

Hierarchies, Institutions and Long Run Development

Sergio Salas¹

(Preliminary-November 2009)

Abstract

In this paper I investigate if the organization of the production process in extractive economies during the colonial period, have lasting effects in the long run development of such economies. In order to analyze this, a model for economies depending on an extractive good is developed. I take the formation of hierarchies as the main organization for production that colonists implemented, extending existing models from the IO literature. Among many features that the hierarchy formation displays is the unequal distribution of income it creates. In the model, once the country loses its colonial status hierarchies will endure, not as labor relationships, but as social structures. Individuals will interact under the risk of expropriation. As a result of these interactions, resources are diverted from productive activities which influence the rate of growth of the economy and the level of welfare.

Keywords: Colonial Period, Hierarchies, Institutions, Interactions.
JEL Classification: O43, C73.

¹PhD Student. Department of Economics, University of Chicago. Email:ssalas@uchicago.edu

Hierarchies, Institutions and Long Run Development

I. Introduction

Recently some authors attempted to construct theories and test hypothesis about the relationship between the different types of institutions that Europeans implemented or foster in countries of the new world in the colonial period and economic performance until the present. See for example: Acemoglu, Jhonson and Robinson (2002) and Acemoglu (2006). These authors analyze the emergence and persistence of two kinds of institutions: *institutions of private property*, defined as institutions that secure property rights for society and *extractive institutions*, which concentrates power in the hands of a small elite and creates a high risk of expropriation for a large segment of the population².

Finding an accepted theory explaining the endogenous formation and persistence of extractive institutions has proven to be difficult. In Acemoglu's (2006) words: "A central question that still remains unanswered is: if institutions matter (so much) for economic performance, why do societies choose or end up with "inefficient" institutions that do not maximize economic growth or aggregate economic welfare?"

In the model I propose, I take into account several features that many regions colonized by European countries shared. These include: the dependence of such economies on a narrow set of goods, the nature of the production process which was mostly an extractive process, the labor relationships that were in most cases based on slavery, and the control of the extractive process which was conducted by the monarchies in Europe. As a main methodological device underlying this features, I consider the formation of hierarchies to organize and control the extractive process. Borrowing from the theory of hierarchy formation in industrial organization, I develop a model, considering the features mentioned above, to explain the organization of labor in an economy with an extractive good. An attempt is also made to analyze if the empirical predictions of the model associated with the formations of hierarchies, match empirical facts about economies during the colonial period.

By analyzing the formation and development of these hierarchies, I explain the formation of unequal wealth distribution during the colonial era. This is important, since this unequal distribution of wealth could have persistent effects even after the independence of these countries are reached and could be at the root of the formation of bad institutions.

I assume that the structure of hierarchies created during the colonial period, endure as social relationships after the colonial period ends. These relationships will allow agents in the hierarchy to expropriate resources of other people in the hierarchy.

When the colonial period ends, agents will have access to a technology of production, that will yield resemble the neoclassical growth model. Agents will interact in the process

²Societies with well established property rights, by providing incentives to private initiatives, will encourage investment, not only in physical and human capital, but also investment in research and development of new technologies and adoption of new technologies that become available in society.

of accumulation with the possibility of expropriation of the returns of investment. As we will see this interactions can create in fact a higher rate of growth towards the steady state compared to the absence of expropriation. But the level of capital in the steady state and welfare will be lower with expropriation. Interestingly, we will see that a pattern of high growth can be displayed under expropriation with an increasing inequality of wealth.

In the next section I present some evidence gathered so far, with respect of the features of countries where Europeans settled. This information will serve to extract the main assumptions of the model that is developed in section 3. In section 4 the long run implications of the hierarchy formation will be analyzed. Section 5 concludes.

II. Some features of countries during the colonial period

So far I have reviewed some literature mainly for Latin America. Some evidence about the social and labor formation of hierarchies is presented, as well as some features of the labor force and technology employed. Also some estimates of the degree of income inequality that was present in that period.

There is some evidence that supply of labor in Latin America was relatively scarce during the colonial period. The main factor that explains this is the diseases that Europeans brought to the new world, that were unknown for the local people. Randall (1977) documents that labor supply declined dramatically after the conquest. According to the author, some 95 per cent of the pre-conquest population died by the 17th century in Mexico and probably as many one or two generations later in Peru. Amazingly, population of Mexico and Peru did not regain their pre-conquest levels until the mid 1940's.

Another important feature is the extent to which slavery was employed by colonizers. According to the same author, only in mining, up to 10 percent of adult population were permitted by the Spanish crown to be subject to work as slaves in Mexico and Peru. According to Randall (1977), by the eighteenth century perhaps as much as 20 per cent of adult population were slaves in the mining sector. The mining process was important in the region. Much of the activity revolved around mining, not only as a direct process of extraction but also, much of the economy emerged as related to the process such as the production of clothes and provision of food to the mines.

With respect to the technology employed in the mining sector, the process is currently capital intensive. In the colonial period, notwithstanding, the main factor of production were workers. There is evidence that there have been technological improvements in the extractive process. For example, the innovation in the production of silver using mercury, is said to be very important for the continuation of the process of extraction that lasted several decades.

Property rights were not applied or fostered in Latin America during the colonial period. The creation of the *encomienda* by the Spanish crown, allowed colonizers to expropriate services, lands and money from the native indians. Moreover many indians themselves owned *encomiendas*, which often corresponded to prior landownership or trib-

ute rights. Randall (1977) illustrates: "For example in the mid 16th century an indian village might pay tribute of 1000 pesos to the Spanish king, and up to 4000 pesos to its own upper class". This illustrates that the colonial process might have maintained the hierarchical social structure of the local people. It is not difficult to argue that this hierarchical structure was pervasive in the society and may have been maintained or created by the colonizers also for the process of extraction. In the lower levels of production this was certainly the case, the form of slavery in mines needed in fact the creation of a hierarchical structure of supervision.

The formation of hierarchies could be understood even in a broader sense. The Spanish crown was entitled to define which persons were assigned as *encomenderos*, their levels of benefits, the wages that they should perceive, how many indians or villages were under their supervision, the level of taxes that they were allowed to charge to the indians and so on. Hence, the structure created by the Spaniard crown could be understood by a big hierarchy.

