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Liquidity Frictions and Wealth Inequality

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Abstract

In this paper I develop a model with uninsurable idiosyncratic shocks that delivers a distribution of wealth that is highly unequal. I show that the combination of idiosyncratic shocks to capital creation or entrepreneurship plus the incorporation of liquidity frictions into an otherwise standard decentralized neoclassical growth model might be more promising than models that emphasize shocks to labor income, in order to understand the unequal distribution of wealth of the U.S. economy.

Keywords: Liquidity constraints, Endogenous grid method, wealth distribution.

JEL Classification: E21, C0, C6.

[1]Email: ssalas@uchicago.edu. Preliminary version, comments welcome. I would like to thank my advisors, who guide me in several stages of this investigation: Veronica Guerrieri, Kenneth Judd and Harald Uhlig.

I. Introduction

Characterizing the inequality of wealth distribution of the U.S. economy has proven to be a difficult task. Initial papers such as Krusell and Smith (1998), have found that uninsurable stochastic shocks to labor earnings are insufficient for replicating the distribution of wealth in the economy. The basic framework has been modified subsequently by several authors. The modifications entail stochastic differences in discount factors, welfare benefits to poor agents, incorporating increasing returns to savings and extremely high wage dispersion. In this paper I show that we do not need to depart by much from the initial framework. It suffices a change in focus from shocks to labor earnings to shocks to capital creation.

I incorporate entrepreneurial shocks, and liquidity frictions into a otherwise standard decentralized neoclassical growth model. The ideas of Kiyotaki and Moore (2008) are introduced in this framework. I interpret idiosyncratic shocks of capital creation as entrepreneurial ideas. This is a reduced form approach to model the entrance into entrepreneurship activities. When agents enter into entrepreneurship, they require to raise funds to finance their projects. Liquidity frictions are introduced as agents cannot sell as much of claims on capital they have to finance the projects. It is shown that the interaction of the liquidity constraints and entrepreneurial shocks are key to obtain a distribution of wealth that is highly unequal.

On a methodological aspect of this paper, I modified the Endogenous Grid Method, to solve models with state dependent constraints. The model is solved numerically, and the following results are obtained: The economy displays a suboptimal equilibrium in the Pareto sense when the liquidity frictions are tight. In a calibration, for yearly analysis, it is shown that approximately agents must be able to sell no more than 30% of their their claims on capital. Under such conditions which are plausible to be satisfied in the data, I find that consumption is 5% lower relative to a frictionless equilibrium. Moreover, these features appear to be relevant in terms of approximating the unequal US wealth distribution. It turns out that the model delivers a more unequal distribution of wealth than what is observed in the data. I interpret this result under a positive lens. The conjunction of shocks to entrepreneurial activity, and liquidity frictions seems to be key important ingredients for characterizing wealth inequality.

Related Literature: This paper is related to several papers that incorporated idiosyncratic shocks to standard models. The seminal reference is Krusell and Smith (1998) who hypothesized discount-rate heterogeneity and show that an small amount of such heterogeneity delivers a realistic wealth inequality. Other papers such as Hubbard, Skinner and Zeldes (1995) considered specific welfare benefits that induce a large mass of agents at lower levels of assets. Other versions such as Quadrini (2000), Campanale (2005) and Ceggeti and De Nardi (2004,2005), investigate versions with increasing returns to saving. Finally Castaneda (2003) generate large wealth inequality by considering a wage process with high dispersion, with a small probability of getting extremely large earnings.

The way liquidity needs and liquidity frictions are modeled in this paper are taken by Kiyotaki and Moore (2008). My paper is a modified setup to their model, where all agents are treated symmetrically and entrepreneurship can be achieved by all agents in the economy, making my model closer to the neoclassical growth model. Another related paper is Angeletos (2005) who considers a model where entrepreneurs are subject to investment risk and can hire labor from workers, his version is similar to Kiotaki and Moore set up.

The rest of the document is organized as follows. In Section II, I present the main model. In Section III, I define the stationary recursive competitive equilibrium, and study a partial equilibrium version of the economy that help to bring insights on the workings of the model. Section IV presents the general equilibrium model and solve for the equilibrium. Section V concludes.

II. The Model

Environment

The economy is populated by a measure one of infinite lived agents indexed by j. At beginning of life each agent has k_0 units of capital. At the beginning of every period t each agent is endowed with one unit of labor to which no disutility is attached. Agents aim to maximize:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^j) \tag{1}$$

where $\beta \in (0,1)$ and $u : \Re_+ \to \Re$ is bounded, strictly increasing, strictly concave and twice differentiable continuous function. I assume that the utility function satisfies the Inada conditions.

Agents come to a new period with a given amount of capital k_t^j, they rent the services of this capital along with their labor endowment to a constant returns to scale firm which pays the rental rate r_t and w_t for capital and labor services respectively in units of the consumable good. The firm also give back the un-depreciated amount of capital to each agent. After all these payments are made, some fraction of the agents receive exogenously and for free an entrepreneurship idea, a constant returns to scale technology of investment, by which they can transform one unit of the consumable good to one unit of new capital[2]. Hence, not all agents can create new units of capital from the general consumption good. I interpret the creation of new units of capital as an entrepreneurship activity. I denote the status of an agent by $z \in Z = \{1, 0\}$. An agent with $z = 1$ will be called an entrepreneur and an agent with $z = 0$ a worker. In each period there is a probability that a given agent change his status. This probability is independent among

[2]This technology is reversible, in the sense that it can be used to transform one unit of capital good into a unit of consumable good. The physical units of new capital are ready to be rented and enter the production process at the beginning of the next period.

individuals and is governed by the Markov chain:

$$Q = \begin{bmatrix} \text{Prob}\{z^j_{t+1} = 1 | z^j_t = 1\}, & \text{Prob}\{z^j_{t+1} = 0 | z^j_t = 1\} \\ \text{Prob}\{z^j_{t+1} = 1 | z^j_t = 0\}, & \text{Prob}\{z^j_{t+1} = 0 | z^j_t = 0\} \end{bmatrix} = \begin{bmatrix} Q(1,1), & Q(1,0) \\ Q(0,1), & Q(0,0) \end{bmatrix} \tag{2}$$

The expectation operator in (1) refers to the idiosyncratic uncertainty expressed in (2).

During period t, after factor payments are made, agents find out their entrepreneurship status, if they are entrepreneurs they decide how much of capital to create. Agents can then trade their capital holdings at price q_t expressed in units of the consumable good[3]. After these transactions are made agents consume and each agent ends up with capital holdings of k^j_{t+1}. We now state the agent's optimization problem.

