The Role of Information in Economic Fluctuations

Javier Nunez[1] and Sergio Salas[2]

(Preliminary-November 2009)

Abstract

A dynamic game of incomplete information is developed to understand the allocative and welfare consequences of asymmetric information between the central bank and the public with regard to the future state of the economy. The framework is a signaling game where the sender, the central bank, uses monetary policy as a signal for the private agents, the receivers, to infer future shocks likely to hit the economy. The value of these shocks being the central bank's private information. Two economies are analyzed that differs only in the degree of price flexibility. One economy is completely flexible where the central bank wants to implement a price level target. The other economy has some degree of price stickiness, firms only adjust partially to changes in the level of money supplied. Our findings are that, as a consequence of the asymmetry of information, monetary policy followed by the central bank is not optimal, being in general more expansive than the socially optimum policy. The equilibrium of the game is such that the central bank conducts its policy in a way that leads to a higher level of prices than what is the optimal level with the resulting level of welfare being lower than with complete, symmetric information.

Keywords: Signaling, monetary policy, asymmetric information.
JEL Classification: E58, C72.

[1]Department of Economics, University of Chile, e-mail: jnunez@econ.uchile.cl
[2]Department of Economics, University of Chicago, e-mail:ssalas@uchicago.edu

1. Introduction

In many instances, firms and the public in general pay a lot of attention to forecasts about future economic conditions made by the central bank. The central bank might have private information regarding the future state of the economy, in particular, if the economy will experience expansionary shocks or negative ones. How should a central bank that cares about society should act in this case? Should the central bank reveal its information? If the economy will experience negative shocks, would the central bank find it optimal to try to mislead agents into believing otherwise?

In an environment with private information, it is natural think about a signal that agents might consider relevant for this transmission of information. In particular, we do not expect that rational consumers and investors be fooled into believing in discourse not backed up with actions that influence the actual behavior of the economy. For this reason, monetary policy may be the instrument that the central bank uses and is perceived by the public as a signal about the future state of the economy. The information transmission via the signal of the central bank would have different effects in the economy depending on how the public perceives and interpret it. To study these issues, we develop a simple signaling model between the central bank and a large group of firms, where at the beginning of the period a central bank privately knows if an expansionary or recessive shock will hit the economy. Firms only have a prior distribution over the possible realizations of the shock. Knowing this information, the central bank sets the supply of money, which is observed by the firms. They use a Bayesian update of their prior beliefs about the future shock to form a posterior distribution of the shocks given the observed monetary policy.

Two different economic structures that differ only on the degree of price stickiness are analyzed. In one economy prices are flexible, monetary policy is neutral but serves as a signal regarding the future state of the economy. In the other economy, we allow for price rigidity. In this last economy firms partially adjust prices upon observation of changes in the money supplied. In both economies, there is an additional role of monetary policy besides the one usually considered. In typical models of monetary policy, under some sort of price rigidities, money affects temporarily real variables. In the model developed in this paper, monetary policy might have a role influencing real variables even without price rigidities. This is because the signal effect of the information transmission is relevant in agent's decisions that influence real outcomes.

Within the context of a single period model, several related issues about money supply, price setting, output and welfare are addressed. First, it is investigated whether the central bank follows a more contractive or expansive monetary policy under symmetric versus asymmetric information about future shocks. We also characterize the level of inflation and product under each case. Finally, we study whether society is better off in a regime of symmetric or asymmetric information.

The findings are the following: For the flexible price economy, I show that the central bank cannot commit to a single price level target independently of the shock hitting the economy. The central bank observing an expansive shock conducts a more expansionary policy compared to the symmetric complete information case which leads to more inflation. This creates a higher price level and on average, the level of welfare is reduced compared to the symmetric complete information case.

When considering the sticky price economy we also find non-pleasant results. Monetary policy is again more expansive under the expansionary shock, and even tough the level of product is also higher, the welfare level is inferior with respect to the symmetric information benchmark.

As a corollary of the results of the model we find an "inflation bias" feature, which arises entirely because of the incomplete and asymmetric information structure of the model and is unrelated to the traditional policy inconsistency result.

Related literature to the specific questions addressed in this paper are scarce. This paper does not deal directly with the issue of time inconsistency in monetary policy on which there is a lot of investigations. One feature that this paper share in common with some papers in the literature has to do with the asymmetry of information between the central bank and the public. Canzoneri (1985), Sleet (2001), Athey et. al. (2004) they all have as main ingredient that the central bank might posses some private information regarding the state of the economy. The issues studied in these papers are related to the time inconsistency problem. These studies consider the important case where the central bank might inflate the economy to try to surprise agents in order to maximize welfare. When the central bank has some private information he can always claim that there was not any attempt to surprises agents, but its change in policy is an optimal response to changes in his private knowledge about the state of the economy. Since the view adopted in these papers is that the central bank is the institution through which society conducts its will, a mechanism design is regarded as the conceptual framework to think about these issues. In particular, these papers design the optimal contract for the central bank in terms of giving the correct amount of discretion to conduct its policy taking into consideration the temptation to surprise agents.

This paper differs from this strand of the literature. At the outset we ruled out the possibility that a contract for the central bank might be written or enforceable. This assumption seems more reasonable, at least for many countries where the institutional framework might not be strong enough for making plausible the view that society might design a contract for the central bank. Instead, we take the point of view that the central bank is regarded as a player in the economy who posses private information and who will signal through its policy the information that he has with the aim of maximizing society's welfare.

The rest of the paper is organized as follows: In section II we present the flexible price model. Section III take the same underlying economy but with the difference that prices are sticky. Section IV presents the conclusions.

II. The Model

The economy is composed of a measure one of households-producers, agents for short, with utility function:

$$U(i) = \left(\frac{C(i)}{\gamma}\right)^{\gamma} \left(\frac{M(i)'/\mathcal{P}}{1-\gamma}\right)^{1-\gamma} - \frac{1}{\psi} N(i)^{\psi}, \quad 0 < \gamma < 1, \; \psi > 1 \tag{1}$$

Agents derive utility from the consumption of an index of aggregate consumption $C(i)$, defined by :

$$C(i) = \left[\int_0^1 c_i(j)^{\frac{\eta-1}{\eta}} dj\right]^{\frac{\eta}{\eta-1}}, \quad \eta > 1$$

Each agent i, is the only producer of good $c(i)$, but derives utility from the consumption of all $[0,1]$ goods produced in the economy. $c_i(j)$ is the level of consumption of agent i of good j.

Agents derive utility from the services of real money balances, $M(i)'$ being the amount of money held by agent i. $\mathcal{P}$ is an index of prices defined by an aggregate of the prices set by all the agents in the economy:

$$\mathcal{P} = \left[\int_0^1 P(i)^{1-\eta} di\right]^{\frac{1}{1-\eta}} \tag{2}$$

They have access to the following technology of production:

$$Y(i) = \theta N(i), \quad \theta > 1 \tag{3}$$

Where we normalize the amount of labor: $N(i) \in [0,1]$. θ is a productivity parameter, the state of the economy. As it will be stated clearly later on, it is assumed that the central bank has private knowledge about the value of this parameter and agents will have to set prices not knowing the value of this shock.

Replacing the production function in the utility function we obtain:

$$U(i) = \left(\frac{C(i)}{\gamma}\right)^{\gamma} \left(\frac{M(i)'/\mathcal{P}}{1-\gamma}\right)^{1-\gamma} - \frac{1}{\psi}\left(\frac{Y(i)}{\theta}\right)^{\psi} \tag{4}$$

Agents incur in costly production that is reflected in the second term in the utility function. ψ will determine how costly is for the agent to produce a given level of output $Y(i)$. In particular, the elasticity of marginal disutility with respect to output is given by $\psi - 1$.

The budget constraint that the agents face, is given by:

$$\int_0^1 P(j)c_i(j)dj + M(i)' = M_0(i) + P(i)Y(i) \equiv I(i) \tag{5}$$

$M_0(i)$ are the initial money holdings of the agent. We denote total income of the agent by $I(i)$.

Let us analyze the optimization problem of the firms that set prices after observing the stock of money supplied by the central bank, but without observing the value of the shock that will hit the economy at the end of the period. To solve their optimization problem let us compute the demand they will face conditional on a given realization for the shock θ.

Utility maximization implies the following values of the variables: The consumption of good j from household i is given by:

$$c_i(j) = \left(\frac{P(j)}{\mathcal{P}}\right)^{-\eta} C(i)$$

The values for the individual's aggregate index of consumption $C(i)$ and money demand $M(i)'$ are given by:

$$\begin{aligned} C(i)(\theta) &= \gamma\frac{I(i)}{\mathcal{P}} \\ M(i)' &= (1-\gamma)I(i) \end{aligned}$$

With these values, producer (i) can compute the partial indirect utility function which is given by:

$$V(i) = \frac{M(i)}{\mathcal{P}} + \frac{P(i)}{\mathcal{P}}Y(i) - \frac{1}{\psi}\left(\frac{Y(i)}{\theta}\right)^{\psi} \tag{6}$$

Market demand for good j is given by:

$$\begin{aligned} Y(j) = \int_0^1 c_i(j)di &= \left(\frac{P(j)}{\mathcal{P}}\right)^{-\eta} \int_0^1 C(i)di \\ &= \left(\frac{P(j)}{\mathcal{P}}\right)^{-\eta} \frac{\gamma}{\mathcal{P}} \int_0^1 I(i)di \end{aligned}$$

where in the second line, we have used the demand for the individual's aggregate consumption derived before.