It is not surprising that the distribution of income of an economy with characteristics described above were highly skewed. Randall (1977) gives a vague but illustrative estimation of the uneven distribution of wealth in the Spaniard colonies in Latin America: "...the poorest survivors in Peru had an annual income of 30 pesos (or its equivalent in food and clothing) and the richest an income of 3200 pesos, the rich minority had an income one hundred times as great as that of the poor majority".

III. The Model

The key feature about hierarchies, is that agents at different hierarchical levels are vertically related. This set up has been analyzed by many authors in the formation of an optimal hierarchy for a firm. Among them: Qian (1994), Calvo and Wellisz (1979,1978), Williamson (1967) and Keren and Levhari (1979). To the best of my knowledge, Qian is the most general treatment as it generalizes and combines the work of previous authors.

Qian (1994) focuses on a static model, with a constant returns to scale technology and assumes a fixed amount of labor and a given level of technology. He is able to derive important results, all endogenous with regard of the span of control, the wage, the number of employees in a given tier, the number of tiers of the hierarchy and total revenue.

In the model that I propose, several considerations must be taken into account, which leads to modification of Qian's setup³:

1. Since I focus on an extractive process, such as the extraction of minerals, the optimal rate of extraction of an exhaustive resource must be explicitly analyzed.
2. The setup must be dynamic since the dependence of extractive economies on exhaustible resources calls for careful consideration of the effects of exhaustion in the

³More investigation is needed to asses if the following features are also present in extractive economies of the colonial period in places other than Latin America.

economy.

3. We need to incorporate a varying labor force through time, to take into account the decrease in population during the colonial period, mentioned in the previous section.
4. We should incorporate the possibility of slavery. This is important in the design of the mechanism of incentives that will yield the compensation to workers in the hierarchy.
5. Finally, we should incorporate the possibility of technological improvements in the extraction process. Which appears to have been significant in determining the evolution in the system of extraction of minerals.

In what follows I sketch the model that considers and incorporates features described previously.

I begin by assuming that the colonist, is the head of the hierarchy it forms. It uses $N(t)$ individuals in the base of the hierarchy in a given time t . In this model, we assume that the base of the hierarchy is composed of local people. As we will see, the model predicts that the number of workers in a given tier is increasing as we move down the tiers. Hence when the hierarchy considers the use of $N(t)$ individuals in the bottom of the hierarchy, it also considers the amount of workers in the entire hierarchy.

According to the colonial history, the number of workers chosen to work in the mines, were designated by the colonist, this was a forced work, hence it is not unreasonable to assume that $N(t)$ is a variable of control for the head of the colony. We also assume that the other workers in upper tiers of the hierarchy were either other local people or foreigners who were sent by the colonist from abroad to work mainly as supervisors in the hierarchy. What is important in the model is that the number $N(t)$ is going to be one of the main determinants of the final product of the hierarchy⁴.

Every agent in the hierarchy has only one superior, we denote tiers in the hierarchy by k , $k = 0, 1, 2, \dots, K(t)$. That is in any given time the number of tiers chosen by the head of the hierarchy is $K(t)$. $x(t)_k$ is the number of agents in time t in tier k . $x(t)_0 = 1$, corresponds to the colonist in our model⁵. The number of agents in t subordinate of a common superior is called the span of control in tier $k - 1$, and we denote by $s(t)_k$. I also assume that the span of control is constant in any given tier, this implies:

$$x(t)_k = x(t)_{k-1} s(t)_k$$

I assume that the utility function of any agent in the hierarchy has the following form:

⁴Even though the number $N(t)$ of workers will be a choice variable, there will be natural bounds on it, that will arise specially as the result of a decreasing level of population.

⁵For simplicity we assume $x(t)_0 = 1$ for all t , there is always only one agent in the top of the hierarchy. Of course, we should not interpret that literally there is only a single individual. The "colonist" could mean an entire group of people at the head of the pyramid.

$$u = c - \phi(e)$$

where c is consumption and $e \in \{0, 1\}$ is the level of effort the agents exerts in production. For simplicity I assume that effort is either 1 or 0. This assumption, coupled with the linearity of the utility function in consumption, permits to obtain simple closed form solutions. I assume: $\phi(0) = 0$ and $\phi(1) = \phi, \phi > 0$.

We assume that the final output (which we could think of as the amount of minerals extracted), depends on the performance of the entire hierarchy. In particular we follow Qian (1994) to assume a recursive technology, where effort of a given agent in any given tier is combined with organizational effectiveness of the superior to produce a level of intermediate input to be used in the following tier. We assume a process for output of a single agent given by: $y(t)_k = e(t)_k y(t)_{k-1}$, where $y(t)_k$ is the level of organizational effectiveness in a given tier k at time t . Given the assumption that effort is either 0 or 1, it does not pay for the hierarchy to implement zero level of effort.

It is assumed that the level of physical output (the amount of minerals extracted) depends on the workforce of the workers in the bottom of the hierarchy. The level of output per capita realized at the bottom of the hierarchy is just $\theta(t)y(t)_K$, where $\theta(t)$ is a technological parameter, potentially changing in time. Notice that $N(t) = x(t)_{K(t)}$, is the number of workers at the bottom of the hierarchy. To obtain the total final output, we must multiply the product of each agent at the bottom of the hierarchy by the number of workers in that tier:

$$\theta(t)N(t)y(t)_K = \theta(t)N(t)$$

The role of any agent in any tier k , in addition to provide its own effort to productive process, is to perform vigilance duties about the effort of agents under his control. We abstract in this period to tackle issues of bribery in the relationships among tiers, or incentives to induce an optimal level of vigilance, and assume that agents in the tiers perform their monitor duties at their best. Monitoring is not perfect though, we assume that any agent has a probability of monitoring individuals under its control that its a decreasing function of the length of its span: $p(t) = 1/s(t)$ for any given tier.

I follow Qian (1994) in considering the following scheme of incentives: to implement the effort level e^* from his subordinates, the superior implements the following scheme:

give c if $e \geq e^*$ is known, or if e is not known
 give 0 if $e \leq e^*$ is known

It is important to point out, that this scheme allows for the possibility of slavery. c could be interpreted as the minimum amount of consumption needed to be given for a worker to produce, and 0 could be interpreted as a punishment the worker receives. It is clear then that incentives and monitoring are important even in the slavery case.

The incentive compatibility condition requires:

$$c - \phi(e^*) \geq p \cdot 0 + (1 - p) \cdot c - \phi(e), \text{ for all } e \leq e^*$$

Using $p(t) = 1/s(t)$, this yields the following compensation:

$$c(t)_k = \phi(e(t)_k)s(t)_k = \phi s(t)_k$$

for the level $e(t)_k = 1$ that the colonist want to implement.

