in stocks compared to older agents. This is in contrast to the standard time separable model, where the fraction of wealth held in stocks is constant and doesn't vary with age.

2 Related Literature

There has been a tremendous amount of work on portfolio choice in the last few decades. Perhaps the most important work, was by Merton (1969) and Samuelson (1969). Merton solved a continuous time model, assuming preferences are given by power utility, stock returns follow a wiener process with a drift, there are no short-sale constraints, labor income can be fully capitalized, and there are no transactions costs. The fraction of wealth invested in stocks is a constant, $\frac{\mu-r}{(1-\gamma)\sigma^2}$. Here $\mu-r$ represents the excess returns of stocks over the risk free rate, γ the coefficient of relative risk aversion (CRRA), σ

the standard deviation of stock returns process.

Samuelson (1969), considered the same model in discrete time, under the same assumptions, however, relaxed the assumption that stock returns follow a normal distribution. He also showed that the proportion of wealth invested in stocks is constant, for any finite or infinite horizon and is given $\alpha^\star$ which solves:

$$\mathbb{E}[(Rf + \alpha^\star(R - Rf)).^{-\gamma}(R - Rf)] = 0$$

Here, Rf represents the risk-free rate, γ is the CRRA, R represents the return on stocks, which can follow any stochastic process. One noteworthy point is, that when R is assumed to be normal and as we calibrate the parameters above for smaller time intervals, the solution converges to Merton's analytical solution. However, this is only true for normality of R, even though the above equation holds for any process. In general, for other distributions the optimal proportion invested in stocks even though constant, will depend on moments of the assumed distribution apart from the mean and variance. Numerical studies often use the merton formula as a benchmark, however, assume different stochastic processes for stock returns.

Several numerical studies consider the effect of adding labor income (stochastic) to the base model. Heaton and Lucas (1997) consider such a model in infinite horizon, Cocco, Gomes and Maenhout (2004) consider a similar model in finite horizon. Empirically, there is little observed systematic risk in labor income, for example, Davis and Willen (2000) examine the correlation structure of labor income risk and sources of systematic risk (S and P 500, other broad based equity indices) by occupation and industry. They find little or no evidence of any such correlation. Overall, labor income tends to resemble implicit holding of a risk-free asset, thereby, crowding out investment in bonds and tilting the optimal portfolios held by young investors more aggressively towards stocks. Even imposing short sale constraints, and borrowing constraints (to prevent agents from borrowing against future labor income) most of these models predict young agents should invest all their wealth in stocks.

Further, several empirical studies have looked at the investment behavior of households and the fraction of wealth they invest in stocks. There is a well-known identification problem of age, time and cohort effects discussed at length in Ameriks and Zeldes (2004). In the U.S, the Survey of Consumer Finances (SCF) is the primary source of data on households. The average fraction of financial wealth held in stocks by agents over the life cycle follows a hump-shape and varies between 0.1 and 0.5 in stark contrast to predictions of these models. Further, the fraction of agents holding equity varies between 0.1 and 0.7 over the life cycle, whereas, standard models predict all agents should enter the stock market right away.

Gomes and Michaelides (2003), numerically solve a portfolio problem with internal habit formation, and stochastic uninsurable labor income along with fixed entry costs in a finite horizon setting. However, they find that incorporating habits actually decreases the ability of the model to match the stylized facts on portfolio choice, and they conclude that the internal habit formation model is 'dominated' by the time-separable counterpart. They attribute this to an decrease in the elasticity of inter-temporal substitution (EIS). They claim this increases agent's desire to smooth consumption, causing them to accumulate more wealth and enter the stock market at young ages and invest their savings entirely in the stock market. Their results are robust to two popular specifications for the way in which habits enter utility, as a difference from current consumption and as a ratio. It does seem counter-intuitive that agents who save to maintain smooth consumption would be willing to invest their entire savings in risky stocks.

Another forthcoming paper, Polkovnichenko (2008), numerically solves a model very similar to Gomes and Michaelides, but reaches a conclusion opposite to their's. There are differences in calibration, evolution of habits and the stock returns process. Habits are restricted to be a constant fraction of the previous period's consumption. The stochastic process for stock returns is assumed to be binomial.

Numerical studies on portfolio choice, including the ones described above, solve finite horizon discrete time models. This involves backward iteration on the value function, through forming a discrete grid for state variables. Further, the choice variables are also restricted to be chosen from a discrete grid. The approximation error generated through iterating backwards 60 to 80 periods, along with discrete choice for choice variables tends to accumulate fast. Consequently, the robustness of results from such models seems questionable.

3 Continuous Time Finite Horizon Model

In this section, I solve a finite horizon model with internal habit formation. Unlike its discrete time counterpart, I am able to obtain analytical solutions to demonstrate the life-cycle effects of aging on portfolio choice. Young agents need to ensure they can support their lifestyle for a longer outstanding horizon, thereby, inducing them to hold more conservative portfolios compared with older agents who needn't worry as much about supporting their lifestyle (or habits) since they have a shorter outstanding horizon. Such life cycle effects are a unique feature of the habit model. In the time-separable (Merton-Samuelson) case, agents hold a constant share of their savings in the stock market independent of age or level of wealth.

The agent lives for a finite time horizon of length $T > 0$. There are two assets agents can invest in, one is a risk-less security returning r per unit of infinitesimal time length dt, the other security is risky and returns $\mu dt + \sigma dz(t)$ Here, r, μ and σ are constants. μ denotes the average stock return, σ the standard deviation of stock returns. $dz(t)$ is a one-dimensional Brownian motion.

The agents sequence problem at time t in $[0, T]$ is:

$$\max_{c(s),\alpha(s)} \mathbb{E}_t \int_t^T e^{-\rho(s)} \frac{[c(s) - x(s)]^{1-\gamma}}{1-\gamma} ds$$

Here, γ is restricted to be > 1. When $\gamma \longrightarrow 1$, the period utility function can be approximated by the natural logarithm, this case is dealt with separately.

Habit stock, x, is defined to be a weighted average of past consumption, given by:

$$x(t) \equiv e^{-at} x_0 + b \int_0^t e^{a(s-t)} c(s) ds$$

Wealth, W is used to finance consumption, and savings are invested in stocks and bonds:

$$W(t) = e^{rt} W_0 + \int_0^t e^{r(s-t)} \{[(\mu - r)\alpha(s) + r] W(s) - c(s)\} ds$$

The parameter a determines how much past habit is discounted, while b

determines how much current consumption affects current habit. If $a = b$, then the habit stock turns out to be a weighted average (summing to one) of past consumption, hence, is in the same units as consumption. If $b < (>)$ a, then the weights in the habit evolution sum to strictly less (greater) than one. The local evolution of the habit stock under this specification is risk-less and deterministic:

$$dx(t) = [-ax(t) + bc(t)]dt$$

The first component is an instantaneous depreciation of the habit stock, the second component is the contribution from current consumption.

$W(t)$ denotes the level of wealth held by the agents, in units of the numeraire consumption good. α denotes the fraction of wealth invested in the risky asset, c denotes consumption per unit of time. The local evolution of wealth then follows:

$$dW(t) = \{[(\mu - r)\alpha(t) + r]W(t) - c(t)\}dt + \sigma\alpha(t)W(t)dz(t)$$

The deterministic (drift) component is the return on the portfolio times the agent's wealth less consumption. The stochastic component is due to the exposure to uncertainty from investment in the risky asset.

The agent's policies must satisfy certain conditions for each $t \in [0, T]$. Firstly, decisions taken at time t must be based on information available at that date. Also, $c(t) > 0$, which requires that $W(t) > 0$, almost surely. Further, $c(t) > x(t)$ i.e. consumption must be greater than the habit level. Lastly, the share of wealth invested in each asset is non-negative i.e. $\alpha(t) \in [0, 1]$.

The problem described is not stationary, in the sense that age, t, matters and is a state variable in addition to wealth level (W) and habit level (x). Let $V(W, x, t)$ denote the associated value function. Then,

$$V(W, x, t) = \max_{admissible-c(s),\alpha(s)} \mathbb{E}_t \int_t^T e^{-\rho(s-t)} \frac{[c(s) - x(s)]^{1-\gamma}}{1-\gamma} ds$$

V turns out to be a homogeneous function of W and x for each t. An important restriction on the ratio of wealth to habit arises endogenously. This ensures that at each t, an admissible policy exists. At each t, for each x, wealth must be sufficiently high to ensure that agents can maintain consumption above their habit level for the outstanding horizon.

Lemma 1: For $t \in [0, T]$, if $\frac{W(t)}{x(t)} > \frac{1}{r+a-b} \left[1 - e^{-(r+a-b)(T-t)}\right] \equiv \underline{y(t)}$, then, there exists a feasible policy $c(s)$, $\alpha(s)$ for all $s \in [t, T]$ such that $c(s) > x(s)$ with probability 1.

Proof: Consider the policy, $\alpha(s) = 0$ and $c(s) = x(t)$. ■

If $W(t)$, $x(t)$ satisfy this restriction, then of course, following the optimal policy guarantees the restriction will be satisfied at all future dates $s \in [t, T]$.

It can be seen that $\underline{y_t}$ is a decreasing function of t, this time variation drives the life-cycle effects in portfolio choice, since, the optimal fraction of wealth invested in stocks will depend on the difference of the wealth-habit ratio, $y(t)$, from $\underline{y(t)}$.

For $t = 0$, this is purely a restriction on model parameters and initial values for $W(0)$ and $x(0)$. Further,

Assumption 1: $r + a - b > 0$, this ensures $\underline{y(t)} > 0$.