The Agent j optimization problem

Each agent has preferences over consumption given by (1), each agent maximizes this utility subject to the flow constraint:

$$c^j_t + i^j_t z^j_t + q_t(k^j_{t+1} - i^j_t z^j_t - (1-\delta)k^j_t) \leq w_t + r_t k^j_t \tag{3a}$$

Note that when the agent is not an entrepreneur in a given period, the investment terms drop from (3a). Such an agent may still obtain more units of capital by buying it in the market at price q. I follow Kiyotaki and Moore (2008), henceforth KM, in modeling the liquidity frictions. Entrepreneurs cannot sell all the capital they have or create. They can sell at most a fraction θ_s of the new units of capital created in a given period, and at most a fraction θ_r of beginning of period capital. Workers face also the same constraint on their beginning of period capital.

[3]Later, I will specify some frictions that are exogenously imposed upon these transactions.

The liquidity constraints can then be expressed as:

$$n^j_{t+1} \geq (1-\theta_s) i_t z^j_t + (1-\theta_r)(1-\delta) n^j_t \tag{3b}$$

The Firm's optimization problem

Firm's optimization problem is standard and simple. They rent capital and labor services from each agent, the aggregate amount of it K_t being used as an input for production in a constant returns to scale technology. Firm's problem is:

$$\max_{K_t, L_t} \left[F(K_t, L_t) - r_t K_t - w_t L_t \right], \quad t = 0, 1, 2, ... \tag{4a}$$

It then follows that the optimality conditions are:

$$r_t = F_K(K_t, L_t), \quad t = 0, 1, 2, \tag{4b}$$

$$w_t = F_N(K_t, L_t), \quad t = 0, 1, 2, ... \tag{4c}$$

III. Stationary Recursive Competitive Equilibrium

In this section I state the recursive representation of the agent's problem and define the stationary recursive competitive equilibrium. To simplify notation I omit the superscript j. Lower case letters denote individual variables. Upper case letters denote aggregates and a superscript prime denotes next period variables.

Appendix A shows that the ergodic set of the distribution of capital will have a lower bound $\underline{k}$, and it will not be bounded upwards. Each agent's individual capital will be restricted then to a set: $k \in B = [\underline{k}, \infty)$.

Since in any given period each agent may have a different amount of capital, and a given investment status $z \in Z = \{1, 0\}$. An agent's individual state is the pair $(k, z) \in B \times Z$. Therefore, the Bellman equation for each agent is defined as[4]:

$$v_z(k) = \max_{c,k',iz} \left[u(c) + \beta \sum_{z' \in Z} v_{z'}(k')Q(z, z') \right] \tag{5a}$$

such that:

$$c + iz + q[k' - iz - (1-\delta)k] \leq w + rk \tag{5b}$$

$$k' \geq (1-\theta^s)iz + (1-\theta^r)(1-\delta)k \tag{5c}$$

I use a probability measure λ defined over subsets of the state space to deal with the heterogeneity of the model. I interpret $\lambda_z(k)$ as a probability measure describing the fractions of agents with the same individual state. The policy functions $k'_z(k), i(k), c_z(k)$ and the Markov process (2) generate a law of motion for the measures:

$$\lambda'_{z'}(\tilde{k}) = \sum_{z \in Z} \int_{\{k \in B : k'_z(n) \leq \tilde{k}\}} Q(z, z')d\lambda_z(k) \tag{6}$$

Equation (6) is the equation of motion of the measures. In the definition of stationary equilibrium this will not change through time.

Definition *A stationary recursive competitive equilibrium consists of constant factor prices (r, w); constant price of capital q, value functions $v_z(k)$, policy functions for consumption $c_z(k)$, next period capital $k'_z(k)$ and investment $i(k)$, probability measures $\lambda_z(k)$, and aggregate level of capital K, such that:*

[4] In Appendix A, I show some proofs for the existence of the value function and properties of it and also properties of the policy functions.

1. *the policy functions* $c_z(k)$, $k'_z(k)$ *and* $i(k)$ *solve each agent's optimization problem,*

2. *at the given factor prices firm's optimization problem is solved:*

$$r = F_K(K, L), \quad w = F_L(K, L) \tag{7a}$$

3. *the probability measures are time invariant,*

$$\lambda_{z'}(\tilde{k}) = \sum_{z \in Z} \int_{\{k \in B : k'_z(k) \leq \tilde{k}\}} Q(z, z') d\lambda_z(k) \tag{7b}$$

4. *the capital and labor markets clear,*

$$K = \sum_{z \in Z} \int k'_z(k) d\lambda_z(k) \tag{7c}$$

$$L = \sum_{z \in Z} \int d\lambda_z(k) \tag{7d}$$

5. *individual investment is consistent with aggregate investment,*

$$\delta K = \int i(k) d\lambda_1(k) \tag{7e}$$

Characterization of the Equilibrium

In order to characterize the equilibrium we begin by finding the first order conditions for the household's optimization problem.

Let us define $\mu_z(k)$ as the multiplier on the liquidity constraint for an agent with state (k, z).

Then, the first order conditions are:

$$c \quad : \qquad u'(c_z(k))q = \beta \sum_{z' \in Z} v'(k', z')Q(z, z') + \mu_z(k) \tag{8a}$$

$$i \quad : \qquad u'(c_1(k))(q-1) = (1-\theta^s)\mu_1(k) \tag{8b}$$

The Karush-Khun-Tucker conditions associated with (3b) are:

$$\mu_z(k)\left[k'_z(k) - (1-\theta^s)i(k)z - (1-\theta^r)(1-\delta)k\right] = 0 \tag{8c}$$

$$k'_z(k) - (1-\theta^s)i(k)z - (1-\theta^r)(1-\delta)k \geq 0, \quad \mu_z(k) \geq 0 \tag{8d}$$

and the envelope condition:

$$v'_z(k) \;\; = \;\; u'(c_z(k))[r + (1-\delta)q] - \mu_z(k)(1-\theta^r)(1-\delta)$$

The Euler equation associated with the household problem is:

$$u'(c_z(k))q = \beta(r+(1-\delta)q) \sum_{z' \in Z} u'(c_{z'}(k'))Q(z,z') - \beta(1-\theta^r)(1-\delta) \sum_{z' \in Z} \mu_{z'}(k')Q(z,z') + \mu_z(k) \tag{9}$$

This equation shows that when the agent expects to be liquidity constrained it saves less today in the form of less capital carried to the future. If he is likely to be constrained in the future, he will be unable to sell all the capital he would like and hence there is no reason to take large amounts of capital to that period.

Equation (8b) describes the optimal amount of investment carried by the investor household. When there is no selling constraint ($\theta^s = 1$), the only value of q that implies an equilibrium for the market of capital is $q = 1$. When $0 < \theta^s < 1$, the value of capital q might be more than one. In this case, for a given value of next period capital, this equation says that the marginal benefit from investment is equal to the marginal cost. It is more revealing to interpret the LHS of (8b) as the marginal benefit. In the event $q > 1$, entrepreneurs sell new units of capital at a price higher than the cost of production and utility increases by $u'(c_1(k))(q-1)$. The marginal cost is given by the RHS of (8b) fraction of investment required to be financed with own funds times the multiplier on the liquidity constraint. A given increase on investment makes more likely that the liquidity constraint binds, which is reflected in the multiplier $\mu_1(k)$.