The total cost of operating the hierarchy in any given time t , is equal to:

$$\sum_{k=1}^{K(t)} \phi s(t)_k x(t)_k$$

Hence we can state the colonist problem as:

$$\max_{s(t)_k, K(t), T} \int_0^T \left[\theta(t)x(t)_{K(t)} - \sum_{k=1}^{K(t)} \phi s(t)_k x(t)_k \right] e^{-\rho t} dt$$

subject to:

$$\begin{aligned} x(t)_k &= x(t)_{k-1} s(t)_k \\ \dot{M}(t) &= -\theta(t)x(t)_K \\ x(t)_0 &= 1 \\ M(0) &= M \\ M(T) &\geq 0 \end{aligned}$$

ρ is the colonist discount rate, $M(t)$ is the amount of stock of the extractive commodity available in time t , which we could think of as minerals. M is the initial level of the stock.

Keren and Levhari (1979), pioneered the solution of optimal formation of hierarchies for the static case, using a continuous approximation, and optimal control techniques. In our case, the the use of a Hamiltonian for the entire problem would involve the use of another Hamiltonian "inside" the Hamiltonian, a non standard problem. The strategy I use to solve this problem, is to optimize first the formation of the hierarchy in a given period of time, *conditional* on a given level of tiers $K(t)$, and find the optimal values of all other variables as a function of $K(t)$. Then replace these values in the original problem, and optimize in time using $K(t)$ as a control variable. Hence, in the next section, I develop the optimal formation of the hierarchy in a given moment of time and conditional to a given level of tiers $K(t)$. The results derived in the next section are similar to those derived in Qian (1994).

III.I Conditional formation of Hierarchy

In this section , we find the conditional (static) optimal hierarchy. We omit the time index for simplicity and use a continuous approximation to state the problem as:

$$\max_{s(k)} \left\{ \theta x(K) - \phi \int_{k=0}^K s(k)x(k)dk \right\}$$

subject to:

$$\begin{aligned} \dot{x}(k) &= x(k)\text{Ln}(s(k)) \\ x(0) &= 1 \\ x(K) &\text{ free} \end{aligned}$$

Hence we treat K as given in the problem. In this formulation of the problem $X(K)$ is free. This is an strong assumption. Recall that this is the amount of labor at the bottom of the hierarchy ($X(K) = N$). The assumption here is that the colonist has access to any amount of workers that he finds optimal to employ in the hierarchy (for a given amount of workers in the bottom N , and a given number of tiers K , the number of workers in the upper tiers is already determined). Once we find the solution to this optimization problem we will put a restriction on the amount of labor that can be used. This is important because of the empirical facts we sketched in section 2, where scarcity of workers appears to have been an important problem in the process of extraction.

Let us state the Hamiltonian as:

$$H = \theta x(K) - \phi s(k)x(k) + \lambda(k)x(k) \ln s(k)$$

The necessary conditions for maximization are:

$$\dot{\lambda}(k) = \phi s(k) - \lambda(k) \ln s(k) \tag{1}$$

$$\lambda(k) = \phi s(k) \tag{2}$$

$$\dot{x}(k) = x(k) \ln s(k) \tag{3}$$

$$\lambda(K) = \theta \tag{4}$$

These are the usual necessary conditions for optimization, equation (4) arises because this problem displays an "scrap value", which is the term $\theta x(K)$ in the hamiltonian.

Sufficient conditions for the existence of a solution, require to find optimal values $s^*(k)$ and $x^*(k)$ such that (1) to (4) holds, plus concavity of $\theta x(K) - \phi s(k)x(k)$ jointly in x and s . This result is known as the *Mangasarian sufficiency theorem*. It can be easily proved

that $\theta x(K) - \phi s(k)x(k)$ is jointly (weakly) concave in x and s for all t . Hence we can proceed to finding the solution, which will turn out to be unique.

From equations (1) and (2) we have:

$$\frac{\dot{s}(k)}{s(k)} = 1 - \ln s(k)$$

The solution for this differential equation is:

$$\ln \left(\frac{s(k)}{e} \right) = Ae^{-k}$$

For some constant A , to find this constant we use (2) and (4):

$$s(K) = \frac{\lambda(K)}{\phi} = \frac{\theta}{\phi}$$

Then we have that:

$$A = e^K \ln \left(\frac{\theta}{\phi e} \right) = e^K \ln a$$

where we have denoted: $a = \left(\frac{\theta}{\phi e} \right)$.

Then we have the solution for the span of control in tier k , $s(k)$ as:

$$s(k) = ea^{e^{K-k}} \tag{5}$$

From equation (3), we have:

$$\dot{x}(k) = x(k) \ln s(k) = x(k) \left[1 + e^{K-k} \ln a \right]$$

This last equation has solution:

$$x(k) = e^k a^{e^{K(1-e^{-k})}} \tag{6}$$

The total number of workers in the bottom of the tier, $x(K) = N$ is given by:

$$N = e^K a^{e^K - 1} \quad (7)$$

The results obtained so far are those that Qian(1994) obtains in his paper, where he analyzes an static formation of a hierarchy. Let us explain some of the features of the hierarchy⁶.

- The span of control in any given tier is increasing in the total number of tiers K . The span of control is decreasing down the hierarchy

$$\frac{\partial s(k)}{\partial k} = -e^{1+K-k} e^{K-k} a \ln a < 0$$

- The number of workers in a given tier, is increasing in the total number of tiers K . The number of workers is increasing down the hierarchy:

$$\frac{\partial x(k)}{\partial k} = a^{e^{-K(-1+e^{-k})}} \left(e^k + e^{-K(-1+e^{-k})} K \ln a \right) > 0$$

- The number of workers in the bottom of the hierarchy N is increasing in the total number of tiers.

$$\frac{\partial N}{\partial K} = e^{K-1} a \left(e^K + e^{2K-1} \ln a \right) > 0$$

By recalling that $a = \left(\frac{\theta}{\phi e} \right)$, it is easy to see the effects of technological innovations in the extractive process (higher θ). The span of control and the number of workers in any given tier is increasing in the parameter θ , the hierarchy will expand. If the distaste for effort is higher (higher ϕ) the hierarchy will shrink, this is because it is more costly to run the hierarchy when agents have a higher distaste of effort To induce agents to exert a given level of effort, more compensation will be needed.