Also notice that demand of money for agent (i) implies:

$$M' = \int_0^1 M(i)'di = (1-\gamma)\int_0^1 I(i)di$$

and hence demand for product j is given by:

$$Y(j) = \frac{\gamma}{1-\gamma}\left(\frac{P(j)}{\mathcal{P}}\right)^{-\eta} \frac{M'}{\mathcal{P}} \tag{7}$$

Also, agents will hold any amount of money supplied: $M = M'$.

Discussion

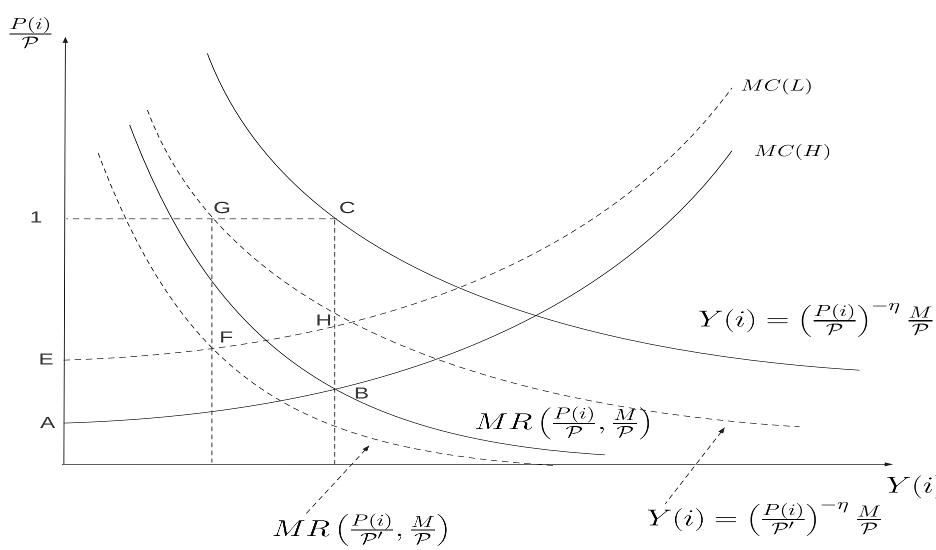
To understand better the issues involved in the following section, let us explain the mechanisms underlying the formation of demand for products and money and their relationship to the assumption that firms are unable to change prices once the shock hit the economy. The situation we want to focus on is the case where firms are planning to set prices for their products that will remain invariant in the near future, and they are unsure about the conditions that will prevail in the economy, summarized in our model by the parameter θ. Firms might draw some inferences about the state of the economy based in the observed monetary policy. Any movements (or lack thereof) in the stock of money supplied might give information to them that is useful in setting an optimal price level. Hence, while any movements in the money supplied by the central bank can be incorporated in their decisions of price setting in the traditional "neutral" way, monetary policy also has a value of conveying information about future economic conditions, to which agents, we assume, are unable to react optimally once those conditions realize.

Let us explain the nature of the costs (and benefits) that arise when agents make a mistake in their prediction of the state of the economy. To explain this, let us derive the optimal price that a firm would charge for a given value of the state θ. This is the solution to the maximization of (6) subject to the demand (7). This gives:

$$\frac{P(i)}{\mathcal{P}} = \left(\frac{\eta}{\eta-1}\frac{1}{\theta^{\psi}}\left(\frac{M}{\mathcal{P}}\right)^{\psi-1}\right)^{\frac{1}{1+\eta(\psi-1)}}$$

This pricing equation is well known in this type of models. The relative price is increasing in the "mark-up" $\eta/(\eta-1)$ and decreasing in the level of the shock θ. Let us assume away for the moment any signaling issues in this economy, and work with a given level of money supply M. We ask: what are the consequences for welfare that arise as firms make a mistake in their prediction of θ? For example what if firms believe that θ will be high H, expansionary, when in fact is recessive L ($L < H$)?

To following figure illustrates the case. First notice that in any situation the relative price implied by the equation above must be 1. Firms will set prices such that believed or expected marginal cost equals marginal revenue. Notice that the demand and hence marginal revenue are independent of the actual realization of the shock. This simplifies the analysis a lot, because there are no wealth effects on the demand for the products as a consequence of changes in the parameter θ. The cost of any mistake will be reflected entirely on the welfare that agents derive from producing at a higher cost. When agents believe that θ is H They will set a level of prices such that the relative price is one. And the aggregate price level is $\mathcal{P}$. In the figure $MC(\theta)$ is the marginal cost when the shock is θ. If H were effectively the value of the shock, then profits make by the firm will be equal to the area $A, B, C, 1$.



When firms are able to change their prices under the new conditions on the state of the economy, they will charge higher prices. Symmetry among producers will imply the same relative prices among firms but a higher price level $\mathcal{P}' > \mathcal{P}$. This will shift demand at the new lower level, depicted with the dashed curve in the figure, and profits will be given by: $EFG1$. Now we ask, what if firms cannot change prices, and have set them believing that a good shock will hit the economy when in fact a low shock does?. Then firms will sell their product at the original prices, and there will be no further movements in the demand that they face. They will produce at a higher cost, and their profits will be given by $EHC1$. Compared to the situation when they are able to change prices, they actually end up with a higher level of profits!. The extra amount compared to the situation when they can change prices is given by $FHCG$. This is a reflection of the aggregate demand externality identified by Blanchard and Kiyotaki (1987). All firms would like to actually make a mistake if all others are making a mistake too. This observation will be the driving force of the tension that exists in the game between the central bank and the agents. Even when the central bank is a Ramsey government, its monetary policy will have an strategic component that affects the allocation of resources in the economy.

II.I. A Flexible Price Economy

In this section we analyze a situation where the economy is characterized by complete flexibility in prices. By flexibility of prices, we mean that all firms will observe the money supplied by the central bank and then they will set prices. They won't know however, the value of the shock that hits the economy, and once the shock hits the economy they are unable to change their prices.

We assume that the central bank wishes to implement a price target $\mathcal{P} = P^T$. When the central bank observes a given value of the shock θ it will adjust the money supplied to meet this target. Let us formalize the structure of the economy as a signaling game between the central bank and the fringe of firms in the economy.

The Game

We define the set of types in the game theoretic language as the possible values the shock can take:

$$\Theta = \{H, L\}$$

We denote the generic value of this shock as θ.

It is common knowledge that agents in the economy have the prior distribution over the shocks given by:

$$\theta \begin{cases} H & q \\ L & 1-q \end{cases}$$

The signal that the sender, the central bank can send is given by $M \in \mathcal{M} \in \Re^{+}$. Where $\mathcal{M}$ is defined as:

$$\mathcal{M} = [M_{min}, M_{max}]$$

Finally the actions that agents can take in this economy is given by:

$$P(i) \in \pi \equiv [0, \infty]$$

Agents in this economy will be the receivers in this signaling game. The utility function of each of this continuum $[0, 1]$ of firms is given by equation (6):

$$V(i)(\theta, M, P(i)) = \frac{M(i)}{\mathcal{P}} + \frac{P(i)}{\mathcal{P}} Y(i) - \frac{1}{\psi} \left(\frac{Y(i)}{\theta} \right)^{\psi}$$

with :

$$Y(i) = \left(\frac{P(i)}{\mathcal{P}} \right)^{-\eta} \frac{M}{\mathcal{P}}$$

Notice that we have explicitly denoted the utility function of the agent as depending on the three relevant quantities for the game: the type of the economy θ, the message M and the action that they can take $P(i)$.

The sender in this signaling game is the central bank, who has a utility function that is the same as the agents in the economy. The central bank takes into account however, that the collective actions of the atomistic firms have influence in the aggregate values of variables in the model, specifically the central bank takes into account that the decisions in price setting of the firms will have an influence in the price level $\mathcal{P}$. Moreover in any equilibrium analyzed in this section, given the symmetry of the agents, they will all take the same action, they will set the same level of prices and hence the utility of the central bank can be expressed as the aggregate utility of society that will be given by:

$$\begin{aligned} V(\theta, M, \mathcal{P}) &= \int_0^1 V(i)(\theta, M, P(i)) di \\ &= \frac{M_0}{\mathcal{P}} + Y - \frac{1}{\psi} \left(\frac{Y}{\theta} \right)^{\psi} \end{aligned}$$

with :

$$Y = \frac{M}{\mathcal{P}}$$

Defining the Nash equilibrium of the game

We restrict attention to the pure strategy Nash equilibrium analysis. The strategies are defined as:

For the sender:

$$M : \Theta \mapsto \mathcal{M}$$

For the receivers, the fringe of firms:

$$P(i) : \mathcal{M} \mapsto \pi$$

Strategies M^* and $P^*(i)$ for the sender and for the receiver respectively form a Nash Equilibrium if and only if:

$$M^*(\theta) = \arg\max_{M \in \mathcal{M}} V(\theta, M, \mathcal{P}^*)$$

where:

$$\mathcal{P}^* = \left[\int_0^1 P^*(i)^{1-\eta} di\right]^{\frac{1}{1-\eta}}$$

and for each M and each agent (i):

$$\begin{aligned} P(i)^* &= \arg\max_{P(i) \in \pi} E_\mu[V(i)(\theta, M^*(\theta), P(i))] \\ &= \arg\max_{P(i) \in \pi} [\mu V(i)(L, M^*(L), P(i)) + (1-\mu) V(i)(\theta, M^*(H), P(i))] \end{aligned}$$

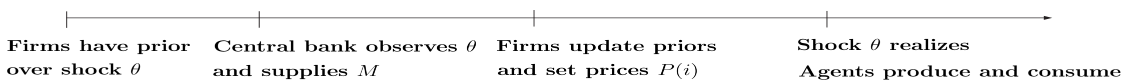
where:

$$\begin{aligned} \mu = Pr(\theta = H|M) &= \frac{Pr(M|\theta = H)Pr(H)}{Pr(M|\theta = H)Pr(H) + Pr(M|\theta = L)Pr(L)} \\ &= \frac{Pr(M|\theta = H)q}{Pr(M|\theta = H)q + Pr(M|\theta = L)(1-q)} \end{aligned}$$

if $Pr(M|\theta = H)q + Pr(M|\theta = L)(1-q) > 0$

We will use this definition to proof the existence of the equilibrium in the following sections. To be more specific about the timing of the game, we present the following figure:

TIMING OF THE GAME



The single crossing condition

To begin characterizing the equilibria of this game, we start by verifying the single crossing condition property. We ask if there is a difference in terms of cost for the central bank observing a high shock to supply a given amount of money versus a central bank observing a low shock. In particular, to verify the single crossing condition property, we compute the marginal rate of substitution for the central bank's utility when both M and $\mathcal{P}$ varies. We verify that these values which are the slopes of the indifference curves in the $M, \mathcal{P}$ plane cross only once when we compute them for the high shock type and for the low shock type respectively.