Assumption 2:$m \equiv \frac{\mu - r}{\gamma\sigma^2} \in [0, 1]$, this ensures the optimal $\alpha(t) \in [0, 1]$.

Next, I analytically derive the optimal policy and value function under the restrictions given above.

The value function must then satisfy the following system of optimality conditions:

The Hamilton-Bellman-Jacobi (HBJ) equation:

$$\max_{c,\alpha} e^{-\rho t} \frac{(c-x)^{1-\gamma}}{1-\gamma} + \frac{\partial V(W,x,t)}{\partial t} + \frac{\partial V(W,x,t)}{\partial W}\{((\mu - r)\alpha + r)W - c\}$$
$$+ \frac{1}{2}\frac{\partial^2 V(W,x,t)}{\partial^2 W}(\sigma^2\alpha^2 W^2) + \frac{\partial V(W,x,t)}{\partial x}(-ax + bc) = 0$$

The first order conditions for the choice variables:

$$e^{-\rho t}(c-x)^{-\gamma} + b\frac{\partial V(W,x,t)}{\partial x} = \frac{\partial V(W,x,t)}{\partial W}$$

$$\frac{\partial V(W,x,t)}{\partial W}(\mu - r)W + \frac{\partial^2 V(W,x,t)}{\partial^2 W}\sigma^2 W^2 \alpha = 0$$

The HBJ equation is the continuous time counterpart to a discrete time Bellman equation. There are two first order conditions for consumption and proportion of savings invested in stocks. The first condition equates the marginal utility for unit of wealth. If the agent increases her wealth it leads to a capital gain ($= \frac{\partial V(W,x,t)}{\partial W}$). Alternately, if the agent decides to consume the extra unit she receives the marginal utility from consumption ($=e^{-\rho t}(c-x)^{-\gamma}$), also, there is dis-utility associated with an increase in the habit stock ($=b\frac{\partial V(W,x,t)}{\partial x}$). The second condition states the proportion of wealth invested in stocks depends on the curvature of the value function, it is the elasticity of marginal utility with respect to wealth.

We obtain a system of equations consisting of two algebraic equations and a second order non-linear partial differential equation with three independent variables. In general, analytical solutions for such systems aren't guaranteed to exist. However, using the economic structure of the problem we guess and verify a solution.

Proposition 1: The optimal value function is of the form:

$$V(W,x,t) = e^{-\rho t}\phi(t)\frac{[W - x\underline{y(t)}]^{1-\gamma}}{1-\gamma} \qquad if,\ \gamma > 1$$

$$= e^{-\rho t}[\phi(t) log\ (W - x\underline{y(t)}) + \kappa(t)]\ if,\ \gamma \longrightarrow 1$$

Where, $\phi(t)$ is a constant function depending on model parameters. and solves the ordinary differential equation (ODE) for $\gamma > 1$,

$$\dot{\phi}(t) = -\gamma[\phi(t)(1 + b\underline{y(t)})]^{1-\frac{1}{\gamma}} + \phi[\rho - \frac{(\mu - r)^2(1-\gamma)}{2\gamma\sigma^2} - r(1-\gamma)]$$

with terminal condition $\phi(T) = \epsilon$.

If $\gamma \longrightarrow 1$,

$$\phi(t) = \frac{1 + (\rho\epsilon - 1)e^{\rho(t-T)}}{\rho}$$

and κ solves the ODE:

$$\dot{\kappa}(t) = \rho\kappa(t) - \phi(\frac{(\mu - r)^2}{2\sigma^2} + r) + log(\phi(t)(1 + b\underline{y}(t))) + 1$$

Further,

$$\phi(t) > 0,\ \phi'(t) < 0,\ \phi''(t) < 0$$

Proof: Guess and Verify. Details in Appendix A. ■

In the case with no-bequests, $\phi(T) = \epsilon$, where ϵ approaches 0. This is merely a mathematical trick to preserve the structure of the problem, while associating zero value with wealth at the terminal date T. Also, ϕ is a decreasing and concave function of time. As T approaches ∞, $\phi(0)$ approaches a constant corresponding to an infinite horizon model.

We can see the value function inherits the functional form of the utility function, as is usually the case. Also, the value depends on the difference between W and $x\ \underline{y}$. The latter denotes the minimum wealth necessary to guarantee consumption greater than habit can be sustained for the remaining life. Hence, the wealth in excess of this lower threshold determines utility.

Proposition 2: The optimal policy function for consumption is of the form:

$$c(W, x, t) = x + \psi(t)[W - x\underline{y(t)}]$$

Using ϕ defined above,

$$\psi(t) \equiv [\phi(t)(1 + b\underline{y(t)})]^{\frac{-1}{\gamma}}$$

Further,

$$\psi(t) > 0,\ \psi'(t) > 0,\ \psi''(t) > 0.\ and\ \psi(t) \to \infty\ as\ t \to T$$

The optimal proportion of wealth invested in stocks:

$$\alpha(W, x, t) = \frac{\mu - r}{\gamma\sigma^2}[1 - \frac{xy(t)}{W}]$$

Proof: Details in Appendix A. ■

The consumption rule is to consume a constant fraction (depending on time) of wealth in excess of minimum required wealth. At the terminal date, consumption approaches ∞, this is merely for mathematical convenience since a flow variable (consumption rate) must equal a state variable (wealth stock). However, this isn't important for the economic structure of the problem, since the value associated this infinite rate of consumption is zero.

Corollary 1: The marginal propensity to consume out of an extra unit of wealth, $\frac{\partial c(W,x,t)}{\partial W} = \psi(t)$. Further, $\psi(t) > 0$ and $\psi'(t) > 0$.

$\frac{\partial c(W,x,t)}{\partial W}$ doesn't depend on the habit level. It is always greater than zero, and further increasing in time. Older agents consume a greater fraction of additional wealth compared with corresponding younger agents since they have a shorter outstanding lifespan and hence, care less about sustaining future habit.

Corollary 2: $\frac{\partial \psi(t)}{\partial a} > 0$. Further, $\frac{\partial \psi(t)}{\partial b} < 0$.

For a fixed value of b, the marginal propensity to consume out of an extra unit of wealth increases with a. Intuitively, if past habit is depreciating at a higher rate then the agent consumes a greater fraction of her wealth today. Similarly, if the contribution of current consumption to habit increases then the agent consumes a smaller fraction of her wealth today.

The optimal stock investment rule has two components. The first one, $\frac{\mu-r}{\gamma\sigma^2}$ is the standard Merton term. The second component is due to habit formation. This term is always less than 1, and depends on how close wealth is to the minimum level of wealth required to sustain habit.

Corollary 3: For $t > s$, and $W(t) = W(s)$, $x(t) = x(s)$ implies $\alpha(W, x, t)$ $> \alpha(W, x, s)$

Proof: Since, $\underline{y(t)} < \underline{y(s)}$, the result follows immediately from the optimal policy, α, defined in proposition 1. ■

The above proposition makes precise the life cycle effects of internal habits on the portfolio allocation rule. With the same wealth and habit levels, older agents optimally invest a higher fraction of their wealth in stocks compared with otherwise identical younger agents.

3.1 Stock Market Participation Cost

I consider adding a one time fixed cost for participation in the stock market. This can be thought of as the effort and costs associated with gathering the relevant information regarding investing and setting up a brokerage (or other) account.

The (optimal) value function, $V(W, x, t, s)$, now includes an additional (binary) state variable,s, which tracks whether the agent has payed participation costs. s = 1 denotes participation, 0 denotes non-participation. K (> 0) will denote the one-time cost. I concentrate on the case, $\gamma > 1$.

The dynamic optimization problem of an otherwise identical agent, without access to the stock market, has the same functional form for the value and policy functions as derived in propositions 1 and 2. Setting the excess return ($\mu - r = 0$) gives the relevant coefficients, which are labeled with subscript 0.

Proposition 3: Given t, W, x, there exists a maximum amount, K which the agent is willing to pay to get access to the stock market:

$$K(W, t, x) = (W - x\underline{y}(t))[1 - (\frac{\phi_0(t)}{\phi(t)})^{\frac{1}{1-\gamma}}]$$

Where, ϕ was defined in proposition 1, and ϕ_0 represents corresponding coefficients for an agent without access to the stock market.

Proof: Since, the value functions (conditional or participation or non-participation) are increasing K is obtained as the unique solution to the equation:

$$V(t, W, x, s = 0) = V(t, W - K, x, s = 1) \blacksquare$$

Corollary 4: $K(t, W, x)$ is a decreasing function of t. $\frac{K(t,W,x)}{(W - x\underline{y}(t))}$ is also decreasing in t.

Proof: Can be shown numerically, by solving the relevant ODEs. $\blacksquare$

The fraction of wealth (in excess of the minimum sustainable amount) which the agent is willing to pay to gain access to the stock market is decreasing with age. Intuitively, since younger agents can enjoy benefits of investing in stocks for a longer time compared with otherwise identical older agents, they are willing to pay more.