With this static framework it is easy to investigate the qualitative predictions of the model. Just as a means of illustration, we investigate the degree of inequality in the hierarchy, something that might be important in light of the reported inequality in the distribution of income in the colonial period, that is documented, and mentioned in section II of the paper.

To give an illustration of the degree of income inequality that the model predicts, we use the following values for the parameters: $\theta = 1$, $\phi = 0.307$ and $K = 4$. With this values, we compute the fraction of population in every tier, and compute a comparison of the relative income of a single individual in each tier, to the income of the head of the hierarchy, the colonist. The results can be seen in the following table.

⁶It is required that $a > 1$ in order for the hierarchy to make at least 0 profits. This will hold if the level of technology θ is sufficiently high or the distaste of labor ϕ sufficiently low.

Table 1

tier	% of workers	% of income of an individual relative to the head of the hierarchy
1	0.001	1.00
2	0.032	0.83
3	0.205	0.69
4	0.762	0.58

The next table compute the same ratios as previous table with the assumption of an improved level of technology, corresponding to $\theta = 1.1$. It is interesting that we find that the distribution of income is more unequal in this case.

Table 2

tier	% of workers	% of income of an individual relative to the head of the hierarchy
1	0.000	1.00
2	0.018	0.76
3	0.182	0.57
4	0.800	0.43

For a given level of total number of tiers K , technological improvements will make possible increase the span of control, the number of workers, the compensations, and the welfare level. These results can be seen in the following graphic, where we have used the same values of the parameters as before. Also, we can see the behavior of these variables as a function of the tiers of the hierarchy.

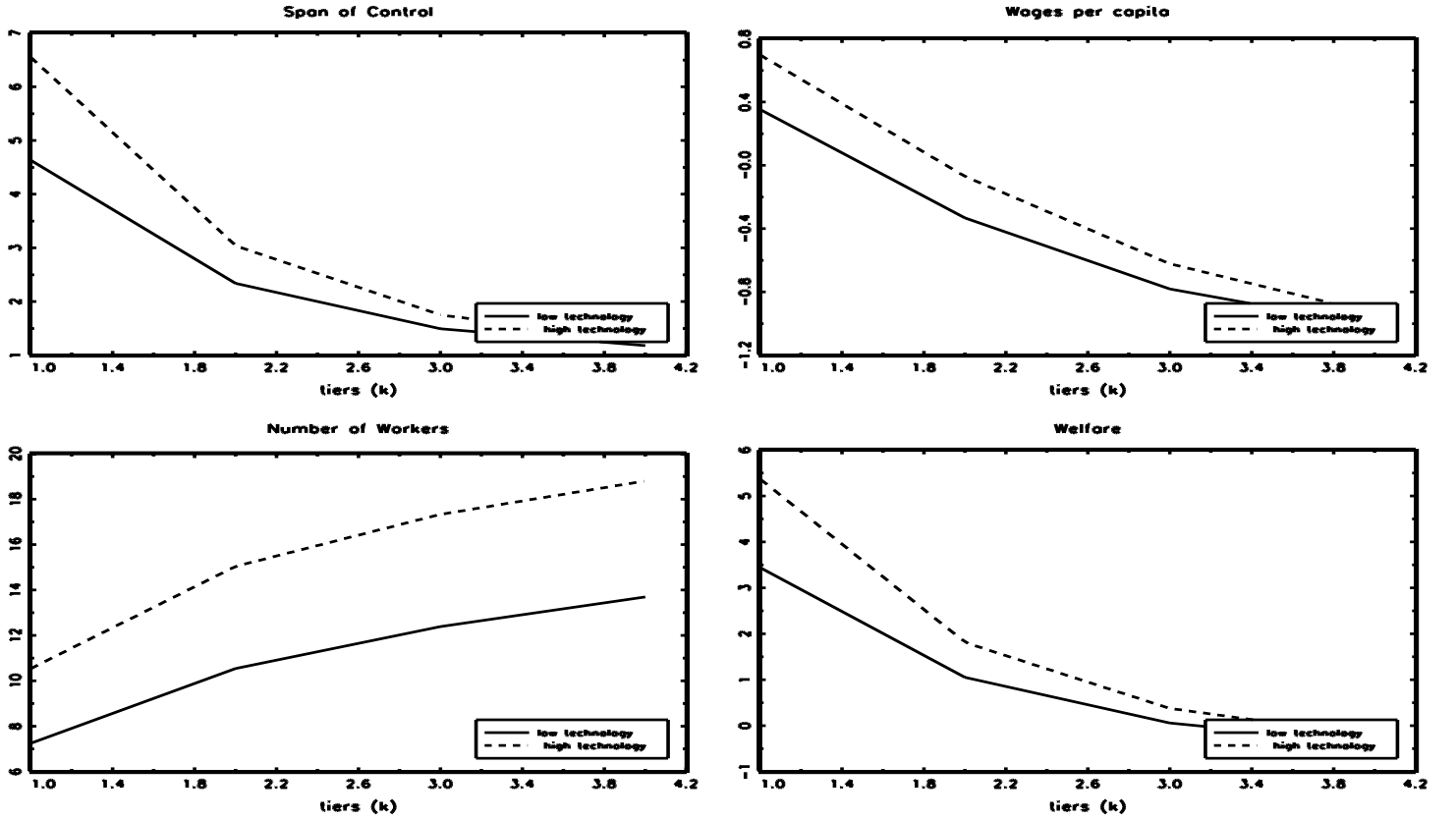


Figure 1. The curves were computed with the parameter values: $\theta = 1, \phi = 0.307, K = 4$. The technological improvement were computed with: $\theta' = 1.1$. Logs have been taken to the variables to facilitate visual comparison.

In the figure we can see that all variables increase, but the increase is not proportional. In particular, the wage of workers in lower tiers increase less than the wage of workers in upper tiers. Whereas the number of workers in lower tiers increase more than the number of workers in upper tiers. This is why we found the increase in income inequality associated to an improvement on technology in table 2.

In the next section we solve the problem of the optimization of the extractive process through time, using the results of this section. The optimization over time will use $K(t)$ as a control variable. Once a solution is found, it will be possible to find the optimized values of variables (5) to (7) as complete solutions over time.

III.II Optimal Rate of Extraction and Hierarchy Formation

In this section , I analyze the second step, which is the optimization over time, using the results of the previous section. We have the variables, $s(k), x(k)$ and N given in equations

(5), (6) and (7) for a given moment in time.