The next Lemma shows that the single crossing condition holds.

Lemma 1 *The single crossing condition holds:*

$$-\left.\frac{\partial V/\partial M}{\partial V/\partial \mathcal{P}}\right|_{\theta=H} > -\left.\frac{\partial V/\partial M}{\partial V/\partial \mathcal{P}}\right|_{\theta=L}$$

. *Proof*:

For any given value of the shock θ, we have:

$$\frac{\partial V}{\partial M} = \frac{1}{\mathcal{P}} - \left(\frac{1}{\theta \mathcal{P}}\right)^{\psi} M^{\psi-1}$$

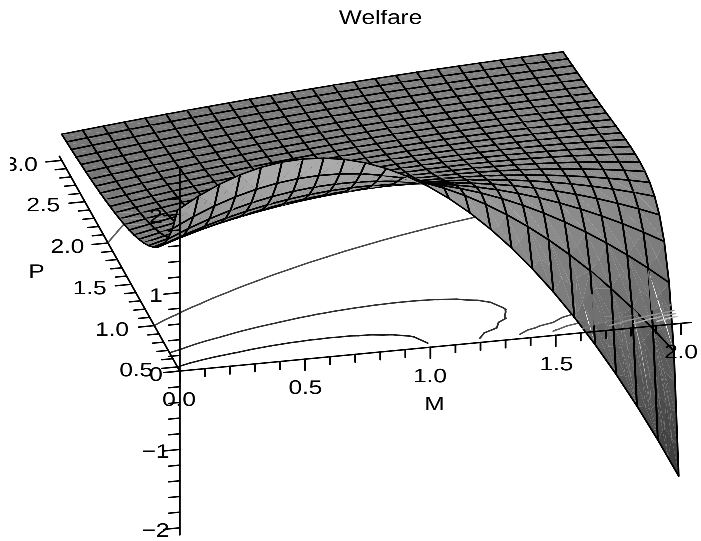
and:

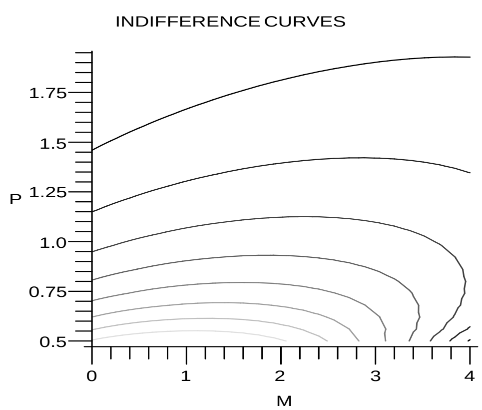
$$\frac{\partial V}{\partial \mathcal{P}} = -\frac{M_0}{\mathcal{P}^2} - \frac{M}{\mathcal{P}^2} + \left(\frac{M}{\theta \mathcal{P}}\right)^{\psi-1} \frac{M}{\theta \mathcal{P}^2}$$

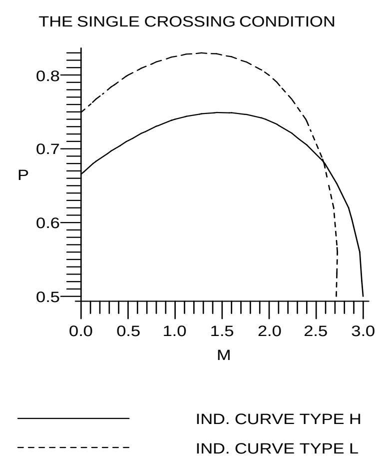
The first equations express the marginal benefit of an increase in money supply. The second equation expresses the marginal benefit of an increase in the price level. Hence the marginal rate of substitution is given by:

$$-\left.\frac{\partial V/\partial M}{\partial V/\partial M}\right|_{\theta} = \frac{\frac{1}{\mathcal{P}} - \left(\frac{M}{\theta \mathcal{P}}\right)^{\psi} \frac{1}{M}}{\frac{1}{\mathcal{P}}\left[\frac{M_0}{\mathcal{P}} + M\left(\frac{1}{\mathcal{P}} - \left(\frac{M}{\theta \mathcal{P}}\right)^{\psi} \frac{1}{M}\right)\right]}$$

Since the same term appears both in the numerator and the denominator: $\frac{1}{\mathcal{P}} - \left(\frac{M}{\theta \mathcal{P}}\right)^{\psi} \frac{1}{M}$ clearly increments in this term will increase the whole expression. The slope of the indifference curves will not be always of the same sign. It is possible to show that the slope as we vary the money supply is positive for low levels of money, then reaches zero, becomes negative and then positive again. The next figures show the welfare function and the single crossing condition property using an example with specific values for the parameters of the problem.







In the next section we characterize optimal monetary policy in the model for the case when both the central bank and the agents in the economy have the same information. We assume that they both observe the value of the shock hitting the economy, this is the symmetric complete information benchmark.

II.II. The Symmetric Information Benchmark

In this section we compute the equilibrium when both the agents and the central bank have the same information about the shock hitting the economy. Both observe the shock and act optimally, the central bank setting the level of money and the agents setting the level of prices. We assume that the central bank commits to maintain a given level of prices, the target P^T.

Agents, observing the value of the shock and the money supplied by the central bank will maximize $V(i)$, equation (6). The optimal price is given by:

$$\frac{P(i)}{\mathcal{P}} = \left(\frac{\eta}{\eta - 1} \frac{1}{\theta^{\psi}} \left(\frac{M}{\mathcal{P}} \right)^{\psi - 1} \right)^{\frac{1}{1+\eta(\psi-1)}} \tag{8}$$

Let us briefly explain this equation. First (8) says that the relative price that firms set is a function of what we can call the mark-up over marginal cost, the term $\eta/(\eta - 1)$. This term will be high when the elasticity of demand of the product is small. Also notice that the relative price is decreasing in the level of the shock. This is the case because a high shock is equivalent to a low cost of production. In this case the cost of production is measured in terms of the disutility of labor. Second, when firms know that a high productivity shock hits the economy they would find optimally to set lower level of prices. Finally the desired optimal price is an increasing function of the money supplied by the central bank M.

Since all firms are symmetric they will charge the same price, and hence the value of real balances will be given by:

$$\frac{M}{\mathcal{P}} = \left(\frac{\eta/(\eta - 1)}{\theta^{\psi}} \right)^{\frac{1}{\psi-1}}$$

Using equation (7) it is possible to see that each firm's product will be given by[3]:

$$Y(i) = \frac{M}{P} = \left(\frac{\eta/(\eta - 1)}{\theta^{\psi}} \right)^{\frac{1}{\psi-1}} \tag{9}$$

From this expression it is evident that money is neutral for the product of each firm, and hence for the product and consumption of all the agents in the economy. The utility function for the central bank can be written in this case as:

[3] For the rest of the paper we assume $\gamma = 1/2$, which corresponds to equal shares of consumption and real money balances in the utility function.

$$V = \frac{M_0}{\mathcal{P}} + \left(\frac{\eta}{\eta-1} - \frac{1}{\psi}\right)\left(\frac{\eta-1}{\eta}\right)^{\frac{\psi}{\psi-1}} \theta^{\frac{\psi}{\psi-1}} \tag{10}$$

The first term in parenthesis is always positive, then a higher shock is always beneficial. It can also be shown that increments in $\eta/(\eta-1)$, which happens as markets become more competitive, increases welfare. This is a reflection of the externality present in the economy, each agent will be better of if all could coordinate in reducing prices and expanding product. See Blanchard and Kiyotaki (1987) for a discussion. As explained in the previous section, the presence of this externality implies that the central bank could have an incentive to misrepresent its information once we consider the asymmetric information case, we will return to this issue shortly.

To begin characterizing the symmetric information case, we point out the relevance of the assumption that the central bank has a price level target P^T. When the central bank commits to maintain the price level at the target it must be the case that:

$$P^T = \mathcal{P}^* = \left(\frac{\eta/(\eta-1)}{\theta^\psi}\right)^{\frac{1}{\psi-1}} M \tag{11}$$

In this signaling game, each firm acts optimizing his utility function. The aggregate behavior it is what matters for the central bank. The optimal action taken by the agents in the economy can be summarized by the reaction function:

$$\mathcal{P}^* = \left(\frac{\eta/(\eta-1)}{\theta^\psi}\right)^{\frac{1}{\psi-1}} M \tag{12}$$

Next, using these results we characterize the symmetric information equilibria of the game where the central bank commits to maintain a price level P^T independently of the type of shock hitting the economy.