At this point I conjecture that in equilibrium with tight enough liquidity constraints, the following assumption with be satisfied:

$$1 < q < 1/\theta^s \tag{10}$$

Proposition 1. *Under Assumption* (10)*, there exist an equilibrium for the entrepreneur, where the amount of investment is strictly positive and finite*

Proof. By inspecting (5b), when $q > 1$ the entrepreneur will want to make investment as high as possible. In this case his liquidity constraint (5c) will be satisfied at equality. By replacing this liquidity constraint at equality into the budget constraint, I obtain:

$$c + \psi[k' - \eta(1-\delta)k] = w + rk \tag{11}$$

where I have defined:

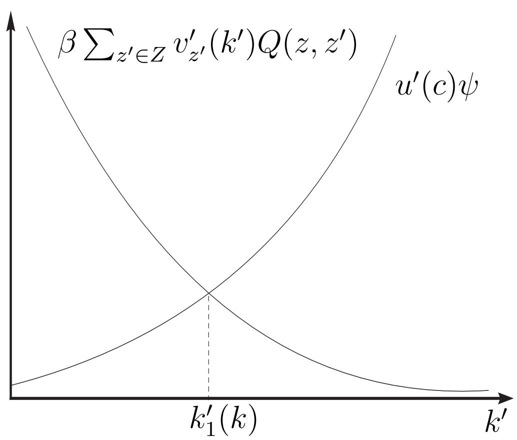
$$\psi \doteq \frac{1-q\theta^s}{1-\theta^s}, \quad \eta = 1 - \theta^r + q\theta^r \frac{1-\theta^s}{1-q\theta^s}$$

This equation makes clear, that there is a trade-off between current and future period consumption only when $1 - q\theta^s > 0$. It this inequality is reversed , then the agent will find optimal to make investment, and hence next period capital, infinite. This will happen in spite of the restriction on resell constraint next period, since the agent would be financing an unbounded future consumption from the renting of an infinite amount of capital to the firms next period. □

To illustrate the equilibrium reached under condition $1 < q < 1/\theta^s$, let us use the first order conditions for the investor's optimization problem, and replace the multiplier on the liquidity constraint from (8b), into (8a); this gives us the following condition of optimality:

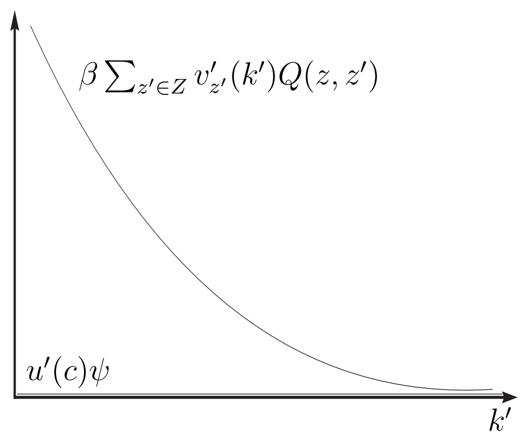
$$u'(c)\psi = \beta \sum_{z' \in Z} v'_{z'}(n')Q(z, z') \tag{12}$$

Assumption (10), guarantees that consumption is strictly decreasing with next period capital, k', and hence marginal utility of consumption is strictly increasing with k'. The same assumption guarantees that the coefficient multiplying marginal utility is positive in equation (12). Furthermore the value function satisfies strict concavity. Figure 1 and Figure 2 depict the solution when Assumption (10) holds, and the limiting case where $q\theta^s = 1$ respectively.



$1 < q < 1/\theta^s$

Figure 1: Existence of optimal solution.



$1 < q = 1/\theta^s$

Figure 2: No solution.

Under Assumption (10), the agents problem can be restated as follows:

The value function for the individual with state (k, z) can be defined as:

$$v_z(k) = \max_{c,k'} \left[u(c) + \beta \sum_{z \in Z} v_{z'}(k')Q(z, z') \right]$$

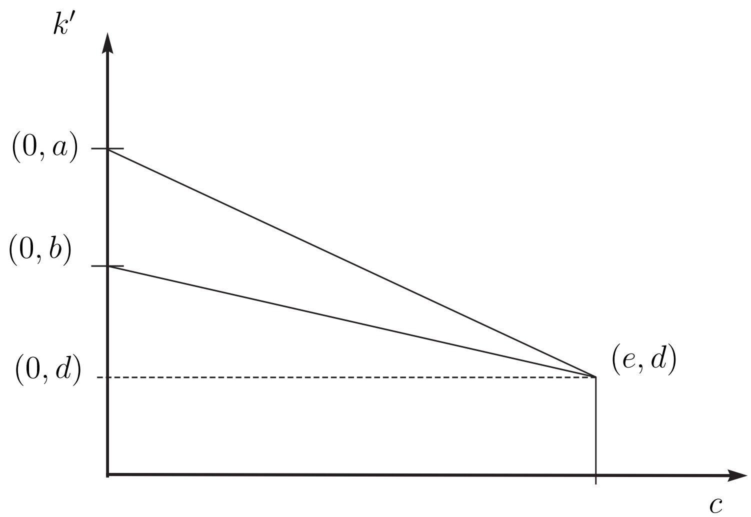
such that:

$$c + \psi[k' - \eta(1-\delta)k] \leq w + rk, \quad k' \geq (1-\theta_r)(1-\delta)k \quad \text{if } z = 1 \tag{13}$$

$$c + q[k' - (1-\delta)k] \leq w + rk, \quad k' \geq (1-\theta_r)(1-\delta)k \quad \text{if } z = 0 \tag{14}$$

Equation (13) express the feasibility set for entrepreneurs, which is composed by the "effective" budget constraint and the non-negativity constraint on investment. Equation (14) express the feasibility set for workers, which is composed by the budget constraint and the liquidity constraint.

For a given value of current period capital, I can express the feasibility set for the agents in the plane (c, k').



$$a = \frac{w+rk}{\psi} + \eta(1-\delta)k, \quad b = \frac{w+rk}{q} + (1-\delta)k$$
$$d = (1-\theta^r)(1-\delta)k, \quad e = w + [r + q\theta^r(1-\delta)]k$$

Figure 3: Feasibility sets in the plane (c, k').

Figure 3 show that the opportunity cost of current consumption is higher for entrepreneurs. If the indifference curves between consumption and next period capital have the usual shape, we can expect different solutions to the problem. It might be possible that some agents are choosing at a corner where the non-negative constraint on investment is binding or when the liquidity constraint is binding, at a point such as (e, d) in the figure. Computationally, this poses some difficulties. I will show that by modifying the Endogenous Grid Method, it is possible to approximate a solution that takes into account such cases.