I assume that the amount of labor that can be used in the bottom of the hierarchy is bounded below and above. This is important, since the scarcity of labor associated with the sharp decline in population described in section II, can have important effects in the formation of the hierarchy.

I assume that there is a maximum level of labor available to be used in the bottom of the hierarchy given by N_{max} , which will be equal to⁷:

$$N_{max} = e^{\bar{K}} a^{e^{\bar{K}} - 1}$$

Hence \bar{K} is the maximum amount of tiers that can be used in any given time. Naturally the lower bound on K is one, and corresponds to the minimum amount of labor N_{min} that the hierarchy will ever use⁸:

$$N_{min} = 1$$

With expressions (5), (6) and (7), it is possible to find the revenue and the cost of the hierarchy in any given time as:

$$\begin{aligned} \text{revenue :} & \quad \theta(t)N(t) = \theta(t)e^{K(t)}a^{e^{K(t)}-1} \\ \text{cost :} & \quad \phi \int_{k=0}^{K(t)} s(t)x(t)dk = \phi e(e^{K(t)} - 1)a^{e^{K(t)}} \end{aligned}$$

By recalling that $a = \frac{\theta(t)}{\phi e} > 1$, we find that profits in any given period of time, are given by:

$$\text{profits : } \theta(t)a^{e^{K(t)}-1}$$

This result states that the (instantaneous) profits are only bounded by the use of the maximum amount of labor that can be used in the bottom of the hierarchy N_{max} , which corresponds to the use of \bar{K} number of tiers, hence if the objective of the hierarchy were to extract the maximum amount of the resource at a given time, \bar{K} will be attained.

⁷It is important to mention that the assumption of slavery in the formation of the hierarchy is showing here, because the level N is assumed to be in control of the colonist (only limited by the physical constraint that $N \in [N_{min}, N_{max}]$). The only cost for the colonist in employing a given amount of workers, is the compensation scheme that arise as a consequence of imperfect monitoring of activities.

⁸I assume that the level N_{min} is available to be used in the bottom of the hierarchy, if there is less number of workers available then there is no possibility of extracting the product at all with the use of the hierarchy.

Although scarcity of labor could well be a problem that actual colonists might have faced, the technological features of increasing returns to scale (absent labor constraints) that we find in this model is odd. In future work it might be convenient to incorporate more reasonable assumptions in the process of extraction that might yield to a more conventional result. Among the features that were omitted for simplicity but could possibly be incorporated in future work are:

- We have assumed that the product of the hierarchy θN is sold at a given price (actually normalized to 1). This assumption might be unrealistic, since the model tries to capture an important and large productive activity of a colony. It is possible to assume that the hierarchy faces a downward demand.
- We have assumed that, at the bottom, product is obtained according to θN . Hence, there is constant returns to scale (if N_{max} is not reached) in the number of workers in the last tier. Incorporating decreasing returns may make the period profit function concave in $K(t)$.
- Increasing costs as resource is depleted. In the model the cost of running the hierarchy does not incorporate that costs might rise as resource remaining for extraction is lower.

I further make the following assumptions: The discount factor (or interest rate) is zero, $\rho = 0$ and the technological parameter $\theta(t) = \theta > 0$ a constant. These assumptions enable us to obtain a simple characterization of the solutions, later we will discuss the properties of the solution under the more general model. Under these conditions, the problem that the hierarchy faces is:

$$\max_{K(t), T} \int_0^T [\theta a^{e^{K(t)}-1}] dt$$

subject to:

$$\begin{aligned} \dot{M}(t) &= -\theta e^{K(t)} a^{e^{K(t)}-1} \\ M(0) &= M \\ M(T) &\geq 0 \\ K(t) &\in [0, \bar{K}] \end{aligned}$$

Notice that when the optimal path of $K(t)$ is found, then all other variables as the number of workers, the span of control, etc., will be determined endogenously.

The Hamiltonian for the above problem is:

$$H = \theta a^{e^{K(t)}-1} - \lambda(t) \theta e^{K(t)} a^{e^{K(t)}-1}$$

where $\lambda(t)$ is the costate variable. The necessary conditions for optimization are:

$$K(t) \quad \text{maximizes } H \text{ s.t. } K(t) \in [0, \bar{K}] \quad (8)$$

$$\dot{\lambda}(t) = 0 \quad (9)$$

$$\dot{M}(t) = -\theta e^{K(t)} a^{e^{K(t)}-1} \quad (10)$$

$$M(0) = M \quad (11)$$

$$\lambda^*(T^*)M(T^*) = 0 \quad (12)$$

$$0 = \theta a^{e^{K^*(T^*)}-1} - \lambda^* \theta e^{K^*(T^*)} a^{e^{K^*(T^*)}-1} \quad (13)$$

To prove that a solution exists, we need to use the existence theorems that were developed in the literature. For this particular case there exists such a theorem that we sketch using the notation of our problem⁹.

The Filippov-Cesari theorem

For each $(M(t), t)$ define the set $N(M, U, t)$ in \mathfrak{R}^2 by:

$$N(M, U, t) = \left\{ \theta a^{e^{K(t)}-1} + \gamma, -\theta e^{K(t)} a^{e^{K(t)}-1} : \gamma \leq 0, K(t) \in U \right\}$$

Consider the problem where (8) to (13) hold. So:

1. An admissible pair $(M(t), K(t))$ is found.
2. $N(M, U, t)$ is convex for each (M, t) .
3. U is closed and bounded.
4. \exists a number b such that $\|M(t)\| \leq b \forall t \in [0, T]$ and $(M(t), K(t))$.

Then there exists an optimal pair $M^*(t), K^*(t)$.

Hence we need to prove that conditions 1 to 4 are satisfied¹⁰. Condition 3 is satisfied because $U = [0, \bar{K}(t)]$. Condition 4 is satisfied because equation (10) is bounded given the bounds on $K(t)$. Condition 2 will be satisfied when $\dot{M}(t)$, equation (10), is linear in $M(t)$, which is obviously satisfied (with zero coefficient) in our case. Hence we need only to take care of condition 1, which we turn to now.

Condition (9) implies that the co-state variable is a constant. Condition (13) says that the hamiltonian at the final time should be equal to zero. This condition helps to

⁹See Seierstad and Sydsaeter 1987. Pag. 131.