Lemma 2 *Let us denote $M^*(\theta)$ the amount of money supplied by the central bank upon observing the shock θ. When the central bank commits to maintain a price level P^T independently of the shock hitting the economy, then:*

$$\begin{aligned} M^*(H) &> M^*(L) \\ V(H, M^*(H), P^T) &> V(L, M^*(L), P^T) \end{aligned}$$

where $V(\theta, M^*(\theta), P^T)$ is the welfare level of society when the central bank conducts its monetary policy to achieve the target P^T.Proof:

First, when the central bank targets P^T it must induce agents to set prices such that this level is attained. This implies that:

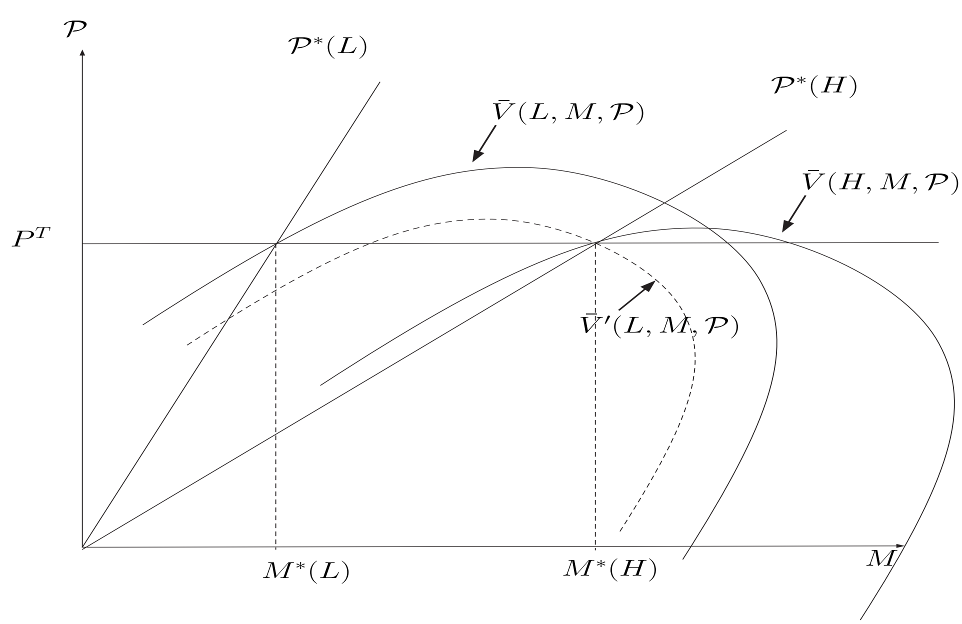
$$M^*(\theta) = \left(\frac{\theta^\psi}{\eta/(\eta-1)}\right)^{\frac{1}{\psi-1}} P^T$$

This proofs the first inequality. Notice also that the higher the monopolistic power of the firms as measured by $\eta/(\eta-1)$, the higher the level of prices that firms will set, and then to achieve the target P^T the central bank must supply a lower level of money.

Next, replacing this level of money into the welfare level we have:

$$\begin{aligned} V(\theta, M(\theta), P^T) &= \frac{M_0}{P^T} + \left(\frac{\theta^\psi}{\eta/(\eta-1)}\right)^{\frac{1}{\psi-1}} - \frac{1}{\psi}\left(\frac{1}{\theta}\left(\frac{\theta^\psi}{\eta/(\eta-1)}\right)^{\frac{1}{\psi-1}}\right)^\psi \\ &= \frac{M_0}{P^T} + \left(\frac{\theta}{\eta/(\eta-1)}\right)^{\frac{\psi}{\psi-1}}\left(\frac{\eta}{\eta-1} - \frac{1}{\psi}\right) \end{aligned}$$

This function is essentially the same as the one derived before in equation (10), but under the price level target. This shows that agents are unable to internalize the externality caused by the monopolistic competition. And the central bank cannot help either with its monetary policy to eliminate this distortion. The next figure shows what the equilibrium look like under the target regime.



Several things are important to explain from the figure. First the two solid curves represent the indifference curves for the welfare function of the central bank for the high and low shocks respectively. The direction of increasing utility is southwest, and, as we have seen before, the level of utility reached by the high shock is always higher than the utility under the low shock. Second, the best response of the agents in the economy are the two upward sloping lines. The central bank when conducting expansionary monetary policy, only influences the price level, so monetary policy will trace this lines. Higher monetary policy will yield higher level of prices under the two shocks, although the increment in prices would be lower for the high shock type. This is so because under the high shock, optimizing agents will set lower prices, and hence expansionary monetary policy is less inflationary in this case. Third, if the central bank wishes to implement the same price level target P^T, it will conduct a more expansionary policy under the high shock yielding to a level $M^*(H)$ which is higher than $M^*(L)$. Forth and most important, notice that the low type L central bank would like to copy the level of money set by the high type H, since in this case welfare improves for the low type central bank. This requires that the reaction function for firms will still be given by the flatter line. As we will see this won't be the case in equilibrium. Next, for illustration we examine conditions under which the central bank would be tempted to make agents believe that a high shock will hit the economy when a low shock does. The central bank might choose $M^*(H)$ in this case, if by doing so effectively induces agents to believe a high shock will hit the economy.

The existence of "Envy"

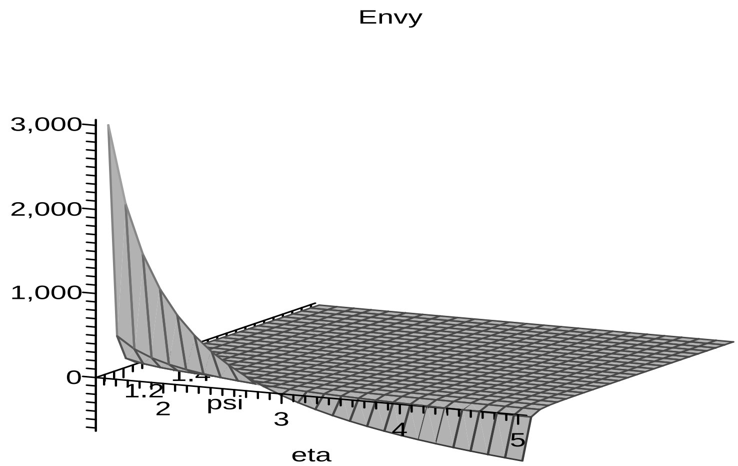
We ask under what conditions the central bank observing a low shock will implement the monetary policy that the central bank observing a high shock if by doing so will induce agents to believe a good shock is coming. This case that we might call "envy" means that:

$$V(L, M^*(H), P^T) > V(L, M^*(L), P^T)$$

Direct computation of the value of these welfare functions gives that the condition will hold if and only if:

$$\frac{\eta}{\eta - 1}\left(H^{\frac{\psi}{\psi-1}} - L^{\frac{\psi}{\psi-1}}\right) > \frac{1}{\psi}\left(\frac{H^{\frac{\psi}{\psi-1}}}{L}\right)^{\psi} - \frac{1}{\psi}\left(\frac{L^{\frac{\psi}{\psi-1}}}{L}\right)^{\psi}$$

In the next figure we graph the difference between this two functions, envy is present when the resulting function is positive.



The graphic was computed with given levels of H and L. We can see that envy will be present for sufficiently low level of elasticity of demand. The higher the difference between H and L the lower is the required value of the elasticity of demand η for envy to be present. The intuition is that the central bank is tempted to mislead agents in order to eliminate the distortion that is produced by the monopolistic environment. By setting $M^{*}(H)$ and if this induces agents believe that in fact a good shock will hit the economy, the central bank induces a reduction in prices that tend to eliminate the dead weight loss associated to the monopolistic environment because this will induce a higher level or product in the economy. If agents reduce prices too much tough, then the cost of the private mistake will translate into a social cost given that firms will incur in costly production in terms of the disutility of labor.