4 Simulation Exercise

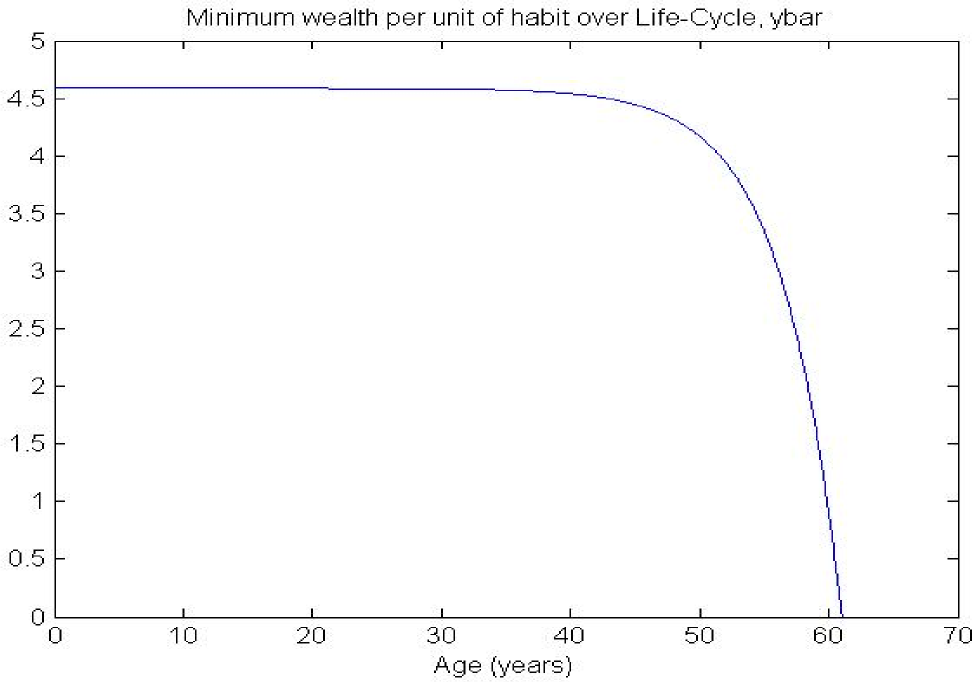
In this section, I calibrate parameter values as per other studies on portfolio choice. Other parameter values don't change the qualitative results substantially. Parameter a captures the fraction of past habit that depreciates, while b captures the fraction of current consumption that contributes to the current habit stock. We must restrict $b \leq a$, to guarantee that the habit stock is in the same units as consumption.

I demonstrate how policy functions evolve over the life cycle. Also, I simulate sample paths to examine the distribution of state and choice variables at different points of the life cycle.

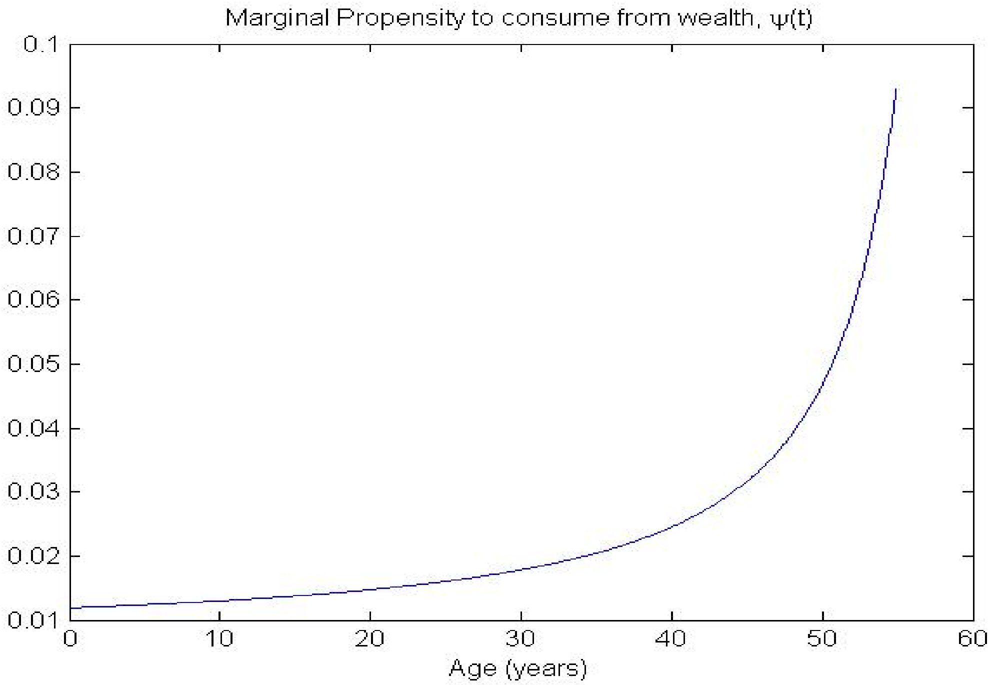
Time horizon	T	61
Time preference	ρ	0.037
Coefficient of Relative Risk aversion	γ	5
Depreciation of past habit	a	0.6
Fraction of consumption in habit	b	0.392
Mean of Stock Return	μ	0.06
Standard deviation of Stock Return	σ	0.165
Risk free rate	r	0.01

Table 1: Per year for continuous time

I plot $\underline{y}$ over the life cycle. $\underline{y}(t)$ denotes the minimum amount of wealth necessary to support habit in the future corresponding to a unit of habit stock. It is a function of time outstanding. We can observe there it is decreasing, however, there isn't much change till the last quarter of the life cycle.



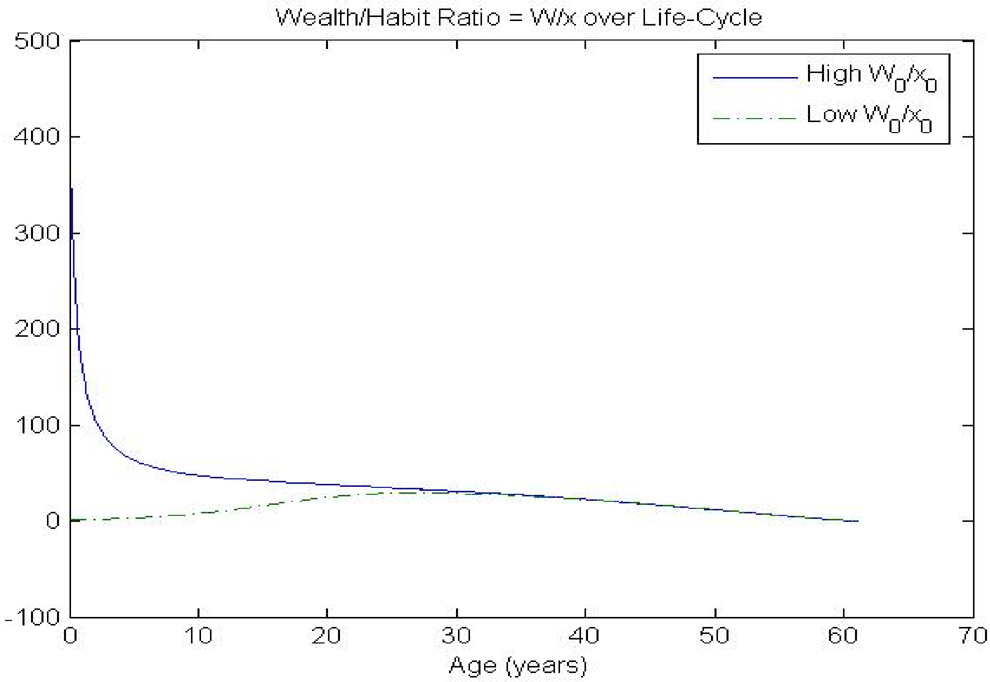
Next, I plot the marginal propensity to consume out of wealth, $\psi(t)$. This is an increasing function of time, however, it doesn't depend on the habit level. It is plotted till age 55 after which it becomes very large. At time T, ψ approaches infinity representing an 'impulse' of consumption required for the flow variable to subsume the stock variable. However, the value associated with this high level of consumption is zero.



To simulate the evolution of variables over the life cycle, I use Ito's Lemma and the policy functions derived in the previous section to obtain a stochastic differential equation which governs the evolution of the state variables over the life cycle, y denotes the ratio of wealth to habit $\frac{W}{x}$:

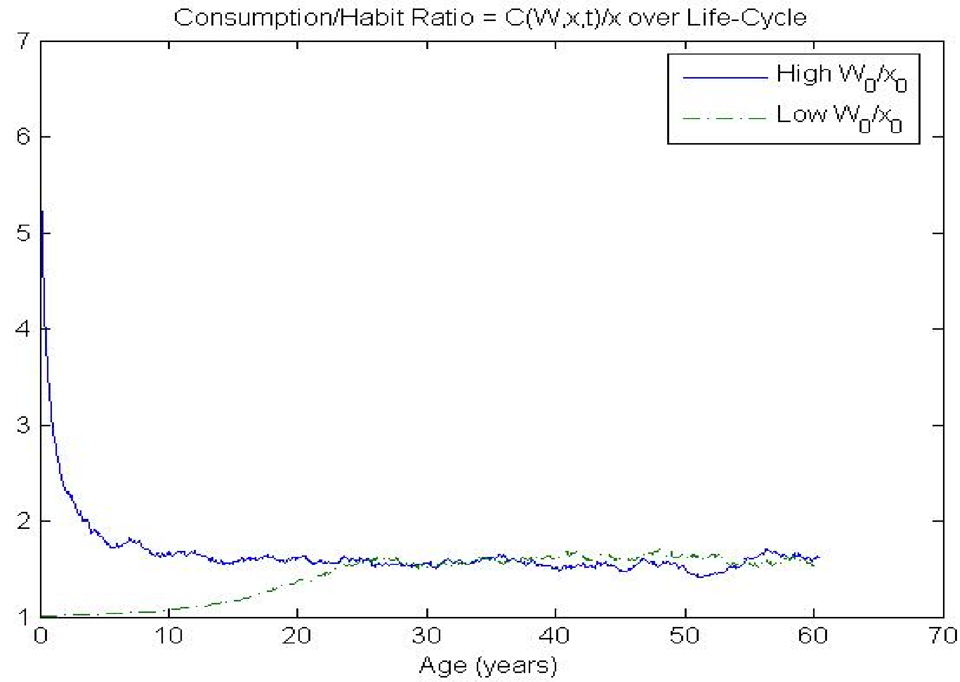
$$d(y(t) - \underline{y}(t)) = [\{(\mu - r)m + (r + a - b) - \psi(t)(1 + b\underline{y}(t))\}(y(t) - \underline{y}(t)) \\ - b\psi(t)(y(t) - \underline{y}(t))^2]dt + \sigma m(y(t) - \underline{y}(t))db(t)$$

The cross-sectional means (conditional on age) are presented for the different histories. Two different initial conditions are considered. First, wealth is very high compared with habit. Second, wealth is close (1.25 times) the minimum required wealth.