¹⁰The theorem is restricted to the case that the final time is fixed, but there is a version of this theorem for free final time problems that requires that the final time belongs to a bounded interval, this will be the case in our problem as we will see shortly. See Seierstad and Sydsaeter 1987. Pag. 145.

pin down T , as an endogenous variable, because the colonist is deciding the duration of the operation of the hierarchy.

The solution to this problem involves analyzing different scenarios that might arise. I investigate two possible cases, the case of an interior solution, and the case of a solution when the bound \bar{K} is attained. There could be a degenerate solution when the lower bound on K is attained, in this case the hierarchy would find optimal to conserve the resource as much as possible, this will entail the use of only one individual in the hierarchy, this is the degenerate solution. We omit the analysis of this case because it is not relevant in light of the facts explained in section II, where the scarcity of labor appeared to have been an important binding constraint in the colonial period.

I begin with the possibility of existence of an interior solution.

Solution when $K^* \in (0, \bar{K})$

I will show by contradiction that there is no interior solution satisfying (8)-(13). If $\lambda^* = 0$, then no interior solution could be found. In this case, the hamiltonian is convex: $H = \theta a^{e^{K(t)}-1}$, hence (8) implies that K^* is \bar{K} . Hence, I assume that $\lambda^* > 0$.

The first order condition corresponding to (8) is given by:

$$\theta a^{e^{K(t)}-1} e^{K(t)} \ln a - \lambda \theta e^{K(t)} a^{e^{K(t)}-1} - \lambda \theta e^{2K(t)} a^{e^{K(t)}-1} \ln a = 0$$

The second order condition is equal to:

$$-\theta a^{e^{K(t)}-1} e^{K(t)} \left(-e^{K(t)} \ln a^2 - \ln a + \lambda + 3\lambda e^{K(t)} \ln a + \lambda e^{2K(t)} \ln a^2 \right)$$

Hence the hamiltonian is not globally concave. The first order condition imply:

$$\lambda e^{K^*} \ln a = \ln a - \lambda \tag{14}$$

which requires $\ln a > \lambda$. To verify that equation (14) delivers the maximum, evaluating the second order condition in this value gives: $-\theta a^{e^{K^*}-1} e^{K^*} (\ln a - \lambda)$, hence equation (14) will deliver indeed a maximum when $\ln a > \lambda$.

With this solution, we find by using equation (10) and (11), that the equation that describes the amount of M in any given moment of time is:

$$M(t) = M - t\theta \left(\frac{\ln a - \lambda}{\lambda \ln a} \right) a^{\frac{\ln a - \lambda}{\lambda \ln a} - 1} \tag{15}$$

Hence the resource is going to be depleted linearly in time. To find the values of λ^* and T^* , we should use (15) and (13) to solve for these values. Replacing (14) in (13) we can solve for λ^* , this gives:

$$\lambda^* = a^{-e^{K^*}} \quad (16)$$

But then, substituting back in (14) we have: $\lambda^* = 0$ which is a contradiction (it is a contradiction both because we have assumed $\lambda^* > 0$ and bounded K^* , while (16) implies that the solution for K^* would be ∞). Hence we conclude that there is no interior solution in the problem.

We now turn to the case of a possible solution in the upper bound of K .

Solution when $K^* = \bar{K}$

There are two possibilities that we analyze in turn, first the case where $\lambda^* = 0$ and when $\lambda^* > 0$. We show that the only solution that can arise is when $\lambda^* > 0$.

$\lambda^* = 0$ case. To verify if this is a solution, we see that equation (8), implies $K^* = \bar{K}$, given that in this case the hamiltonian is convex. Equation (10) and equation (11) imply that the amount of M in any given moment of time is:

$$M(t) = M - t\theta e^{\bar{K}} a^{e^{\bar{K}-1}}$$

For this equation to be a solution to the problem it must be the case that:

$$M(T) = M - T\theta e^{\bar{K}} a^{e^{\bar{K}-1}} \geq 0$$

The value $M(T)$ can be greater than zero, as $\lambda = 0$ and hence (12) will be satisfied. The problem arises when we try to verify condition (13), since λ^* is assumed to be zero, $H = \theta a^{e^{K(t)}-1}$, and hence the only solution to (13) implies $K^* = -\infty$ which is a contradiction.

$\lambda^* > 0$ case. In this case the first order condition corresponding to (8) would be:

$$\theta a^{e^{K(t)}-1} e^{K(t)} \ln a - \lambda \theta e^{K(t)} a^{e^{K(t)}-1} - \lambda \theta e^{2K(t)} a^{e^{K(t)}-1} \ln a > 0$$

which implies:

$$\lambda e^{\bar{K}} \ln a > \ln a - \lambda \quad (17)$$

equations (10) and (11) will imply:

$$M(t) = M - t\theta e^{\bar{K}} a^{e^{\bar{K}}-1}$$

Hence the optimal time length of operation of the hierarchy is:

$$T^* = \frac{M}{\theta e^{\bar{K}} a^{e^{\bar{K}}-1}}$$

To find the optimal value of λ we use (13) to find: $\lambda^* = e^{-\bar{K}}$. This value will verify equation (17):

$$\ln a > \ln a - e^{-\bar{K}}$$

Hence we have found the solution to the problem. In this solution, the amount of resources will be depleted linearly over time. This implies that the total number of tiers K^* is unchanged and equal to \bar{K} . Technology improvements will accelerate the pace at which the resource is depleted, and will increase the number of total tiers of the hierarchies. The effects on the rest of the variables will be similar to what we found in **Figure 1.**

The next figure shows the behavior of $M(t)$ over time, and the case where the hierarchy operated under a better technology (higher θ).