II.III. The Separating Equilibrium

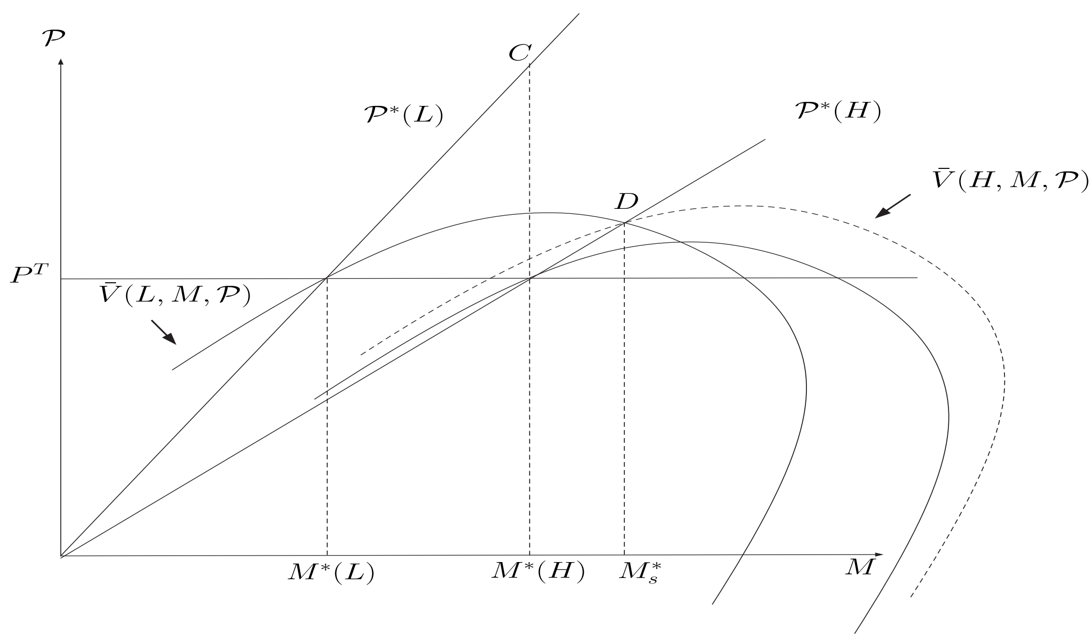
In this section we address the issue of equilibrium in the signaling game. Recall from the previous section that the central bank observing a low shock will try to supply the symmetric information level of money that the other type would have chosen if the economy were to be hit by a low shock. This will be the case if the agents believe that actually a good shock will hit the economy. This situation will not happen in equilibrium. The reason is that agents are atomistic. If they could somehow could coordinate to "fall for the trick" they will all be better off. They would all set a lower level of prices that what is privately optimal, but since there is an externality, the benefit in terms of higher production and consumption will more than compensate the private costs and they will get a higher welfare. The problem is that if any given agent set a low level of prices while the others don't, then the private consequences are very negative. This inability of coordination will imply that agents believe that a low shock will hit the economy when observing the level of money that a high type central bank would have set. The consequences for welfare of such an outcome will not be desirable. One alternative is that the central bank distorts with respect to symmetric information, the level of money supplied in the economy. This we will see will support the existence of a separating equilibrium. To discuss this issues before formally analyzing the existence of a separating equilibrium, we show in the following graphic what the separating equilibrium would look like.

The next figure is similar to the one presented before. Here it can be seen that if the central bank observing a high shock H supplies $M^*(H)$ to the economy, and agents assign zero probability that a low shock will hit the economy, the outcome is point C, which implies much lower level of welfare than the one implied when setting $M^*(L)$. Also, even when actually a high shock hits the economy while the central bank is supplying $M^*(H)$, the level of welfare will be the one implied by point C.

We want to examine properties of a separating equilibrium, in such an equilibrium the level of the shocks are revealed completely once agents observe the signal. So let us denote $P^*(i)(\theta)$ as the value that firms set when the shock is θ, and the induced aggregate level of prices as $\mathcal{P}^*(\theta)$. Then:

$$\mathcal{P}^*(\theta) = \left(\frac{\eta/(\eta-1)}{\theta^{\psi}}\right)^{\frac{1}{\psi-1}} M$$

Notice that this value reflects that the belief assigned to a given shock θ hitting the economy is correct, and since the money supplied is observed, firms adjust prices to any movements in money.



We will see shortly that the equilibrium implies that the central bank supplies a higher level of money than $M^*(H)$. This is optimal because the high type central bank wants to signal that is actually a high type, by sending a higher signal M it makes more costly for the other type (low shock L) to imitate him. When the high type central bank convinces agents that he is actually a high type, the equilibrium should be read along the $\mathcal{P}^*(H) = \left(\frac{\eta/(\eta-1)}{H^\psi}\right)^{\frac{1}{\psi-1}} M$ curve, and then the least cost way to separate from the low type is to implement the allocation D. In this point, the central bank observing a low shock has no incentive to copy the high type central bank, and hence the doubts of agents regarding the possibility that the central bank wants to mislead them are dissipated. The cost associated at this outcome can be seen by the lower level of welfare that is attained when the economy is hit by a high shock, the dashed indifference curve in the figure higher than the symmetric information one. Next we show formally the existence of a separating equilibrium. We will specify condition under which such an equilibrium will arise giving more intuition for the results.

As explained in the previous section, the posterior distribution over the shock is given by:

$$\theta|M = \begin{cases} H & \mu \\ L & 1-\mu \end{cases}$$

with this definition we are ready to investigate the existence of a separating equilibrium.

Proposition 1 *There exists a continuum of separating equilibria defined by the nonempty set S, such that:*

$$V(L, M^*(L), P^T) \geq V(L, M_s^*, P^s) \tag{13}$$

$$V(H, M^*(L), P^T) \leq V(H, M_s^*, P^s) \tag{14}$$

where $M_s^* \in S$, and P^s is associated price level. Agent's equilibrium beliefs are given by:

$$\mu = \begin{cases} 0 & \text{if} \quad M = M^*(H) \\ 1 & \text{if} \quad M = M_s^* \end{cases}$$

and out of equilibrium beliefs:

$$\mu = \begin{cases} 0 & \text{if} \quad M < M_s^* \\ 1 & \text{if} \quad M \geq M_s^* \end{cases}$$

and with individual prices yielding the aggregate:

$$\mathcal{P}^* = \begin{cases} \mathcal{P}^*(L) = \frac{\eta/(\eta-1)}{L^\psi} M & \text{if} \quad M < M^s \\ \mathcal{P}^*(H) = \frac{\eta/(\eta-1)}{H^\psi} M & \text{if} \quad M \geq M^s \end{cases}$$

Proof:

Let us begin by adding $V(L, M(H), \mathcal{P}^*) = \frac{M_0}{P^T} + \left(\frac{H^\psi}{\eta/(\eta-1)}\right)^{\frac{1}{\psi-1}} - \frac{1}{\psi}\left(\frac{1}{L}\left(\frac{H^\psi}{\eta/(\eta-1)}\right)^{\frac{1}{\psi-1}}\right)^\psi$ to the negative of both sides of inequality (1):

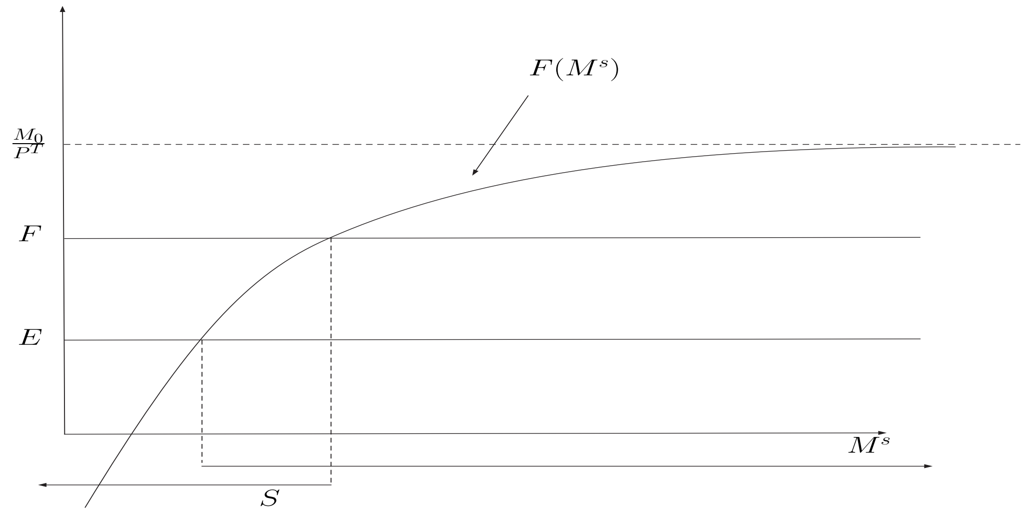
$$E \equiv V(L, M(H), P^T) - V(L, M(L), P^T) \leq \frac{M_0}{P^T} - \frac{M_0}{((\eta/\eta-1)/H^\psi)^{\frac{1}{\psi-1}} M^s} = f(M^s)$$

The value E is strictly positive under the "envy" assumption. Also as M^s is increased the right hand side increases. To guarantee that the right hand side is higher than E we require that P^T is sufficiently low (see the following figure). This condition is intuitive. The inequality (13) is requiring that the low type central bank does not copy the monetary policy M^s of the high type, this is more likely to be met when the opportunity cost of copying, the value $M_0/\mathcal{P}^*$ lost, is high. This value will be high in turn when the level of prices under the low shock scenario is targeted at a low value (recall that setting M^s requires always an increasing level of prices).

Next, let us add $V(H, M(L), P^T) = \frac{M_0}{P^T} + \left(\frac{L^\psi}{\eta/(\eta-1)}\right)^{\frac{1}{\psi-1}} - \frac{1}{\psi}\left(\frac{1}{H}\left(\frac{L^\psi}{\eta/(\eta-1)}\right)^{\frac{1}{\psi-1}}\right)^\psi$ to the negative of both sides of inequality (2):

$$F \equiv V(H, M(H), P^T) - V(H, M(L), P^T) \geq \frac{M_0}{P^T} - \frac{M_0}{((\eta/\eta-1)/H^\psi)^{\frac{1}{\psi-1}} M^s} = f(M^s)$$

The value F is always positive because $M(H)$ implies a higher level of product under the same level of prices in this case. It is clear that what is needed for the existence of a non-empty set S of separating equilibria, is that $F > E$. But this is an equilibrium version of the single crossing condition property, because this condition is saying that the high type H gains more than the low type L, by changing the message from $M(L)$ to $M(H)$. It is straightforward that the condition $F > E$ is equivalent to: $\left(\frac{1}{H}\left(\frac{H^\psi}{\eta/(\eta-1)}\right)^{\frac{1}{\psi-1}}\right)^\psi - \left(\frac{1}{H}\left(\frac{L^\psi}{\eta/(\eta-1)}\right)^{\frac{1}{\psi-1}}\right)^\psi < \left(\frac{1}{L}\left(\frac{H^\psi}{\eta/(\eta-1)}\right)^{\frac{1}{\psi-1}}\right)^\psi - \left(\frac{1}{L}\left(\frac{L^\psi}{\eta/(\eta-1)}\right)^{\frac{1}{\psi-1}}\right)^\psi$ which says precisely that the cost from producing more, is smaller for the high type than for the low type.