Some Partial Equilibrium Comparative Statics

In this section I present some comparative statics, for given aggregate stock of capital, prices of inputs and price of capital. The objective here is two fold. First, to assess how changes in the liquidity parameters θ^s, θ^r and changes in the transition matrix Q influence the policy functions of the agents. Second, to illustrate the performance of the Endogenous Grid Method. Note that the model displays state dependent constraints[5].

I use the values for the parameters of the model and for the aggregate stock of capital and price of capital presented in Table 1.

Table 1: **Parameters and Aggregate Values**

Parameter	Value	Description
β	0.95	rate of time preference
α	0.36	share of capital
δ	0.075	depreciation rate
γ	2	elasticity of the marginal utility of consumption
K	0.9	aggregate stock of capital
q	1.5	price capital

The rest of the parameters of the model are presented in each of the figures below.

Figure 4 show the policy functions for next period capital, consumption and investment. Several features of the solution are also present in the general equilibrium framework to analyze later. Note first that for low levels of current value of capital, entrepreneurs bind their non-negativity

[5] A summary of the implementation of the Endogenous Grid Method is presented in Appendix B.

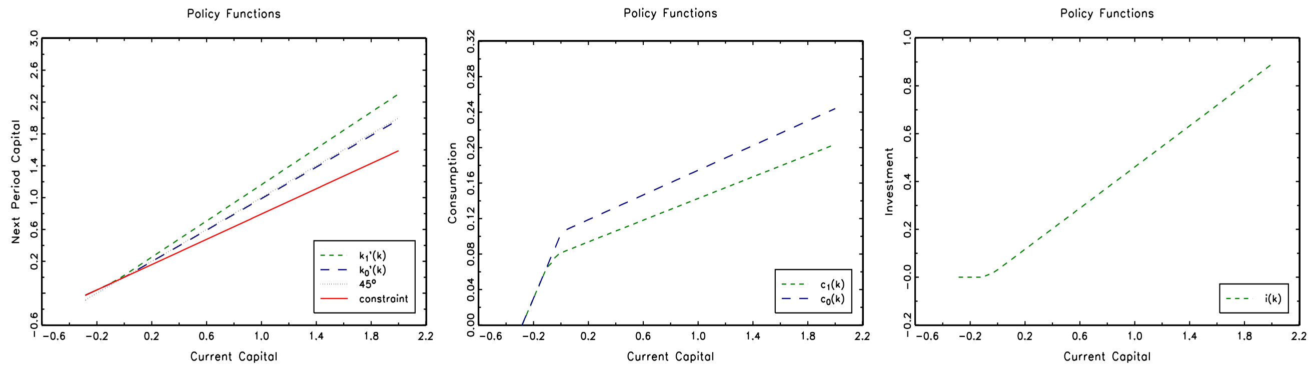


Figure 4: $\theta^s = \theta^r = 0.2, Q(1,1) = Q(0,1) = 0.2$

constraint on investment, whereas workers bind their liquidity constraint. When this happens, both agents have the same value for their policy functions (both for capital and consumption). Entrepreneurs policy for capital is higher than that of workers. They face a high return on savings which also explains that their policy for consumption is lower than that of workers. With some small probability an entrepreneur will end up accumulating capital even as time goes to infinity, the distribution of wealth as measured by capital holdings is not bounded above in its support.

Next, I vary the parameter of reselling constraint, to see how the policy functions change. Figure 5 show the result.

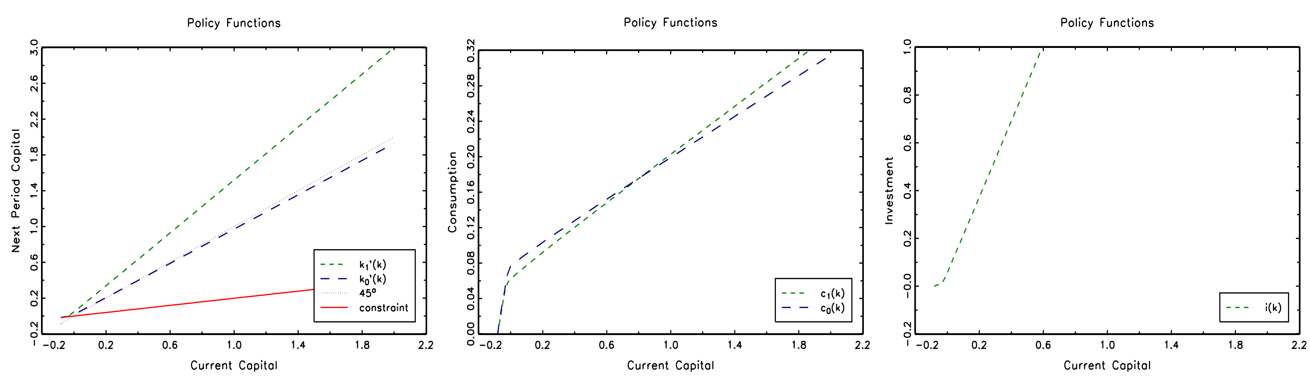


Figure 5: $\theta^s = 0.2, \theta^r = 0.8, Q(1,1) = Q(0,1) = 0.2$

Figure 5 show that when the reselling constraint is higher, agents invest more and consume more. Interestingly the policy functions for consumption for investors and savers now cross for high levels of capital. When agents can resell high amount of capital, the policy of entrepreneurs for consumption is higher for high levels of capital. In this case the income effect is high enough that allows them to use more of their resources in current consumption.

I also investigate the effect of a high persistence in the status of agents. Figure 6 show the result.