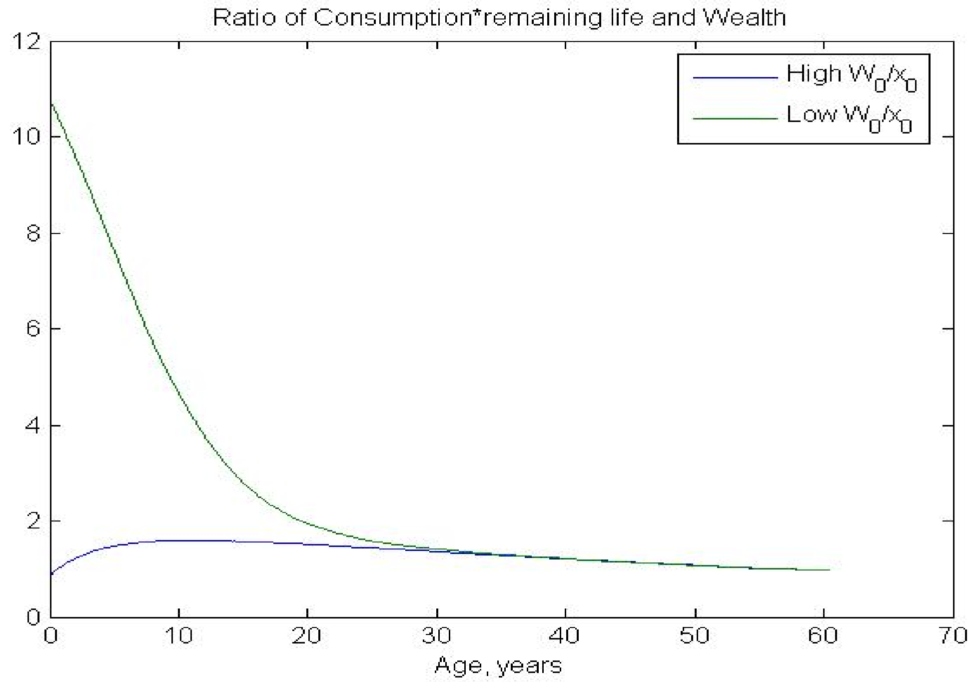


We can see that the ratio of consumption to habit, which is always greater than 1, is much greater initially in the first case. The agent's wealth is very large compared with her habit, and she consumes at a very high rate for roughly the first 20 years (first third) of her life cycle. For the remaining 40 years, consumption is roughly 1.4 to 1.6 times the habit stock. Near death consumption becomes infinite. For the second case, the agent's wealth is small compared with the minimum required wealth. The agent starts by consuming a very small fraction above habit, roughly 1.1 to 1.3. From age

25 onwards, the consumption rate in the two cases is very similar.



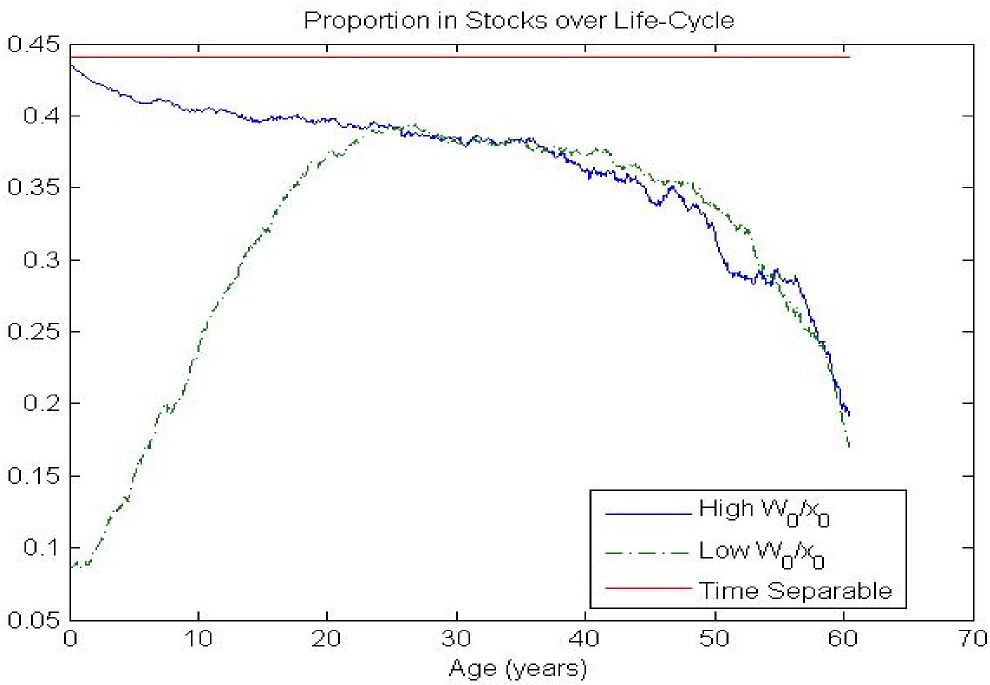
Consumption times remaining life is plotted against wealth. This shows the ratio of cumulative consumption to wealth, if the agent was to consume at the same rate for her outstanding life. We can see the different initial conditions matter for the first 25 years. Initially, the high wealth individuals consume at a very high rate as opposed to low wealth individuals who consume conservatively.



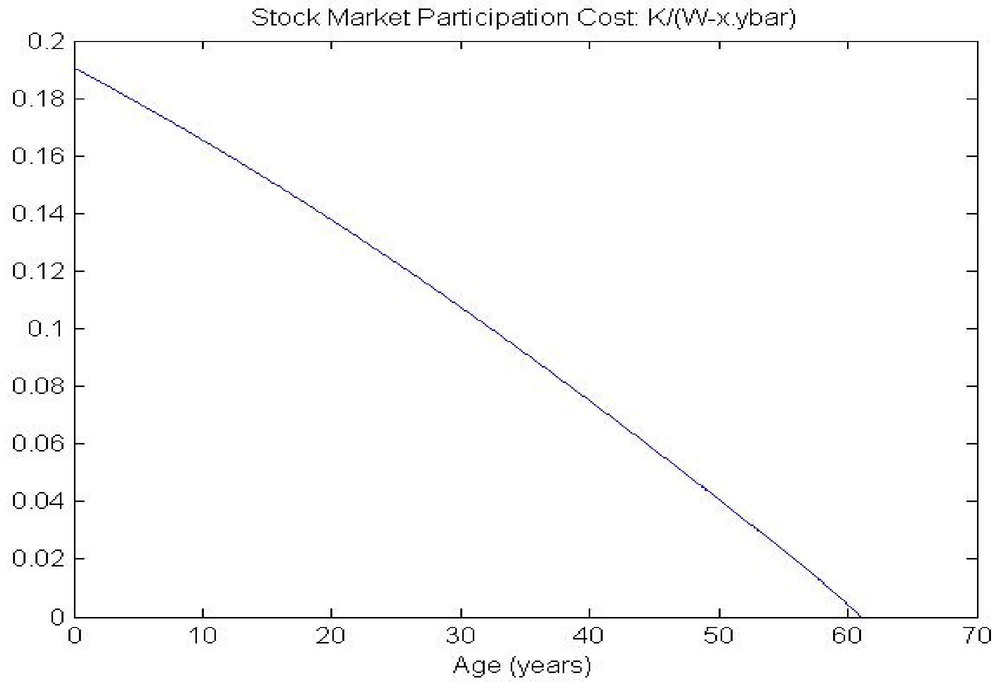
We can see the proportion of wealth invested in the risky asset over the life cycle for the two cases. For the first (high wealth) case, the agent invests a large fraction, nearly m (time separable), in the risky asset. However, as the agent's habit stock increases she becomes more risk averse starts investing a smaller fraction of her wealth in stocks.

For the second (low wealth) case, the agent's wealth level is close to the minimum sustainable wealth level, and the agent is very risk averse. She invests a very small fraction of her wealth in the risky asset. Also, she consumes cautiously, her habit stock decreases for the first 22-25 years of her life cycle. She becomes less risk averse and more aggressive in the stock market, this peaks at roughly age 23. After that the consumption rate peaks, the habit stock starts increasing. The agent becomes more risk averse and starts decreasing her fraction of wealth in stocks. The minimum is attained near the end of the life cycle.

The heterogeneity from the high initial wealth lasts for roughly first 20 years (first thirds) of the life cycle.



Lastly, I plot the fraction of their wealth that (non-participating) agents are willing to pay to get access to the stock market.



We can see younger agents are willing to pay a much higher fraction (roughly 20%) of their wealth compared with older agents.