In the figure, the arrow in the bottom pointing to the right denotes all the points of M^s such that equation (13) is satisfied. The arrow pointing to the left indicates the values of M^s such that equation (14) is satisfied. The set of pooling equilibria is the intersection of those points denoted by S in the diagram. Clearly a necessary condition for existence is that $F > E$.

We have established that the existence of a separating equilibrium requires certain conditions regarding the degree of monopolistic power of firms. In this case the consequences of asymmetric information are negative. When the economy is hit by a low shock then the central bank would conduct its monetary policy in the same way as in the case of symmetric information, but when the economy is hit by a high shock the central bank conducts a more expansive policy, this means that the commitment to the price level target is not possible, if the central bank does not want to make firms to believe that a low shock is coming it must allow for the price level to rise, and hence welfare is lower in this case with respect to symmetric information case.

If the central bank is forced to abandon its target of prices then the question is whether it would find optimal to induce two level of prices in each case. Probably this will not be the case, a pooling equilibrium could be welfare improving. This may happen if, in any circumstance the central bank always supplies the minimum level of money M_{min}. In this case firms are not able to tell if a high or a low shock will hit the economy and they will be unable to update their prior distribution over the shock. Since the price level is as low as possible in this case we expect that the average level of welfare will be higher that the one implied by the equilibrium found in this section.

In the next section we modify the economy that we worked with above, by specifying that the adjustment of prices when firms observe a change in money supply, will be partial. This may be more realistic than the case we just developed, because we expect that some fraction of firms to be unable to adjust prices upon observing a monetary policy shock.

III. A Sticky Price Economy

In this section we work with an economy where the degree of price adjustment when firms observe an increase in money supplied is only partial. We assume that firms will adjust only a fraction α of the change in money supplied.

At the beginning of the period firms will observe the money supplied by the central bank. Different amounts supplied will have associated different level of prices set by firms, but there will only be a partial adjustment to movements in money. Monetary policy will be useful to update their prior distribution of the shock, hence firms will also set a different level of prices depending on the value of money observed.

The partial adjustment assumption implies that there will be a single level of prices such that $P(i) = \mathcal{P}$, and:

$$\frac{d\mathcal{P}}{\mathcal{P}} = \alpha \frac{dM}{M} \tag{15}$$

The level of relative prices will be equal to one, and the amount of the good i produced will be given by:

$$Y(i) = Y = \frac{M}{\mathcal{P}}$$

Under this conditions the level of welfare that the central bank will maximize is given by:

$$V(\theta, M, \mathcal{P}) = \frac{M_0}{\mathcal{P}} + Y - \frac{1}{\psi}\left(\frac{Y}{\theta}\right)^{\psi}$$

This is the same welfare function that we have in the previous section. The single crossing condition will again be satisfied. Let us repeat it here: The marginal rate of substitution between M and P is given by:

$$\left.\frac{dP}{dM}\right|_{\bar{V}} = -\frac{\partial V/\partial M}{\partial V/\partial M} = \frac{\frac{1}{P} - \left(\frac{M}{\theta P}\right)^{\psi}\frac{1}{M}}{\frac{1}{P}\left[\frac{M_0}{P} + M\left(\frac{1}{P} - \left(\frac{M}{\theta P}\right)^{\psi}\frac{1}{M}\right)\right]} \tag{16}$$

The single crossing condition implies that:

$$-\left.\frac{\partial V/\partial M}{\partial V/\partial P}\right|_{\theta=H} > -\left.\frac{\partial V/\partial M}{\partial V/\partial P}\right|_{\theta=L}$$

Let us discuss the relevance of the assumption that firms adjust only partially the level of prices when they face changes in the money supplied. The maximization of the utility of each individual firm in the economy will imply that they set the level of prices:

$$\frac{P(i)}{\mathcal{P}} = \left(\frac{\eta}{\eta-1} E_{\mu} \frac{1}{\theta^{\psi}} \left(\frac{M^{\alpha}}{P}\right)^{\psi-1}\right)^{\frac{1}{1+\eta(\psi-1)}} \tag{17}$$

In this equation the assumption of partial adjustment of prices stated in equation (15) have been incorporated. All firms will set the same level of prices, this will imply that the optimal aggregate level of prices will be given by:

$$\mathcal{P}^{*} = \left(\frac{\eta}{\eta-1} E_{\mu} \frac{1}{\theta}\right)^{\frac{1}{\psi-1}} M^{\alpha} \tag{18}$$

Notice that this equation implies that the level of output is given by:

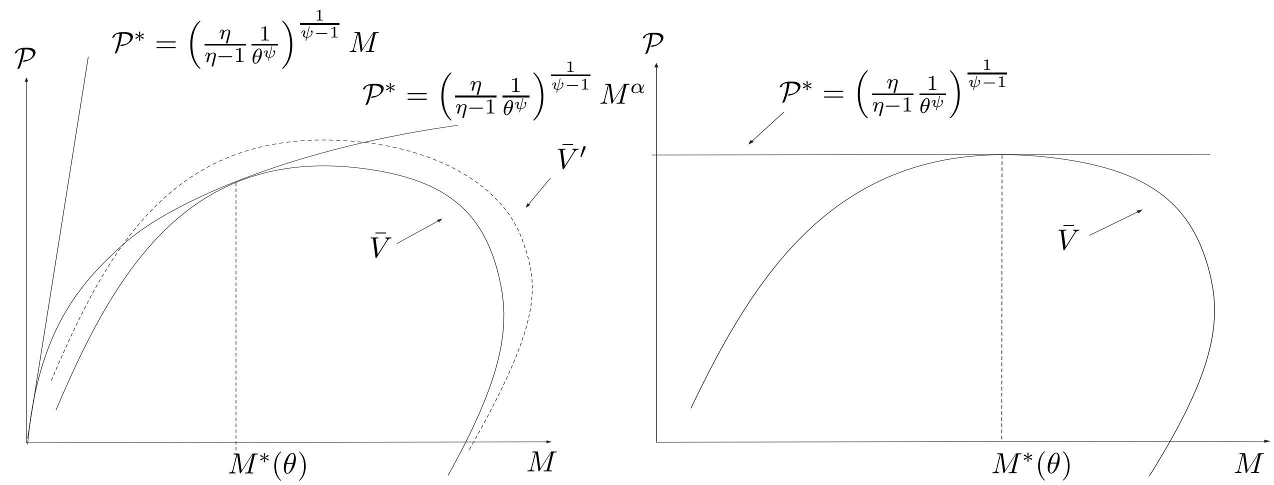
$$Y = \frac{M^{1-\alpha}}{\left(\frac{\eta}{\eta-1} E_{\mu} \frac{1}{\theta}\right)^{\frac{1}{\psi-1}}}$$

When α is equal to one and hence the adjustment is complete, then money is neutral as in the previous section. Let us denote as $M^{*}(\theta)$ the value set by the central bank in any equilibrium upon observing the shock θ.

The central bank will maximize utility subject to the constraint imposed by the optimal behavior of firms. Let us illustrate, for a given value of θ and under symmetric information, what would be the optimal monetary policy. The next graphic illustrate the indifference curves in the plane M, P for the society's welfare that the central bank wants to maximize. Also in the figure we graphic the equation (18) which gives the reaction function for the agents, and will determine the equilibrium of the economy. we also show in the graphic different values for α the degree of price adjustment.

In the graphic we can see the optimal level of money supply given by $M^{*}(\theta)$. Notice that if $\alpha = 1$, which gives the straight line with positive slope in the figure, then the optimal level supplied is the minimum M_{min} available to the central bank. This shows that the central bank is willing to tolerate higher prices when prices are sticky. This is so, because the central bank can influence the level of output in this case.

Notice that we have assumed that agents do not adjust prices completely when observing a change in their demand for their products, or equivalently when observing a change in the level of money supplied. Notwithstanding, we assume that the change in money is useful for them to



$\hat{V} > \hat{V}'$ $\qquad$ $\alpha \to 0$

infer the shock that will hit the economy at the end of the period, since they must set prices at the beginning of the period, we are assuming implicitly that they charge the best possible price conditional on the rigidity they face regarding the incomplete adjustment in prices. Equation (17) says that for different values of the money supplied M, the level of prices charged will be different than those that arise when $\alpha = 0$ even if those different values of M induce different values of the expected value of the shock. So price rigidity will affect also the optimality of prices even when money supplied reveal the true value of the shock, a case that will arise under the separating equilibrium we will find later.

In the next section we characterize the equilibrium for the symmetric, complete information case.

III.I The Symmetric Information Benchmark

In this section we derive the results for the symmetric, complete information benchmark. The next lemma characterizes the equilibrium. We again use the notation $V(\theta, M, \mathcal{P})$ to express the welfare function for the central bank when the shock is θ, the signal is M and the action of the agents lead to a price level $\mathcal{P}$.