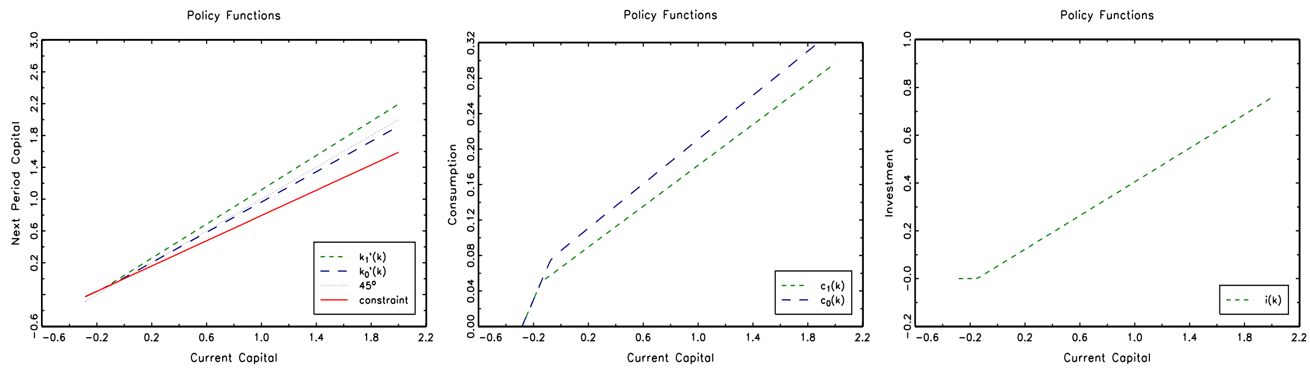


Figure 6: $\theta^s = \theta^r = 0.2, Q(1,1) = 0.99, Q(0,1) = 0.4$

Figure 6 shows that when entrepreneurs expect to remain being entrepreneurs with high probability, while there is a fair chance that workers may switch status, then there agents consume more. Note that the policy for capital accumulation for workers is less close to the 45 degree line. They are not saving as much as before, they anticipate that when they change status they will enjoy an expanded opportunity set and hence they do not need to save as much as before.

Finally, I explore the implications of modifying the degree of risk aversion. I consider much more risk averse individuals than in the base case. Figure 7 shows the result. It can be seen that when there is a high degree of risk aversion, the policy functions for consumption are very similar among agents, agents dislike swings in their consumption more than before, and hence their policies are similar.

In the previous analysis I have computed different comparative static exercises. This help us

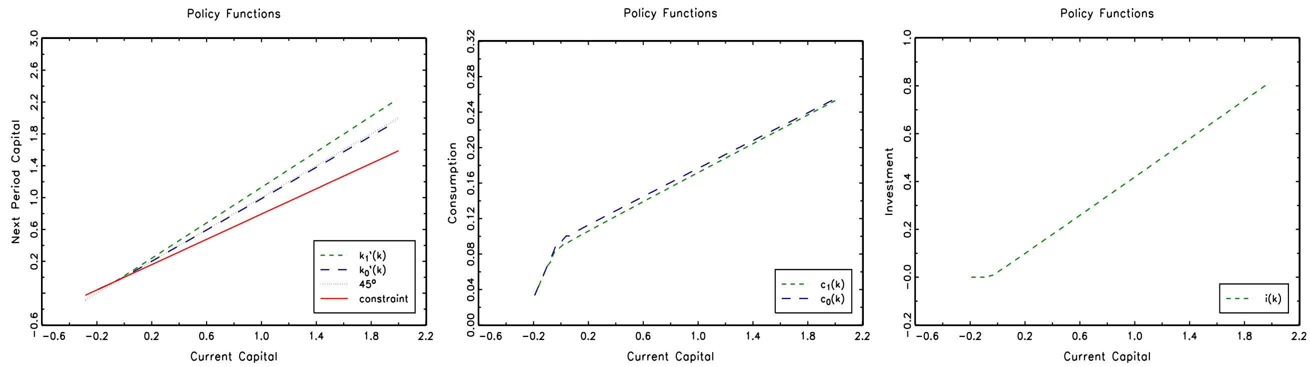


Figure 7: $\theta^s = \theta^r = 0.2, Q(1,1) = Q(0,1) = 0.2, \gamma = 6$

to gain some intuition on the workings of the model, and also illustrates the flexibility of the Endogenous Grid Method. The next task is to solve for the general equilibrium of the model. This requires implementing a method to find the measures of agents in the economy. I implement iterations on the distributions along to the Endogenous Grid Method, to find the solution of the model.

III. Solving for the Stationary Equilibrium

Most of the parameters of the model are standard. The non-standard parameters are the probability of changing status from investor to saver, $Q(1,0)$, the probability of changing status from saver to investor $Q(0,1)$, the constraint on borrowing θ^s and the constraint on sales of capital θ^r. The following table shows the values for the parameters used in the following simulations.

When the liquidity constraint parameters are relatively high (roughly above 0.4), then I have found that the price of capital is 1. This implies that the Pareto Optimal allocation is found.

I explore quantitatively the implications of having different degrees of liquidity constraints. I use the parametrization given in the Table 2. And compute the equlibrium for different values of

Table 2: **Parameters and Aggregate Values**

Parameter	Value	Description
β	0.95	rate of time preference
α	0.36	share of capital
δ	0.075	depreciation rate
γ	2	elasticity of the marginal utility of consumption
$Q(1,0)$	0.816	probability of changing status from investor to saver
$Q(0,1)$	0.033	probability of changing status from saver to investor
θ^s	0.2,0.3,0.4,1	selling constraint
θ^r	0.2,0.3,0.4,1	reselling constraint

the liquidity constraint parameters. I found that there is a region determined by the parameters θ^s and θ^r such that the economy displays a suboptimal behavior in terms of the capital stock in the steady state. In particular, there is no need for the parameters to be equal to one to sustain the optimal allocation. I find that, the tighter the selling constraint is, as expressed in a lower value of the parameter θ^s, the looser the reselling constraint as expressed in the parameter θ^r must be for the economy to attain a Pareto Optimal allocation. Figure 8 show the results.

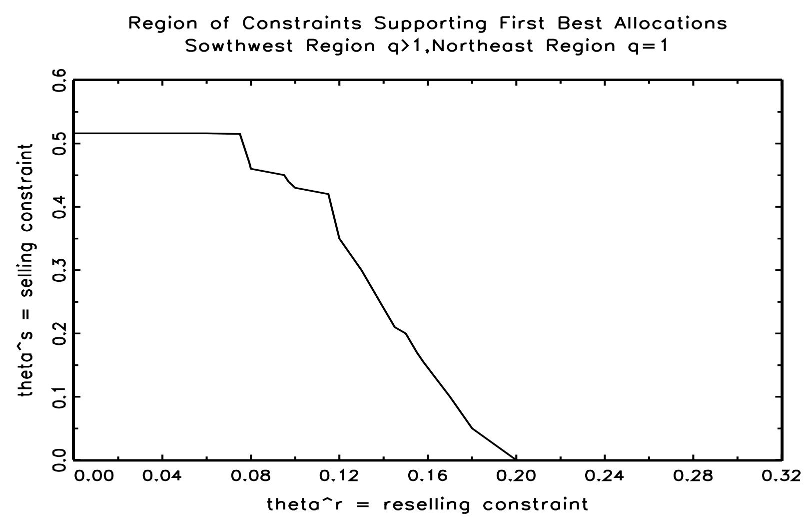


Figure 8: Southwest region, suboptimal equilibrium.

What are the effects on aggregate variables of the liquidity constraints? Are there sizeable welfare costs? The answer to this question is of course, quantitative. We know that in the southwest region of Figure 8 allocations are not Pareto optimal. But which level of liquidity constraint is regarded as a good approximation of liquidity constraints in reality is an open question. I rely on recent calibrations conducted by other authors that worked with a related model with this type of liquidity frictions. Del Negro et.al. (2010) and Nezafat and Slavick (2009) calibrate the values of θ^s and θ^r to be approximately 0.1-0.2 for the former and 0.1 for the later. I take the values $\theta^s = \theta^r = 0.1$ for the computation of table 3.