4.1 Portfolios of Retired Households

The simulation exercise thus far, was interesting to observe the behavior, magnitude and evolution of various variables over the life cycle in the model. However, in order to successfully compare predictions from the model with the data we need to map variables from the theoretical model to quantities we can measure and observe in the data. Wealth in the model above is a measure of total wealth, and includes human capital (a present discounted value of future labor income) as well as financial wealth. This quantity is hard to construct using data since labor income is risky, and forming a present discounted value of future labor income isn't feasible. Also, in the real world agent's aren't able to borrow against their future labor income without incurring substantial costs. Various papers such as Viciera, discuss the effects of non-tradeable labor income on portfolio choice. Cocco, Gomes and Maenhout (2005) investigate the impact of risky labor income on portfolio choice. Other important determinants of portfolio choice, especially for young agents, are their housing decision (for example, Yao and Zhang 2005).

To compare predictions of the theoretical model with the data, we look at portfolios of retired individuals. The main advantage of this is old agents don't receive labor income, hence, we can measure their total wealth as defined in the model from the data. Further, younger households' portfolio decision interacts in important ways with other decisions such as housing, education. Older agents are less concerned with these decisions. Further, older agents have been consuming throughout their (young) life and are therefore more accustomed to a certain level of consumption or lifestyle (habits). Thus, it seems logical to study the portfolio choice of old agents to evaluate the effects of habit formation on portfolio choice in the data.

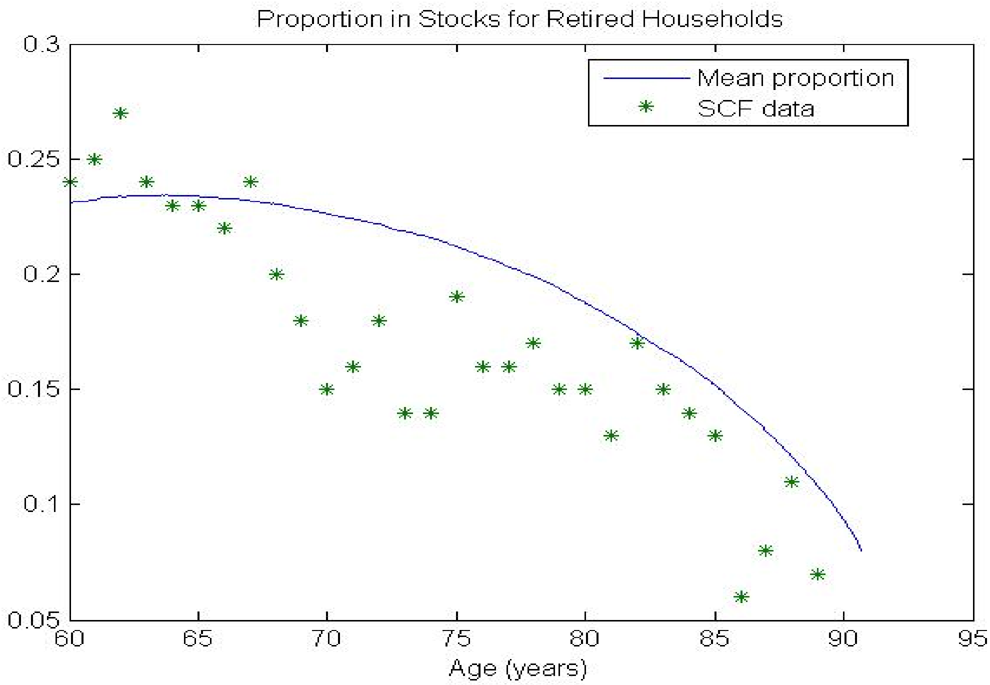
From the data, I obtain average consumption conditional on age for agents from ages 20 through 60 (see Appendix A). Using this consumption profile as a representative agent's consumption through her work life and an initial value for the habit stock I obtain a value of the representative agent's habit stock at retirement (old) age. The habit stock at retirement isn't sensitive to the choice of initial value of habit. Next, wealth is measured using the average financial wealth of 60 year old agents. Construction of the financial wealth variable from SCF data is discussed in the next section.

Parameters are calibrated to match existing studies on portfolio choice. For habit evolution parameters, I use Constantinides' parameter values which resolve the equity premium puzzle.

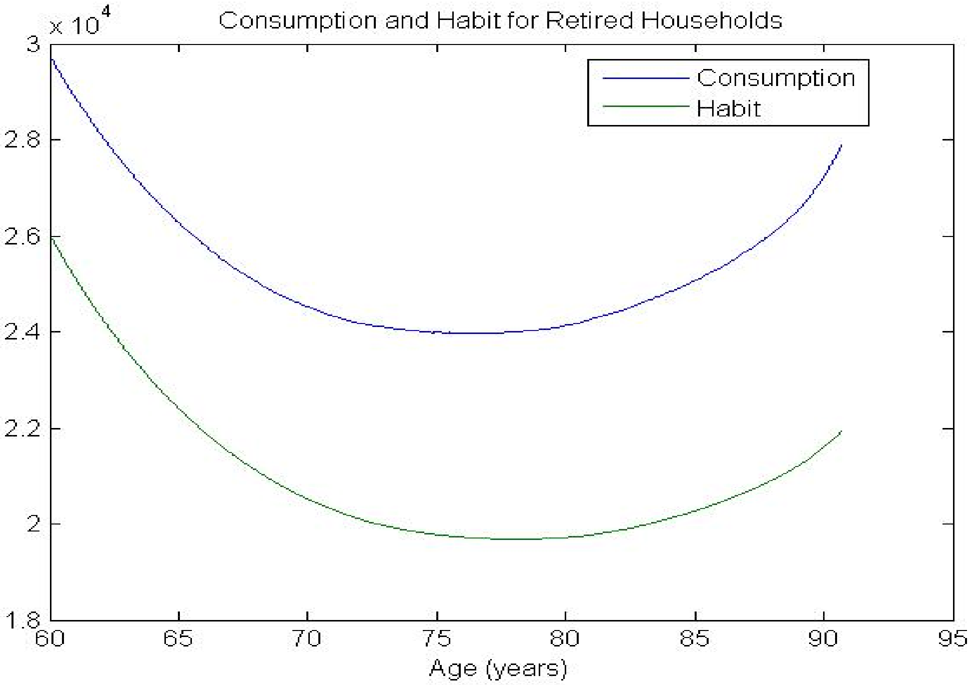
Time horizon	T	31
Time preference	ρ	0.037
Coefficient of Relative Risk aversion	γ	5
Depreciation of past habit	a	0.6
Fraction of consumption in habit	b	0.492
Mean of Stock Return	μ	0.05
Standard deviation of Stock Return	σ	0.165
Risk free rate	r	0.01

Table 2: Per year for continuous time

Next, I present the mean proportion of wealth invested in stocks as predicted by the model and as observed in the data.



Evolution of consumption and habit for retired households is presented next.



Retired households might face substantial mortality risk. We examine robustness of the results by adding mortality risk. Conditional survival probabilities from the life tables compiled by the Social Security Adminstration are used to augment the households' consumption, savings, portfolio choice decision over the life-cycle. Hurd and McGarry (2002) shows that these true survival probabilities indeed reflect the estimates used by agents in decision making under uncertainty.

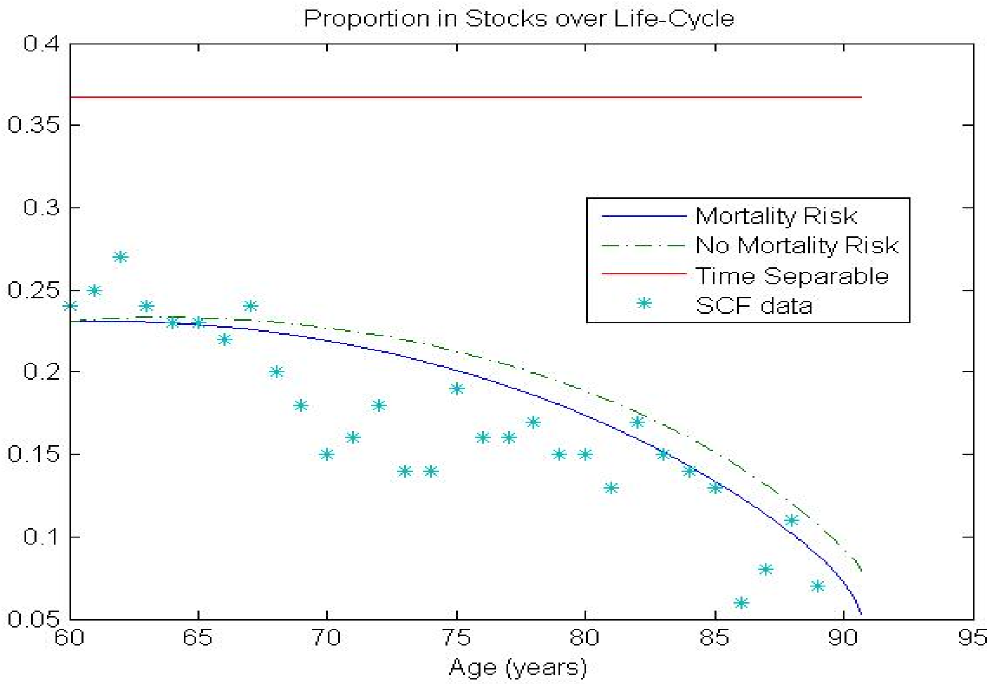
In this case, the discount factor in the value function is augmented by the conditional probability of survival.

$$V(W,x,t) = \max_{admissible-c(s),\alpha(s)} \mathbb{E}_t \int_t^T e^{-\rho(s-t)} p(s) \frac{[c(s)-x(s)]^{1-\gamma}}{1-\gamma} ds$$