Lemma 3 *Let us denote $M(\theta)$ the amount of money supplied by the central bank upon observing the shock θ. Let $\mathcal{P}(\theta)$ be the aggregate price level that agents induce by setting prices observing M and θ, then:*

$$\begin{aligned} M(H) &> M(L) \\ V(H, M(H), \mathcal{P}(H)) &> V(L, M(L), \mathcal{P}(L)) \end{aligned}$$

where $V(\theta, M(\theta), \mathcal{P}(\theta))$ is the welfare level of society as defined above.Proof:

To proof this lemma, let us state the problem of the central bank: He will maximize:

$$V = \frac{M_0}{\mathcal{P}} + \frac{M}{\mathcal{P}} - \frac{1}{\psi}\left(\frac{M/\mathcal{P}}{\theta}\right)^{\psi}$$

subject to the reaction function of the agents that yield the level of prices, as a constraint:

$$\mathcal{P}(\theta) = \left(\frac{\eta}{\eta - 1}\frac{1}{\theta^{\psi}}\right)^{\frac{1}{\psi-1}} M^{\alpha}$$

The lagrangian for this problem is:

$$\mathcal{L} = \frac{M_0}{\mathcal{P}} + \frac{M}{\mathcal{P}} - \frac{1}{\psi}\left(\frac{M/\mathcal{P}}{\theta}\right)^{\psi} + \lambda\left[\mathcal{P}(\theta) - \left(\frac{\eta}{\eta - 1}\frac{1}{\theta^{\psi}}\right)^{\frac{1}{\psi-1}} M^{\alpha}\right] \tag{19}$$

The first order condition yield:

$$\left.\frac{dP}{dM}\right|_{\bar{V}} \equiv \frac{\frac{1}{\mathcal{P}} - \left(\frac{M}{\theta\mathcal{P}}\right)^{\psi}\frac{1}{M}}{\frac{1}{\mathcal{P}}\left[\frac{M_0}{\mathcal{P}} + M\left(\frac{1}{\mathcal{P}} - \left(\frac{M}{\theta\mathcal{P}}\right)^{\psi}\frac{1}{M}\right)\right]} = \alpha\left(\frac{\eta}{\eta - 1}\frac{1}{\theta^{\psi}}\right)^{\frac{1}{\psi-1}} M^{\alpha-1} \equiv \frac{dP(\theta)}{dM} \tag{20}$$

Given that the right hand side of this equation is always positive, this condition states that the benefit of an increasing money supply is equal at the margin to the cost which is given by the higher price level implied by the reaction function of the agents. Notice also that if $\alpha = 0$ then no interior solution could be found, this will imply the least possible amount of money supplied as an optimizer quantity of money.

Let us state the first order condition as:

$$G \equiv \left.\frac{dP}{dM}\right|_{\bar{V}} - \frac{dP(\theta)}{dM} = 0$$

Then by the implicit function theorem:

$$\frac{dM(\theta)}{d\theta} = -\frac{G_{\theta}}{G_M} \equiv -\frac{\partial \left.\frac{dP}{dM}\right|_{\bar{V}}/\partial\theta - \partial\frac{dP(\theta)}{dM}/\partial\theta}{\partial \left.\frac{dP}{dM}\right|_{\bar{V}}/\partial M - \partial\frac{dP(\theta)}{dM}/\partial M}$$

This expression can be shown to be positive. The numerator is always positive because the first term is positive and the second term negative. The numerator is negative since the first term is negative and the second term is negative but smaller than the first term in absolute value. This shows that:

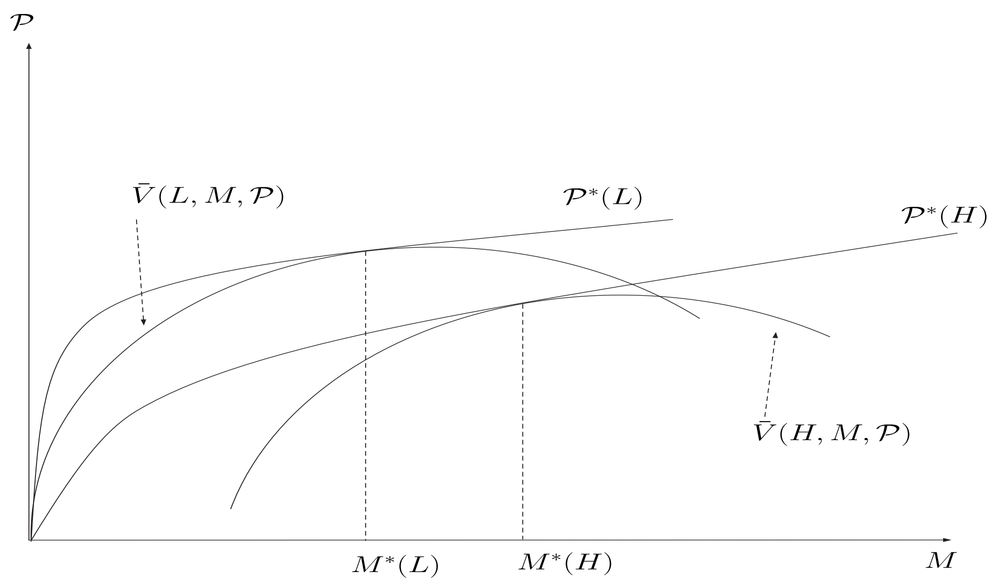
$$M^*(H) > M^*(L)$$

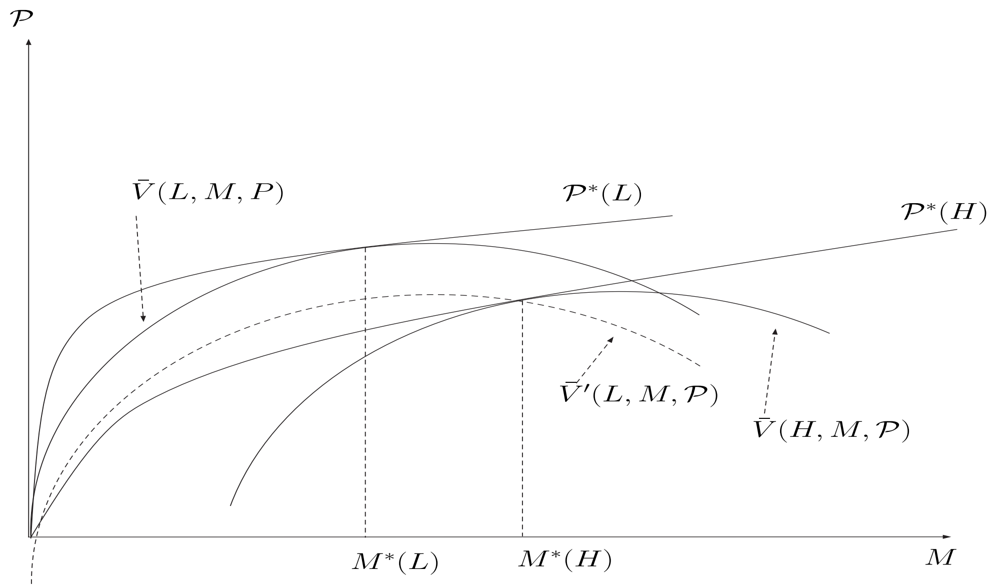
To show that the level of welfare is higher under the high shock, we use the envelope theorem applied to (19):

$$\frac{\partial \mathcal{L}}{\partial \theta} = \left(\frac{M/\mathcal{P}}{\theta}\right)^{\psi-1} \frac{M/\mathcal{P}}{\theta^2} + \frac{\lambda}{\psi-1}\left(\frac{\eta}{\eta-1}\right)^{-\frac{\psi}{\psi-1}} \theta$$

which is always positive.

In the following figure we present the situation in the symmetric information case. We can see that money supplied, is higher under the high shock than under the low shock. Also in the figure in the bottom, we show that the case depicted corresponds to the existence of "envy". The central bank observing a low shock could supply the level $M(H)$. If firms observing $M(H)$ would believe that a high shock will hit the economy, they will set prices along $\mathcal{P}(H)$, this will increase the level of welfare.





$$\bar{V}'(L, M, \mathcal{P}) > \bar{V}(L, M, \mathcal{P})$$

The existence of "Envy"

The central bank observing the low shock will pretend have been observed the high shock when

$$V(L, M^*(H), \mathcal{P}^*(H)) > V(L, M^*(L), \mathcal{P}^*(L))$$

This is equivalent to:

$$\frac{M_0}{\mathcal{P}^*(H)} + \frac{M^*(H)}{\mathcal{P}^*(H)} - \frac{1}{\psi}\left(\frac{1}{L}\frac{M^*(H)}{\mathcal{P}^*(H)}\right)^{\psi} > \frac{M_0}{\mathcal{P}^*(L)} + \frac{M^*(L)}{\mathcal{P}^*(L)} - \frac{1}{\psi}\left(\frac{1}{L}\frac{M^*(L)}{\mathcal{P}^*(L)}\right)^{\psi}$$

We will assume that this condition hold. It is expected that this condition will hold when the elasticity of substitution between goods η is low enough, as was the case in the flexible price economy.

In the next section we show the existence of the separating equilibrium.