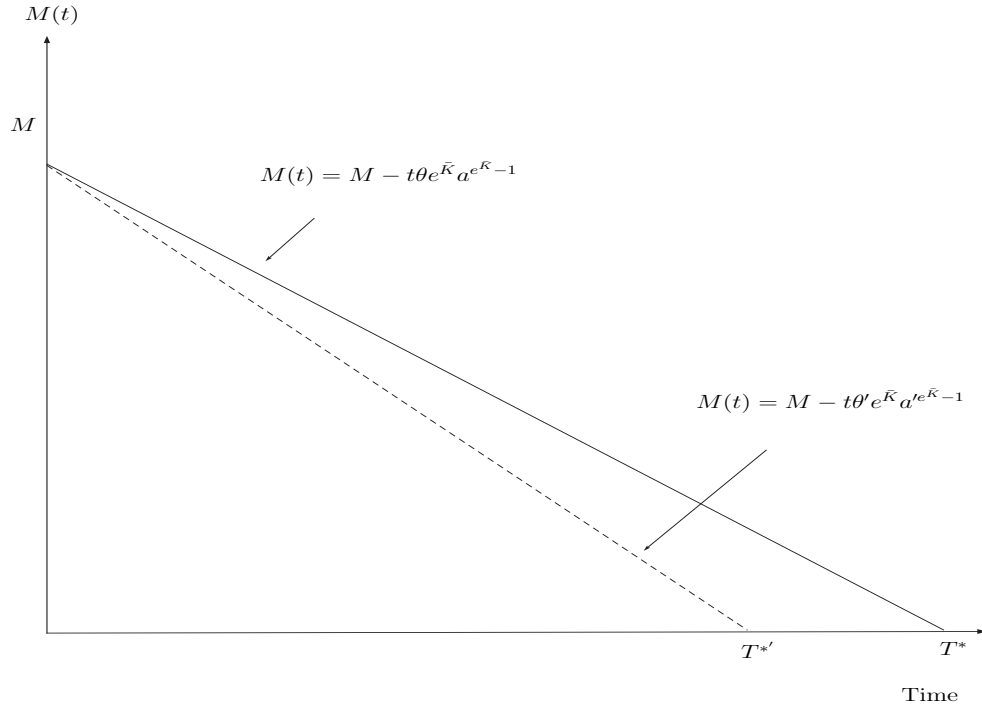


Figure 2. The behavior of $M(t)$. The resource will be depleted over time, with a constant number of tiers and workers. If the technology is better $\theta' > \theta$, there is less conservation, and the hierarchy ends sooner, when $T^{*'}$ is reached.

In the next section I incorporate the feature of a decreasing level of workers or population. The objective in analyzing this case, is to incorporate the fact of scarcity of labor in extractive economies of the colonial period explained in section II.

The case of shrinking labor

In this section we analyze the properties of the solution when the maximum amount of labor available to be used in the bottom of the hierarchy is shrinking over time. The motivation to analyze this case, is the empirical facts that colonial economies experienced. The sharp decline of population that were associated with the diseases that local people experienced.

We assume that the maximum number of laborers available to be used in the bottom of the hierarchy is given by:

$$N_{max} = N_0 e^{-\eta t}$$

Where N_0 is the initial amount of workers, and $-\eta < 0$ is the rate of decrease of workers available to be used in the bottom of the hierarchy. It is assumed that $N_0 > 1$. With this setup, there will exist a given time period (t^*) where the minimum level of maximum tiers ($K = 0$) associated with the minimum amount of labor needed to run the hierarchy ($N = 1$) is reached. This period of time is going to be given as:

$$N_0 e^{-\eta t^*} = e^0 a^{e^0 - 1} = 1 \rightarrow t^* = \frac{\ln N_0}{\eta}$$

In this case the optimal path of the number of tiers must satisfy:

$$K^*(t) \in [0, \bar{K}(t)]$$

So, in any given time the control will belong to a compact set, the solution procedure is the same as before. Hence we know that a solution must satisfy: $\lambda > 0$ and $K^*(t) = \bar{K}(t)$. The first order condition associated with equation (8) is:

$$\lambda e^{\bar{K}(t)} \ln a > \ln a - \lambda \tag{18}$$

Equation (10) implies:

$$\dot{M}(t) = -\theta N_0 e^{-\eta t}$$

The solution to this differential equation can be easily found. There is going to be two process, describing the solution, one up to t^* and the other from t^* until T^* . After time t^* the maximum rate of extraction is going to be given by the use of only one worker $N = 1$ which corresponds to $K = 0$. We state the solution for completeness of the mathematical procedure, but we should stress that after t^* is reached, the hierarchy would have collapsed and the further extraction would be just given by a single individual on the "degenerate" hierarchy. After time t^* the process governing the amount of $M(t)$ remaining is:

$$\dot{M}(t) = -\theta$$

The solution for the entire span of time $[0, T^*]$ is given by:

$$M(t) = \begin{cases} M - \frac{\theta N_0}{\eta} [1 - e^{-\eta t}] & t \in [0, t^* = \frac{\ln N_0}{\eta}] \\ M - \frac{\theta(N_0-1)}{\eta} + \frac{\theta \ln N_0}{\eta} - \theta t & t \in [t^*, T^*] \end{cases} \quad (19)$$

To find the optimal value of λ we use equation (13), which gives us: $\lambda^* = 1$. We must verify that equation (18) is satisfied:

$$e^{\bar{K}(t)} \ln a > \ln a - 1 \rightarrow e^{\bar{K}(t)} = 1 - \frac{1}{\ln a}$$

which will hold as the minimum value of $e^{\bar{K}(t)}$ is 1, precisely when $\bar{K} = 0$.

To find the optimal value of the final time T^* we use (19)¹¹:

$$T^* = \frac{M}{\theta} - \frac{N_0 - 1}{\eta} + \frac{\ln N_0}{\eta}$$

In the next figure we can see the behavior of $M(t)$ over time.

¹¹We have assumed throughout that M is sufficiently large.