Table 3: **Equilibrium values as percentage of frictionless steady state**

Capital	K	0.863
Output	Y	0.948
Consumption	C	0.957
Levels		
Price of Capital	q	1.15

In table 3, we can see the detrimental effects of the liquidity constraints. The stock of capital is 15% less of its frictionless counterpart. Output is more than 5% less and consumption is close to 5% less. These are steady state values, and hence the welfare loss in sizeable.

The Equilibrium and the Distribution of Wealth

In this section I explore the quantitative implications of having different degrees of liquidity constraints on the distribution of wealth.

I have conducted the following exercise. I computed the equilibrium values for the economy for a highly constrained economy. When $\theta^s = 0.1$ and $\theta^r = 0.05$. Then I computed the equilibrium for the values $\theta^2 = 0.2$ and $\theta^r = 0.1$. The equilibrium measures are shown in figures 9 and 10.

Even though is hard to see, with a little care we can see that when the economy is less liquidity constrained, the economy is more egalitarian (look at the mass above 2 and the mass of workers for low levels of capital). It is important also to emphasize that the economy displays a highly

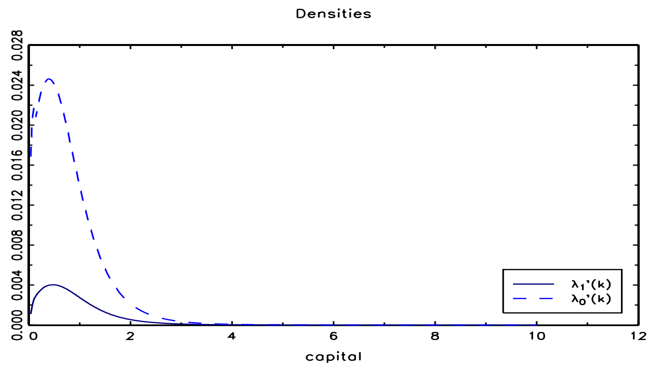


Figure 9: $\theta^s = 0.1, \theta^r = 0.05$ A highly constrained economy

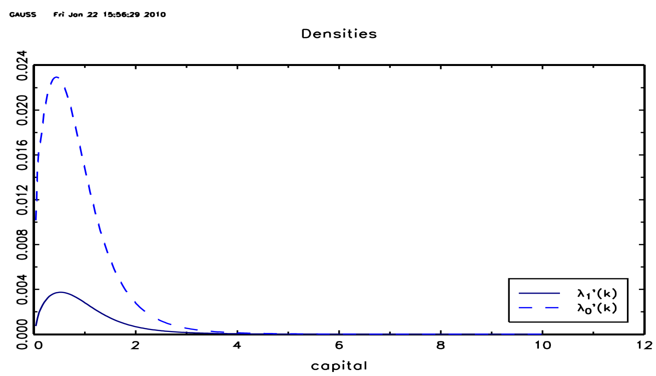


Figure 10: $\theta^s = 0.2, \theta^r = 0.1$ A less constrained economy

unequal distribution of wealth in aby event. In fact, when a comparison is made with the data, this model does not capture well the wealth inequality. It predicts too much of it. Table 4 shows the wealth inequality in the data versus the one predicted by the model. For the transition probabilities of changing status from entrepreneur to worker, I used a computation by Quadrini (2000), who computes the fraction of entrepreneurs who remain being entrepreneurs in the next year and the fraction of workers who become entrepreneurs. The values I calculate are: $Q(1,1) = 0.816$ and $Q(0,1) = 0.033$.

Table 4: **The Distribution of Wealth**

	Top Percentiles				
	1%	5%	10%	20%	30%
SCF 1989	35.7	58	70.1	83.7	91.8
Model	66	77	82	86	89

The table 4 shows different percentiles of wealth computed with the Survey of Consumer Finances for the year 1989 and the percentiles given by the model. The model over-predicts the wealth inequality. I interpret this findings under a positive lens. Given the modest success of other models in explaining wealth inequality, it seems that liquidity constraints of the sort analyzed in the paper in conjunction with shocks to capital creation or entrepreneurship, are important ingredients of a model that aims to replicate the wealth inequality of the U.S. It seems that incorporating idiosyncratic shocks to labor earnings is not the best route to take if we want to explain wealth inequality.

V. Conclusions

In this paper I have developed a model similar to the decentralized neoclassical growth model. Idiosyncratic shocks to entrepreneurial activity were introduced where only some agents can create capital in the economy. I showed that the interaction of this sort of shocks and liquidity constraints can create a large wealth inequality. The reason is that agents that are entrepreneurs face a high return on capital accumulation, a feature that arises directly from the liquidity constraints they face.

When the model is compared with the data in terms of the inequality of wealth. It was found that the model over-predicts inequality. I interpret this under a positive view. It seems that shocks to capital creation in conjunction with liquidity frictions are key ingredients of a model that aims to replicate the U.S. wealth inequality. Shocks to capital creation are interpreted as entrepreneurial shocks. This is a reduced form that avoids modeling explicitly the decision of individuals to become entrepreneurs. This is left for future work. I also showed that the welfare costs of the liquidity constraints are sizeable, they may represent approximately 5% of consumption in the steady state.

On a methodological aspect of this paper. I showed that adapting the Endogenous Grid Method to deal with state dependent inequality constraints can be reliable and easy to program.

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Appendix A. Proofs of properties of the solution

Assumption 1: $1 < q < 1/\theta_s$.

The value function for the individual with state (k, z) can be defined as:

$$v(k, z) = \max_{c,k'} \left[u(c) + \beta \int v(k', z') Q(z, dz') \right]$$

such that:

$$c + \psi[k' - \eta(1-\delta)k] \leq w + rk \quad \text{if} \quad z = 1 \tag{15}$$

$$c + q[k' - (1-\delta)k] \leq w + rk \quad \text{if} \quad z = 0 \tag{16}$$

and:

$$k' \geq (1-\theta_r)(1-\delta)k$$

The opportunity sets can be stated in terms of correspondences for both agents:

$$\Gamma(k, 1) = [\underline{k}', \overline{k}_1'] \equiv \left[(1-\theta_r)(1-\delta)k, \frac{w}{\psi} + \frac{r}{\psi}k + \eta(1-\delta)k \right]$$

$$\Gamma(k, 0) = [\underline{k}', \overline{k}_0'] \equiv \left[(1-\theta_r)(1-\delta)k, \frac{w}{q} + \frac{r}{q}k + (1-\delta)k \right]$$

The next figure shows the correspondences.