III.II The Separating Equilibrium

Proposition 2 *There exists a continuum of separating equilibria defined by the nonempty set* S*, such that:*

$$V(L, M^*(L), \mathcal{P}^*(L)) \geq V(L, M_s^*, \mathcal{P}^*(H)) \tag{21}$$

$$V(H, M^*(L), \mathcal{P}^*(L)) \leq V(H, M_s^*, \mathcal{P}^*(H)) \tag{22}$$

where $M^s \in S$*. Agent's equilibrium beliefs are given by:*

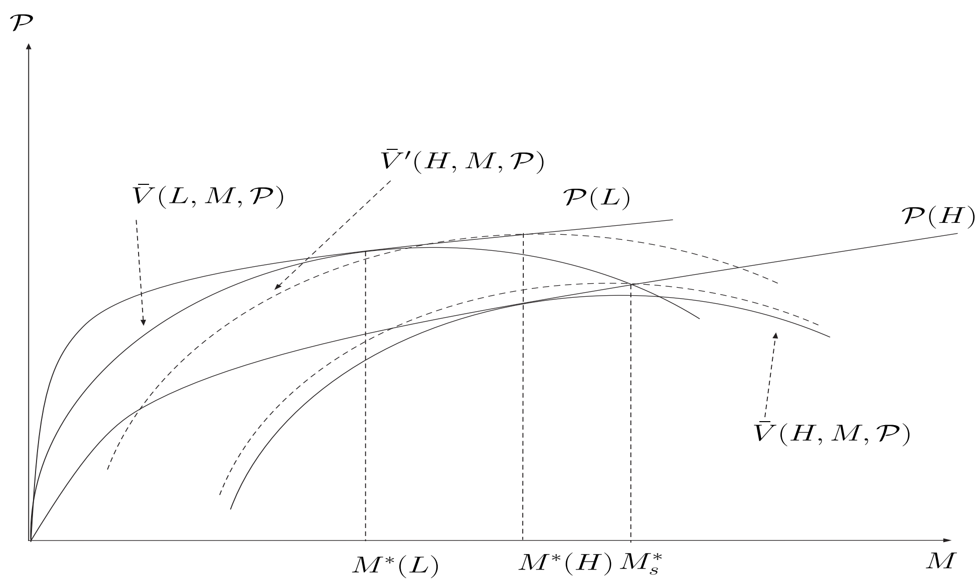
$$\mu = \begin{cases} 0 & \text{if} \quad M = M^*(L) \\ 1 & \text{if} \quad M = M_s^* \end{cases}$$

and out of equilibrium beliefs:

$$\mu = \begin{cases} 0 & \text{if} \quad M < M_s^* \\ 1 & \text{if} \quad M \geq M_s^* \end{cases}$$

Proof:

Before showing the proof let us explain in a graphic, what the separating equilibrium would look like. In the following graphic we can see that the central bank observing a high shock would have to distort its symmetric information supply of money, in fact supplying more money. The level of prices would increase along the $\mathcal{P}^*(H)$ curve until at least a point where there is no gain for a central bank observing a low shock to copy the money supplied by the high shock central bank. This distortion is necessary since if the central bank insists in supplying quantity $M(H)$ when observing a high shock, agents would assign probability zero to a high shock occurring and hence they would use the rule $\mathcal{P}^*(L)$ which would make the indifference curve of the high type agent to be the one labeled $V'\bar{(}H)$ which yields a strictly lower welfare level than the one that arise in the distortion at M_s^*.



$$\bar{V}'(H, M, \mathcal{P}) < \bar{V}(H, M, \mathcal{P})$$

To formally proof the existence of a separating equilibrium, let us state conditions (21) and (22) as:

$$V(L, M^*(H), \mathcal{P}^*(H)) - V(L, M^*(L), \mathcal{P}^*(L)) \leq V(L, M^*(H), \mathcal{P}^*(H)) - V(L, M^*_s, \mathcal{P}^*(H)) \quad (23)$$
$$V(H, M^*(H), \mathcal{P}^*(H)) - V(H, M^*(L), \mathcal{P}^*(L)) \geq V(H, M^*(H), \mathcal{P}^*(H)) - V(H, M^*_s, \mathcal{P}^*(H)) \quad (24)$$

The right hand side of equation (23) can be expressed as a function of M as:

$$f(M) = \frac{M^*(H) - M}{\mathcal{P}^*(H)} - \frac{1}{\psi}\left(\frac{1}{L}\right)^{\psi} \frac{M^*(H)^{\psi} - M^{\psi}}{\mathcal{P}^*(H)^{\psi}} \quad (25)$$

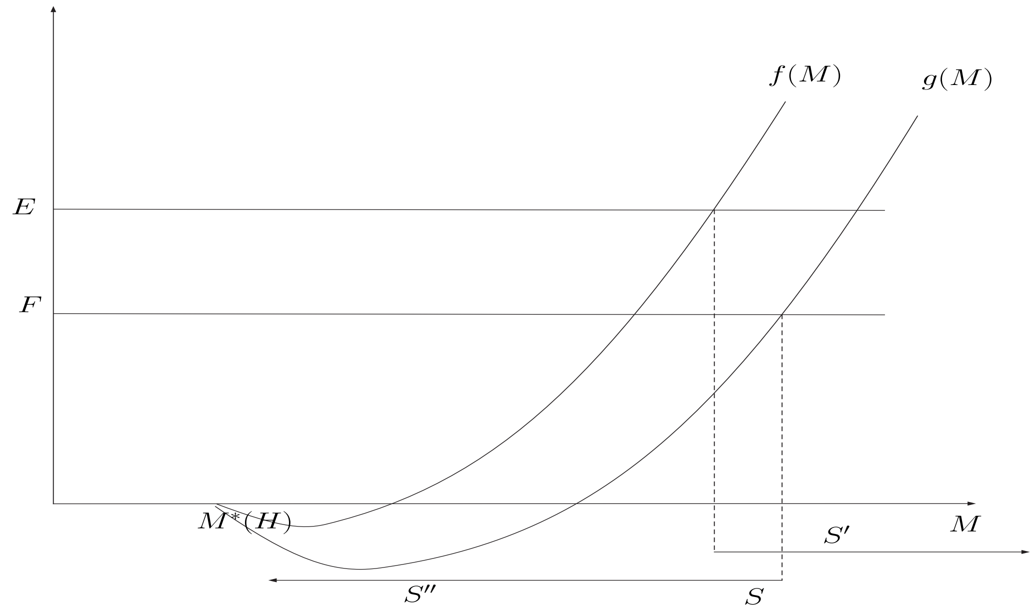
Since it will be the case that in any separating equilibrium $M^*_s > M(H)$ by the arguments stated above, this function has the shape drawn in the figure below.

Also in the figure the line $E = V(L, M^*(H), \mathcal{P}^*(H)) - V(L, M^*(L), \mathcal{P}^*(L))$ is strictly positive under the "envy" assumption. This is the left hand side of equation (23). So all the points M such that $f(M) > E$ will satisfy equation (21). This set is labeled S' in the figure.

The right hand side of equation (24) is equal to:

$$g(M) = \frac{M^*(H) - M^*_s}{\mathcal{P}^*(H)} - \frac{1}{\psi}\left(\frac{1}{H}\right)^{\psi} \frac{M^*(H)^{\psi} - M^{\psi}}{\mathcal{P}^*(H)^{\psi}} \quad (26)$$

This is almost the same curve $f(M)$. The difference is that the shock is H not L. Then the g curve will be always to the right of the f curve as drawn in the figure. The left hand side of equation (24) is $F = V(H, M^*(H), \mathcal{P}^*(H)) - V(H, M^*(L), \mathcal{P}^*(L))$, this value is always positive since the high type never gains by pretending being a low type. Furthermore $F < E$ because the high type gains less by changing from $\mathcal{P}^*(L), M^*(L)$ to $\mathcal{P}^*(H), M^*(H)$ than does the low type. The set satisfying condition (22) then is given by the one denoted as S''. In the figure we assume that the intersection is nonempty. This would require that E is not much higher than F.



IV. Conclusions

In this paper we have developed a signaling game between the central bank and the agents in the economy. Agents are unsure about the future state of the economy. The central bank posses more information about it than the public. Monetary policy is regarded by the central bank and the agents as a signal that might reveal the future conditions.

We worked specifically with a productivity shock as the state of the economy. We have shown that when the central bank has superior information about this shock that will hit the economy, it can no longer set the monetary policy as in the case of symmetric information. In general in the economies analyzed we have found that the central bank observing a high shock, does more expansionary monetary policy than observing a low shock, the reason being that it wants to avoid the public believe that it wants to mislead them, and hence induce negative expectations about the future.

We have considered two economies. In the first economy, there is complete flexibility of prices and the central bank wants to implement a price level target. We have shown that, as a consequence of the asymmetry of information, the central bank is unable to maintain the target. Its policy imply a higher level of prices upon observing a high, expansionary shock.

In the second economy, we considered a sticky price economy. We also found that the predictions of the separating equilibrium are not pleasant. The central bank in equilibrium does a more inflationary monetary policy, and the level of welfare is lower compared to the symmetric information case. It is important to emphasize that the "inflation bias" result comes from a different source than was typically analyzed. In our model the timing of events are such that monetary policy gives a signal about the future supply shock hitting the economy. That is the central bank moves first, the "inflation bias" comes directly as a consequence of the asymmetry of information regarding the shock.

References

- Athey S.; Atkenson A. and Kehoe P. "The optimal degree of monetary policy discretion". Econometrica 73 (5), September, 2005, 1431-1476.

- Blanchard O and Kiyotaki N. "Monopolistic Competition and the Effects of Aggregate Demand". The American Economic Review. Vol. 77, No. 4 (Sep., 1987), pp. 647-666

- Carroll C.; Fuhrer J. and Wilcox D. "Does Consumer Sentiment Forecast Household Spending? If So, Why?". The American Economic Review Vol. 84, No. 5 (Dec., 1994), pp. 1397-1408

- Cooper R. "Coordination Games. Complementarities and Macroeconomics" (1999) Cambridge University Press.

- Romer C. and Romer D. "Federal Reserve Information and the Behavior of Interest Rates". The American Economic Review Vol. 90, No. 3 (Jun., 2000), pp.429-457

- Sleet C. "On Credible Monetary Policy and Private Government Information". Journal of Economic Theory 99,338-376 (2001)