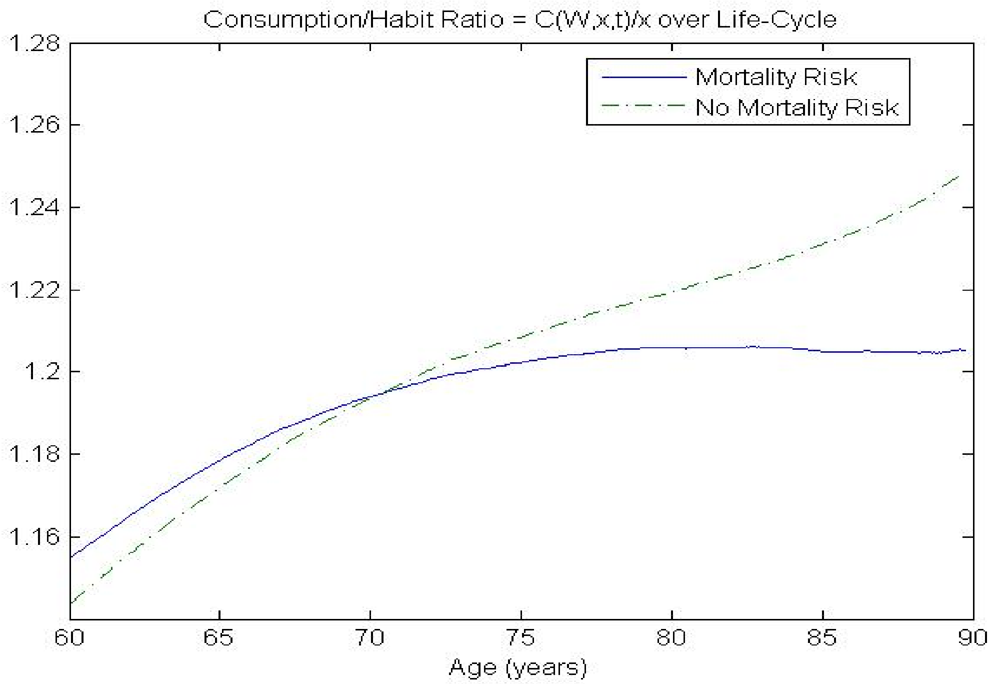
Here, $p(s)$ denotes the probability of survival conditional on being alive at retirement. The solution to the HBJ and optimality conditions is similar, details are provided in Appendix B.

Mortaility risk augments the agents discount factor, and she discounts future periods at a higher rate compared with the no-mortality risk case. Consumption is at a higher rate.

Next, we present the mean proportion of wealth invested in stocks as predicted by the model with and without mortality risk and compare with the time separable case and SCF data.



Consumption-habit ratios over the life cycle as predicted by the model with and without mortality risk are compared. With mortality risk, the agent consumes at a higher rate. Hence, the agent's habit stock increases rapidly, consequently the consumption-habit ratio is smaller for the agents facing mortality risk later in the life-cycle.



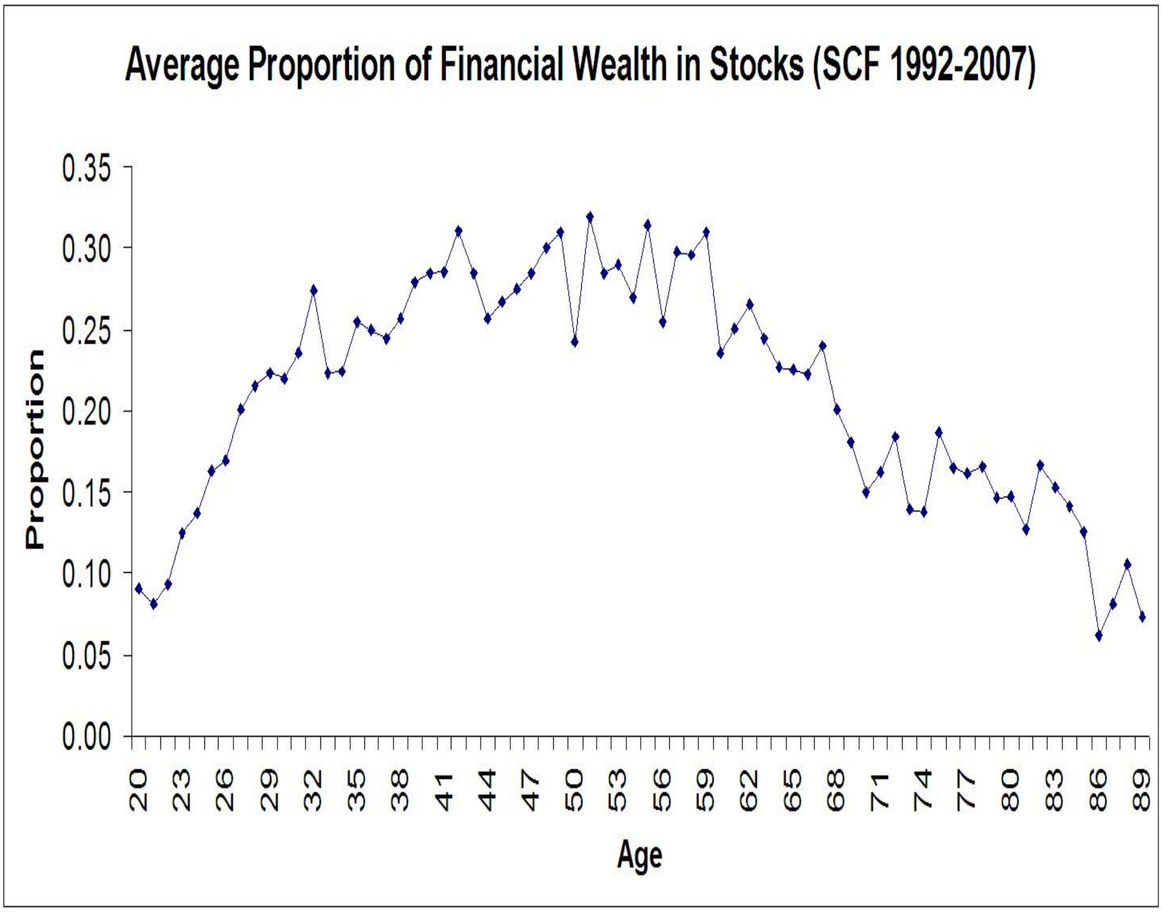
5 Empirical Evidence

In this section, I present empirical evidence using data from the Survey of Consumer Finances (SCF). The SCF is maintained by the Federal Reserve Board. It is a triennial survey of the balance sheet, income and other characteristics of households in the United States. Approximately $4,500$ families are interviewed randomly, and a strong attempt is made to interview families from all economic strata to obtain a representative picture of households in the US. One major advantage of the SCF is that rich households are oversampled. Given the vast inequalities in the wealth distribution in the US (and most other countries), this is important to obtain an accurate picture of household finances.

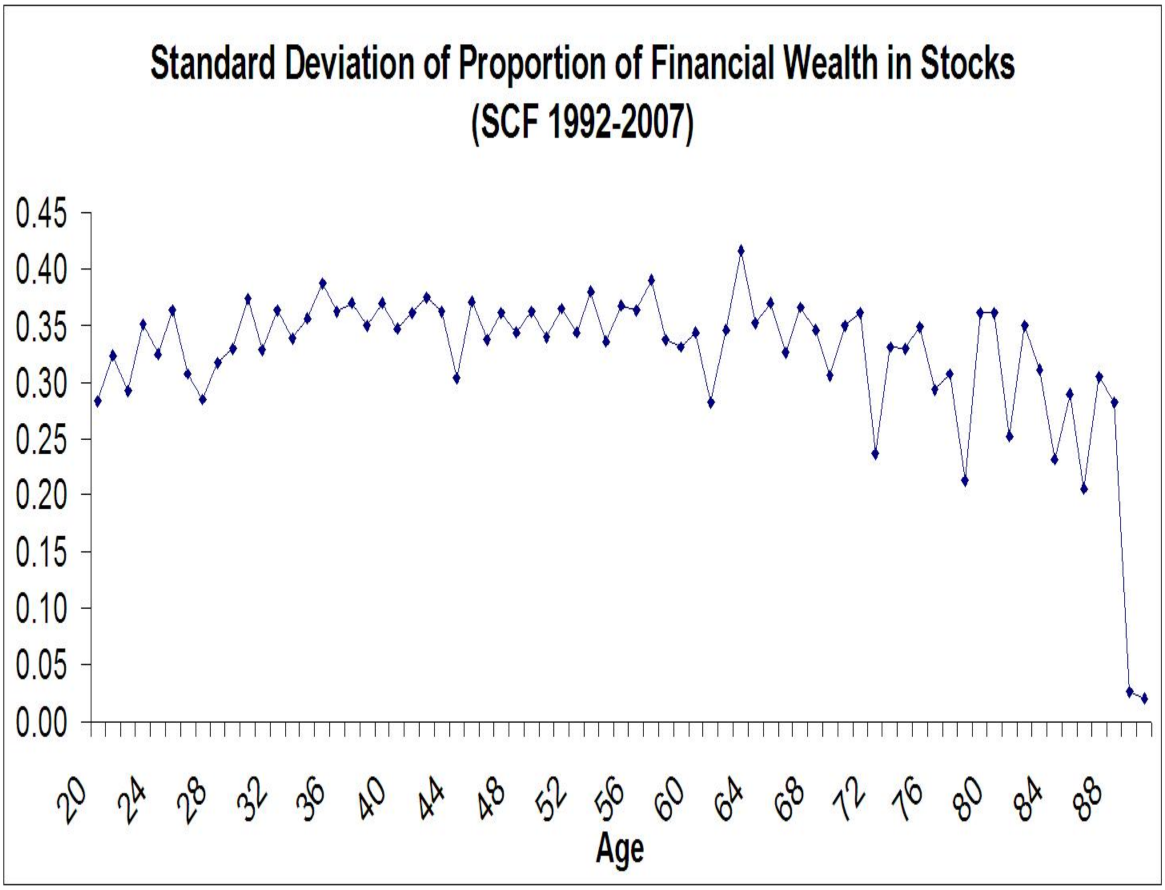
The SCF provides sample weights to ensure the computed statistics are representative of the population in the US. Also, missing observations are multiply (5 times) imputed using a multivariate multiple imputation procedure.