It is clear from the restrictions of the problem, that the set of allowable capital has a lower bound defined by either $\underline{k_1}$ or $\underline{k_0}$. I choose $\underline{k_0}$ since the investor agent will always want to accumulate more capital than any value between $\underline{k_1}$ and $\underline{k_0}$. Hence we have that $B = [\underline{k_0}, \infty)$.

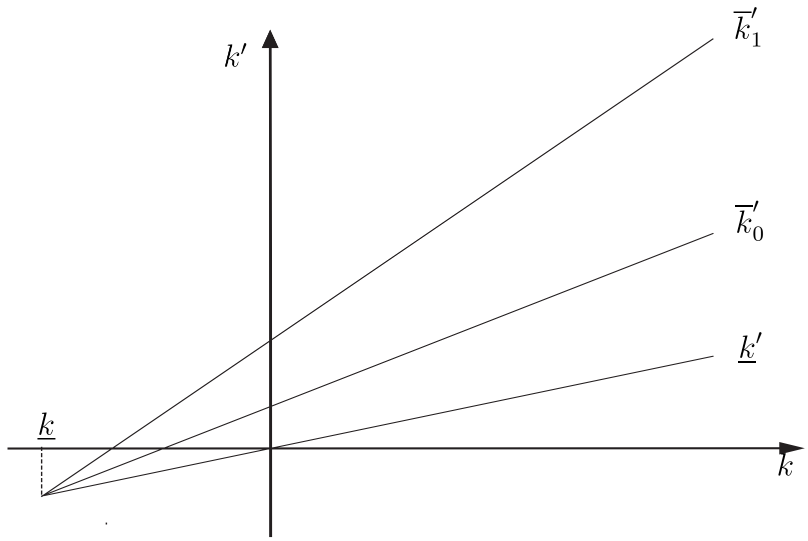

Figure 11: Correspondences.

Let me denote:

$$
\begin{aligned}
F(k, k', 1) &= u(w + [r + (1 - \delta)]\, k - q(\theta) k') \\
F(k, k', 0) &= u(w + [r + q(1 - \delta)]\, k - q k')
\end{aligned}
$$

Let $(B, \mathcal{B})$ and $(Z, \mathcal{Z})$ be measurable spaces of possible values for the endogenous and exogenous state variables respectively. The following properties are satisfied for our economy:

1. B is a convex Borel set in $\Re$, with its Borel subsets $\mathcal{B}$.

2. Z is a countable set defined by $\{0, 1\}$. Then $\mathcal{Z}$ is a σ-algebra containing all subsets of Z.

3. The correspondences $\Gamma(k, z) : B \times z \to B$ are nonempty, compact-valued and continuous.

4. The function $F : A \to \Re$ is bounded and continuous, where $A = \{(k, k', z) \in B \times B \times Z : k' \in \Gamma(k, z)\}$ is the graph of the correspondence. And $\beta \in (0, 1)$.

5. For each $(k', z) \in B \times Z, F(\cdot, k', z) : A_{k'z} \to \Re$ is strictly increasing.

6. For each $z \in Z, \Gamma(\cdot, z) : B \to B$ is increasing.

Given the properties above, by Theorem 9.7 of SLP. We know that: for each $z \in Z$, $v(\cdot, z)$: $B \to \Re$ is strictly increasing.

In order to prove strict concavity of the value function with respect to k, we define for the sequence problem.

Take the sequences of shocks $z^t = \{z_0, z_1, ...\}$. And take two initial values: $k_0 > \bar{k}_0$, and the associated optimal consumption sequences: $\{c_t(z^t)\}_{t=0}^{\infty}$ and $\{\bar{c}(z^t)_t\}_{t=0}^{\infty}$. Let $\{c_t^{\tau}(z^t)\}_{t=0}^{\infty}$ be the optimal consumption sequence associated with the initial $\tau k_0 + (1-\tau)\bar{k}$. Then:

$$
\begin{aligned}
v(\tau k_0 + (1-\tau)\bar{k}, z_0) &= E\left[\sum_{t=0}^{\infty} \beta^t u(c_t^{\tau}(z^t))\right] \\
&\geq E\left[\sum_{t=0}^{\infty} \beta^t u(\tau c_t(z^t) + (1-\tau)\bar{c}_t(z^t))\right] \\
&> E\left[\sum_{t=0}^{\infty} \beta^t (\tau u(c_t(z^t)) + (1-\tau) u(\bar{c}_t(z^t)))\right] \\
&= \tau E \sum_{t=0}^{\infty} \beta^t u(c_t) + (1-\tau) E \sum_{t=0}^{\infty} \beta^t u(\bar{c}_t) \\
&= \tau v(k_0, z_0) + (1-\tau) v(\bar{k}_0, z_0)
\end{aligned}
$$

which shows that $v(k, z)$ is strictly concave.

It is straightforward to verify that A_z is convex for each z. There are no increasing returns. Hence by Theorem 9.8 of SLP we have that: For each $z \in Z$, $v(\cdot, z) : B \to \Re$ is strictly concave and the policy is a continuous function not a correspondence. Let us denote this policy as $g(k, z)$. We also need to establish differentiability of the value function. This is not trivial as the liquidity constraints may bind for some states or some individuals once we analyze the market equilibrium with heterogeneity. If the problem can be stated with unique Karush-Khun-Tucker multipliers, then the value function is differentiable, in this case the problem can be represented as a static one in the same dynamic programming fashion as usual. We assume differentiability and proceed, leaving for the future to check this result.

Next, we verify the following: For each $(k, k') \in B \times B, F(k, k', z) : A_{kk'} \rightarrow \Re$ is strictly increasing.

Since $u(\cdot)$ is strictly increasing, we need to verify that: $w + [r + \psi\eta(1-\delta)] - \psi k' > w + [r + q(1-\delta)] - qk'$. This is means that $k' > (1-\theta)(1-\delta)k$. This condition is satisfied for the investor since his liquidity constraint is binding with a positive amount of investment. For the saver we need to assume that he will not be liquidity constrained for the property above to be satisfied.

The following assumption is trivially satisfied for our case as is illustrated in the figure above: For each $k \in B, \Gamma(k, \cdot) : Z \rightarrow B$ is increasing. Also we assume that the transition matrix Q satisfies the monotonicity property: for any increasing function $f(z)$, the function $Tf(z) = \sum_{z'} f(z')Q(z, z')$ is increasing in z.

Hence by Theorem 9.11 of SLP, we conclude that for each $k \in B$, $v(k, 1) > v(k, 0)$.