I examine how the proportion of financial wealth invested in equities varies over the life-cycle. However, there is an important identification problem which arises in the data due to the multi-collinearity between age, time and cohort effects (Age = Time − Cohort). Ameriks and Zeldes (2004) discusses the problem in detail and demonstrates that variation of these portfolio shares with age depends critically on the identification assumptions made. It isn't possible to identify all three effects, at most two independent linear combinations of these effects can be identified in the data.

Nevertheless, I present the average proportion of financial wealth invested in stocks across the life cycle. We can observe this follows a familiar hump shape, increasing for young agents and peaking around the age of 60 around retirement. This has been discovered by previous empirical studies on portfolio choice.

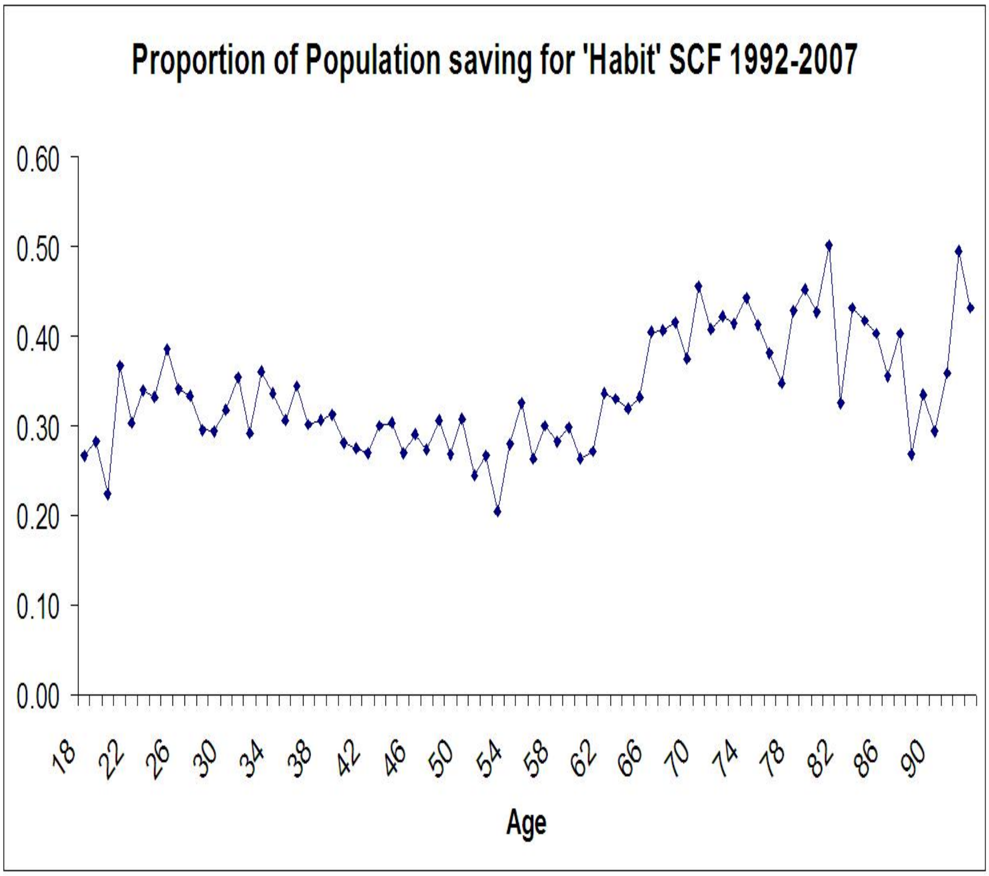


To compute the means for each age, I first compute the mean (for each age) for each implicate for the given year using the sample weights provided. Averaging the means across different implicates yields a mean (for each age) for the given year. The mean of each of these yearly means (for each age) is presented above. Since the sample weights provided reflect the population distribution at a particular point in time, it isn't appropriate to simply pool different observations across time.



The standard deviation of the proportion of financial wealth invested in stocks is presented. There is substantial variation conditional on age, it is roughly constant around 0.3 through the life cycle. To compute standard deviations, the variation between implicates is also incorporated as described in Montalto and Sung (1996).

Next, I provide some casual evidence which suggests that maintaining habit are indeed an important reason for individual agents to save. The SCF interviewers ask households what is their main reason for saving. Households are asked to state their six most important reasons, in order of importance, from roughly 35 possible answers. I choose 6 of these possible answers, corresponding closely to avoiding a decrease in future consumption, as maintaining habit. The graph presents the fraction of the population conditional on age, who gave 'maintaining habit' as their single most important reason to save.



We can observe, roughly 30-35% of the population for each age from ages 18 through 60 states habit as their main reason for saving. This number increases to roughly 40% after age 60, coinciding with retirement age for most persons. During their working and younger years agents save primarily for four reasons: for retirement, for habits, for purchasing assets and investing in education. After retirement, the two most important reported reasons for saving are for habits followed by for old age.

6 Conclusion

The introduction of habits to preferences has shown promise in explaining asset pricing puzzles such as the equity premium puzzle of Mehra and Prescott. Since puzzles in the portfolio choice literature are essentially partial equilibrium counterparts of asset pricing puzzles, I introduce habit formation in a finite horizon framework to study the effect on portfolio choice. The habit formation is internal, agents take into account the effect their consumption will have on future habits. The interpretation of habits is of a 'minimum acceptable standard of living' that the agent ensures he can sustain even if the stock market tanks. Habits evolve as a weighted average of past consumption.

I obtain analytical solutions for the optimal value and policy functions. Younger agents invest a smaller fraction of their wealth in stocks since they need to sustain their habits for a longer outstanding lifespan compared with otherwise identical older agents. The risk aversion of agents varies through the life cycle depending on how much wealth agents have in excess of the minimum required level of wealth to sustain habits in the future. Simulations of life cycle paths of variables (ratio of wealth to habit, ratio of consumption to habit, proportion of wealth invested in stocks) shows how their distributions evolve over the life cycle. The effects of different initial conditions lasts for roughly the first third of the life cycle.

Empirical evidence using data from the SCF is presented. Habits seem to be an important motive for saving by various households. The average proportion of financial wealth invested in equities by age is presented. For retired households, implications of the model are compared with this data.

7 Appendices

Appendix A: Model

Proof of Proposition 1: Assume the policy functions stated in Proposition 2 are indeed optimal. We will prove this in Proposition 2.

Substitute these policy functions and the conjectured value function into the HBJ equation. Using the definition of $\underline{y}(t)$, derived in Lemma 1, and substantial algebra (omitted for the sake of brevity) we are able to verify that the conjectured value function solves the HBJ equation and the coefficients $\phi(t)$ and $\kappa(t)$ must satisfy the stated ordinary differential equations.

We have shown that if the policy functions are as stated in Proposition 2 then the value function must as in Proposition 1.

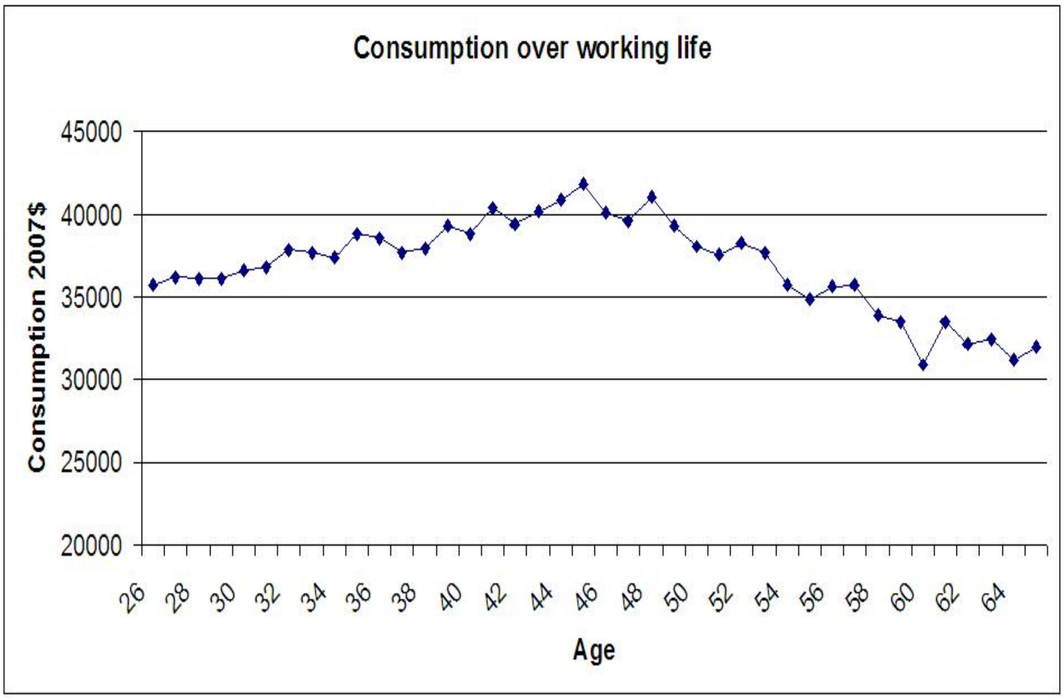
The properties for the coefficients, $\phi(t)$ can be verified analytically for the log case and numerically for the general case.