Proposition: $g(k, z)$ is strictly increasing in k. *Proof:* I will show the proof only for the case $z = 1$, the other case is analogous. Denoting by $\lambda(k)$ the multiplier on the constraint for minimum capital holdings, we have that the Bellman equation can be written as:

$$v(k, 1) = \max_{k'} \left[u(w + [r + \psi\eta(1-\delta)]k - \psi k') + \beta \int v(k', z')Q(z, dz') + \lambda(k)(k' - (1-\theta)(1-\delta)k) \right]$$

The first order condition is given by:

$$-q(\theta)u'(w+[r+\eta\psi(1-\delta)]k-\psi g(k,1))+\beta\int v'(k',z')Q(z,dz')+\lambda(k)=0$$

By way of contradiction assume that for $k>\bar{k}$, $g(k,1)\leq g(\bar{k})$. Then:

$$\begin{aligned}
q(\theta)u'(w+[r+\psi\eta(1-\delta)]k-\psi g(k,1)) &= \beta\int v'(g(k,1),z')Q(z,dz')+\lambda(k)\\
&> \beta\int v'(g(\bar{k},1),z')Q(z,dz')+\lambda(\bar{k})\\
&= q(\theta)u'(w+[r+\psi\eta(1-\delta)]\bar{k}-\psi g(\bar{k},1))
\end{aligned}$$

where the strict inequality follows from the strict concavity of the value function and the assumption that $\lambda(k)\geq\lambda(\bar{k})$, which accords with the assumption that the policy is strictly decreasing in current capital. Since $u(\cdot)$ is strictly concave, it follows that necessarily $g(k,1)>g(\bar{k},1)$, a contradiction.

Appendix B. The Endogenous Grid Method

The two Euler equations are:

$$\begin{aligned}
& \psi u'(w+rk-\psi[k'-\eta(1-\delta)k])-\lambda_1(k)\\
= & \beta Q(1,1)[r+\psi\eta(1-\delta)]u'(w+rk'-\psi[k_1'(k')-\eta(1-\delta)k'])-\beta Q(1,1)(1-\theta_r)(1-\delta)\lambda_1(k')\\
+ & \beta Q(1,0)[r+q(1-\delta)]u'(w+rk'-q[k_0'(k')-(1-\delta)k']-\beta Q(1,1)(1-\theta_r)(1-\delta)\lambda_0(k')
\end{aligned}$$

$$
\begin{aligned}
& qu'(w + rk - q[k' - (1-\delta)k]) - \lambda_0(k) \\
= \;& \beta Q(0,1)[r + \psi\eta(1-\delta)]u'(w + rk' - \psi[k_1'(k') - \eta(1-\delta)k']) - \beta Q(0,1)(1-\theta_r)(1-\delta)\lambda_1(k') \\
+ \;& \beta Q(0,0)[r + q(1-\delta)]u'(w + rk' - q[k_0'(k') - (1-\delta)k'] - \beta Q(0,0)(1-\theta_r)(1-\delta)\lambda_0(k')
\end{aligned}
$$

The implementation of the endogenous grid method requires to fix a grid on the capital state space: $\{k_1, k_2, ..., k_M\}$, which won't change through iterations.

In the equations above, it is replaced the next period capital assumed to be on the fixed grid. Given that tomorrow's capital is on the grid, a policy function for the next period is guessed on the future capital, both for investors and savers: $k_1^j(k_i)$ and $k_0^j(k_i)$ for $j = 0$. Also a guess for future values of the multipliers is introduced: $\lambda_1^j(k_i)$ and $\lambda_0^j(k_i)$ for $j = 0$. With this functions defined, the aim is to compute the current value of capital for both agents from their Euler equations. The algorithm is as follows:

1. Given the fixed grid, guess values $k_1^j(k_i)$,$k_0^j(k_i)$,$\lambda_1^j(k_i)$ and $\lambda_0^j(k_i)$ for $j = 0$.

2. Solve for the vectors k_1^* and k_0^* using the Euler equations:

$$
\begin{aligned}
& \psi u'(w + rk_1^* - \psi[k_i - \eta(1-\delta)k_1^*]) - \lambda_1(k_1^*) \\
= \;& \beta Q(1,1)[r + \psi\eta(1-\delta)]u'(w + rk_i - \psi[k_1^j(k_i) - \eta(1-\delta)k_i]) - \beta Q(1,1)(1-\theta_r)(1-\delta)\lambda_1^j(k_i) \\
+ \;& \beta Q(1,0)[r + q(1-\delta)]u'(w + rk_i - q[k_0^j(k_i) - (1-\delta)k_i] - \beta Q(1,1)(1-\theta_r)(1-\delta)\lambda_0^j(k_i)
\end{aligned}
$$

$$
\begin{aligned}
& qu'(w + rk_0^* - q[k_i - (1-\delta)k_0^*]) - \lambda_0(k_0^*) \\
= \; & \beta Q(1,1)[r + \psi\eta(1-\delta)]u'(w + rk_i - \psi[k_1^j(k_i) - \eta(1-\delta)k_i]) - \beta Q(1,1)(1-\theta_r)(1-\delta)\lambda_1^j(k_i) \\
+ \; & \beta Q(1,0)[r + q(1-\delta)]u'(w + rk_i - q[k_0^j(k_i) - (1-\delta)k_i] - \beta Q(1,1)(1-\theta_r)(1-\delta)\lambda_0^j(k_i)
\end{aligned}
$$

In order to do that assume $\lambda_1(k_1^*) = \lambda_0(k_0^*)$. Call the values that solve the equations: $\hat{k}_1^*$ and $\hat{k}_0^*$.

3. For each k_i on the fixed grid check: if

$$
\begin{aligned}
k_i \geq (1-\theta_r)(1-\delta)\hat{k}_1^* \;&\Rightarrow\; k_1^* = \hat{k}_1^* \\
k_i < (1-\theta_r)(1-\delta)\hat{k}_1^* \;&\Rightarrow\; k_1^* = \tfrac{k_i}{(1-\theta_r)(1-\delta)} \\
k_i \geq (1-\theta_r)(1-\delta)\hat{k}_0^* \;&\Rightarrow\; k_0^* = \hat{k}_0^* \\
k_i < (1-\theta_r)(1-\delta)\hat{k}_0^* \;&\Rightarrow\; k_0^* = \tfrac{k_i}{(1-\theta_r)(1-\delta)}
\end{aligned}
$$

4. With the values k_1^* and k_0^* find the values of the multipliers $\lambda_1(k_1^*)$ and $\lambda_0(k_0^*)$, directly from the Euler equations. Taking the fixed grid as the value for next period capital and the obtained values k_1^* and k_0^* as arguments of this functions, we have the new policy functions. We also have the new functions for the multipliers.

5. Using interpolation, find $k_1^{j+1}(k_i), k_0^{j+1}(k_i)$ and $\lambda_1^{j+1}(k_i)$ and $\lambda_0^{j+1}(k_i)$, for $j = 0$. Then we have the new policies and the new multipliers evaluated in the fixed grid. Then we iterate on j until we have an approximate convergence of all policies and multipliers.