Proof of Proposition 2: We have to show that the stated policy functions are indeed optimal. Substituting the conjectured consumption policy function into the corresponding first order condition yields the definition for $\psi(t)$ in terms of $phi(t)$. The conjectured policy for α satisfies the corresponding first order condition.

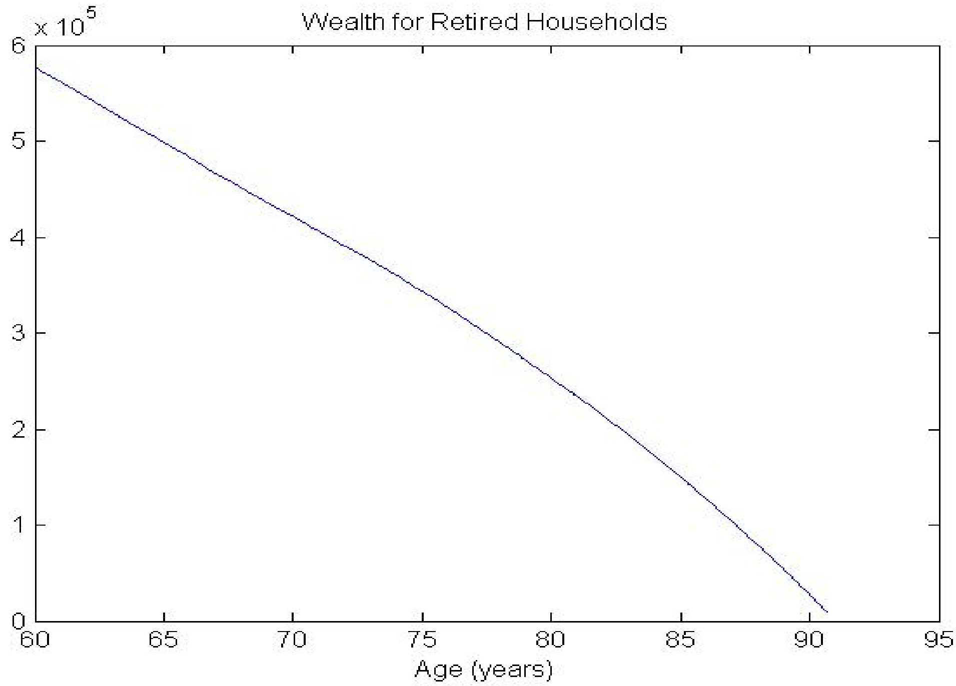
Properties of $\psi(t)$ follow from its definition and corresponding properties of $\phi(t)$.

Appendix B: Retirement Portfolios

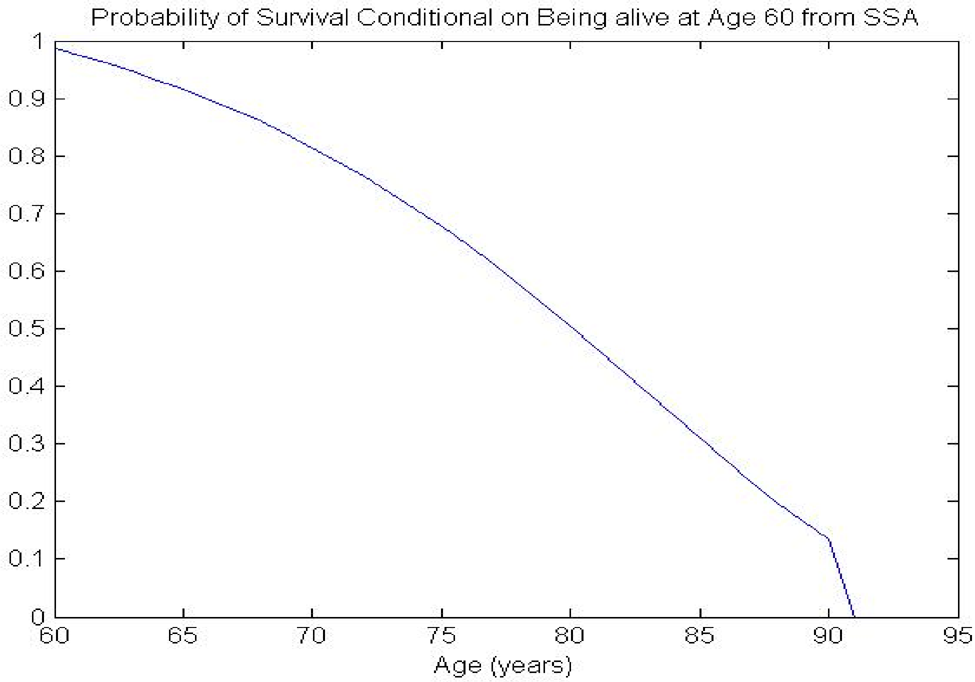
I use consumption profiles constructed by Gourinchas and Parker (2002), and adjust these for inflation using CPI estimates from the BLS website. For robustness, I also used the family level extracts from NBER Consumer Expenditure Survey (CEX) to construct the same consumption profile, the difference is negligible.



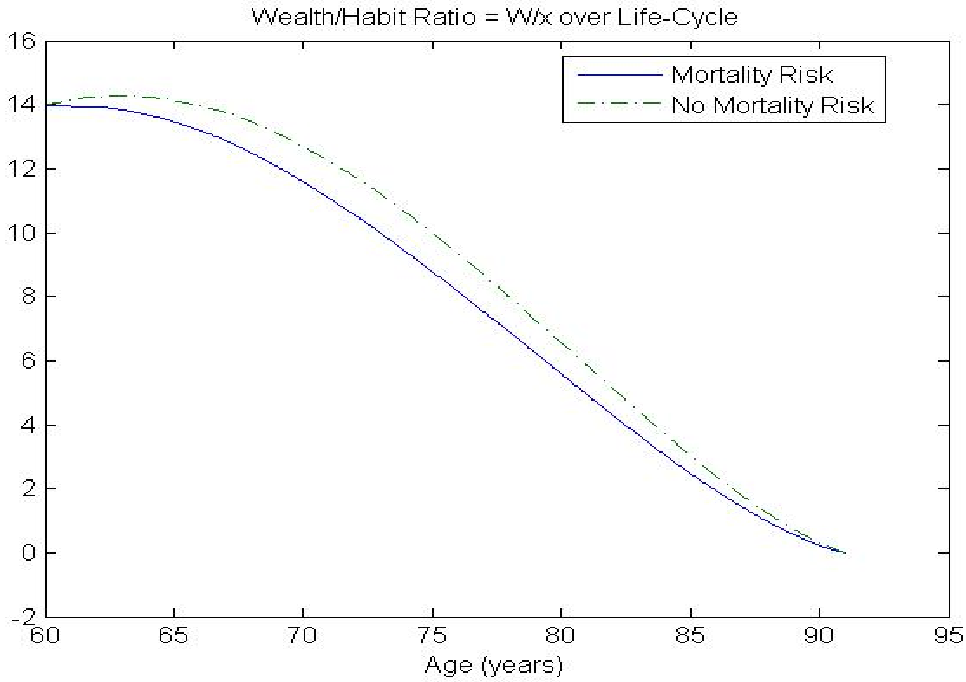
Evolution of wealth for retired households is presented next.



The survival probailities obtained from the life cycle tables compiled by the Social Security Administration (SSA) are presented.



The wealth-habit ratios are compared for the case with and without mortality risk.



Appendix C: SCF DATA

Financial wealth is computed as the sum of the following components:

- All liquid assets: Checking accounts, Saving accounts, Money-market accounts, Call accounts at brokerages. ,
- Certificates of deposits
- Total directly held mutual funds excluding money market mutual funds.
- Amount held in publicly traded stocks.
- Total bonds, not including bond funds or savings bonds.
- Savings Bonds.
- Total Quasi Liquid assets: sum of IRAs, thrift accounts, and future pensions includes currently received benefits. All IRA type accounts (401K, 403b etc) are included. No attempt is made to distinguish between accounts against which agents can/cannot borrow/withdraw.

- Cash value of whole life insurance.
- Other managed assets. Trusts, annuities and managed investment.
- Other financial assets: includes loans from the household to someone else, future proceeds, royalties, futures, non-public stock, deferred compensation, oil/gas/mineral invest., cash.

Equity is computed as the sum of the following components:

- directly-held stock
- stock mutual funds: full value if described as stock mutual fund, $\frac{1}{2}$ value of combination mutual funds
- IRAs/Keoghs invested in stock: full value if mostly invested in stock, $\frac{1}{2}$ value if split between stocks/bonds or stocks/money market, $\frac{1}{3}$ value if split between stocks/bonds/money market
- other managed assets w/equity interest (annuities, trusts, MIAs): full value if mostly invested in stock, $\frac{1}{2}$ value if split between stocks/MFs and bonds/CDs, or 'mixed/diversified' $\frac{1}{3}$ value if 'other'
- thrift-type retirement accounts invested in stock full value if mostly invested in stock $\frac{1}{2}$ value if split between stocks and interest earning assets;

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