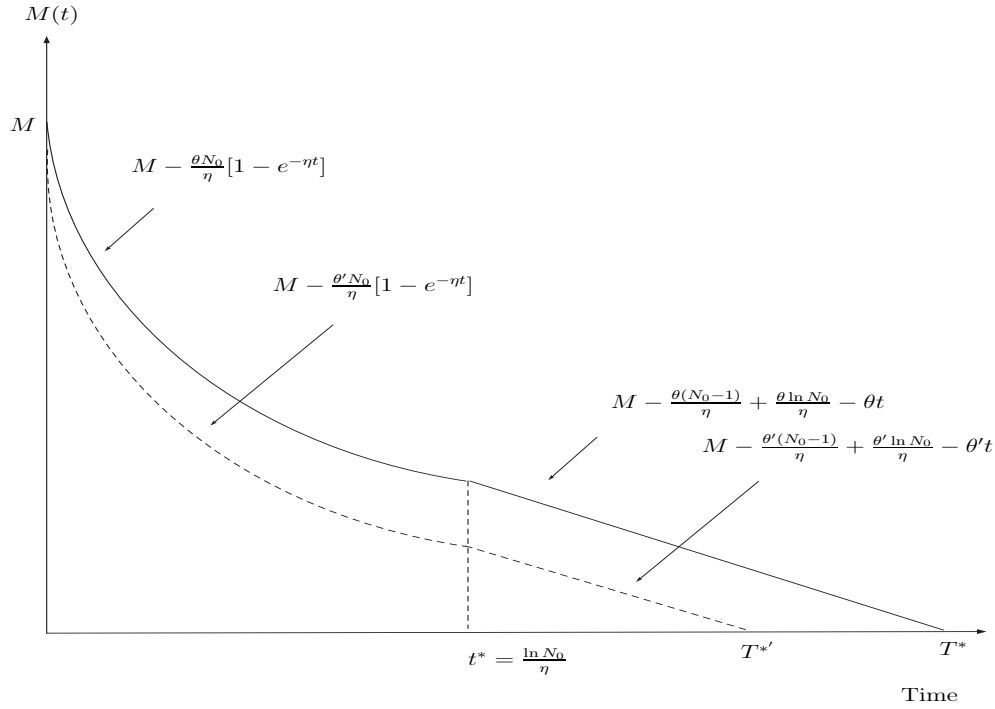


Figure 3. Until period t^* the depletion of resources will be exponential, when period t^* is reached the hierarchy would have collapsed to a single individual. In the figure we also show the behavior of $M(t)$ when a better level of technology is available $\theta' > \theta$.

It is possible to compute a measure of income inequality as we did in the previous section. In the next table we show when the number of tiers is $K = 6$, the fraction of workers of the hierarchy in each tier and the level of income of an individual relative to the income of the head of the hierarchy.

Table 3

tier	% of workers	% of income of individual relative to head of hierarchy
1	0.000	1.00
2	0.003	0.83
3	0.021	0.69
4	0.077	0.58
5	0.234	0.48
6	0.665	0.40

As time passes and the number of available workers diminishes, we have that the hierarchy will also shrink. The effects on income inequality can be seen in the following table.

Table 4

tier	% of workers	% of income of individual relative to head of hierarchy
1	0.00	1.00
2	0.14	0.83
3	0.86	0.69

We can see in table 4 that the income inequality is reducing over time. In figure 3, there is a given time t^* where the hierarchy collapses. This level is irrespective of how good the technology is. It depends on the rate of population decrease. If the rate η is higher, the hierarchy collapses sooner. The increases in the level of technology makes income distribution more unequal, but over time the hierarchy presents a more egalitarian structure as can be seen in table 4.

IV. Long run consequences

In this section we link the results developed previously, to propose an answer to the following question: If it is true that a reasonable model to explain production and welfare features of colonial extractive economies; is the model of hierarchies in production developed before, could this structure lead to detrimental effects in the long run rate of growth of such countries, after the colonial process of extraction is over?

We propose that the distinctive feature of the formation of the hierarchy endures after the process of colonization is over. Hierarchies themselves endure, not as labor relationships anymore, but as social structures. The exact nature of these hierarchies is not made explicit, but one obvious candidate is the government structure. It is not rare to find former colonies having a very centralized form of government with a hierarchical structure. We argue that the hierarchies developed in the process of extraction, endure and make less costly for agents in the upper tiers to involve in expropriation activities directed towards the lower tiers. Not only government may be part of the hierarchical structure that endures, but also any group related to power in society. Recall that we have found that agents in the upper tiers of the hierarchy enjoy more income than people in the lower tiers. In as much as wealth is related to power in the formation of independent nations after the colonial period, it is not perhaps unrealistic to assume that agents in the upper tiers had an influence in the endurance of the hierarchical structure as a way to secure their privileged positions in society. This could lead to the construction of democracies with a fragmented, hierarchical structure, since their inception as independent nations carried the flaws of the colonial structure.

We assume that the colonial economy ends before the hierarchy collapses in the model of shrinking labor explained in the previous section. So the hierarchical structure is maintained. We assume that with the end of the colonial period an opportunity of investment is opened. In particular we assume that agents in any tier k have access to a production technology of the form:

$$F(H_k, x_k) \tag{20}$$

F is a constant returns to scale production function, where H_k is the aggregate amount of capital in tier k and x_k is defined as before, equal to the number of agents in tier k . We assume, for the sake of simplicity that tiers are totally separate in their decisions of production. In each tier there exists a representative firm with the technology expressed in (20), that only uses capital and labor from the tier where it belongs. The only interaction between tiers is in the possibility of expropriation. We assume that any agents in a given tier, can expropriate a percentage of the investment of an agent who is below him in the hierarchy and belongs to his span of control. As a shortcut, we assume that the bigger the span of control, the lower the probability that expropriation actually takes place. Let $1/s_k$ be the probability of expropriation. Hence if b_k is the level of assets of an agent in tier k , the amount that he actually receives is given by:

$$r_k \left(1 - \frac{\tau}{s_k}\right) b_k, \quad 0 < \tau < 1$$

τ is the fraction of assets that is expropriated. r_k is the rate of return on assets to be determined in equilibrium.

We assume that agents supply inelastically a unit of labor or effort $e = 1$. Hence the utility function of an agent in tier k is given by:

$$U_k = c_k - \phi(e) = c_k - \phi$$

which is the same utility function that agents had in the colonial period. We want to maintain the same assumptions as before to isolate the effect of the risk of expropriation in the hierarchy from other changes, so we do not modify preference assumptions.

The budget constraint that agents face is given by:

$$\dot{b}_k = s_{k+1} \frac{\tau}{s_{k+1}} b_{k+1} + r_k \left(1 - \frac{\tau}{s_k}\right) b_k - c_k + w_k \tag{21}$$

\dot{b}_k is the derivative with respect to time of the assets of the individual. $\frac{\tau}{s_{k+1}} b_{k+1}$ is the extra income that the individual in tier k gets from expropriating an individual's assets who is a member of his span of control in the lower tier $k + 1$. Total income from expropriation is obtained by multiplying this value by the number of people that belong to the span, s_{k+1} . w_k is the wage rate prevailing in tier k .

For $k = 0$, that is the head of the hierarchy we have the budget constraint:

$$\dot{b}_0 = s_1 \frac{\tau}{s_1} b_1 + r_0 b_0 - c_0 + w_0$$

Since there is no one in an upper tier that could expropriate the return on his assets.

Also for $k = K$, that is agents in the bottom of the hierarchy, the budget constraint is given by:

$$\dot{b}_K = r_K \left(1 - \frac{\tau}{s_K}\right) b_K - c_K + w_K$$

Since there is no one below to whom expropriate assets.

Let us index the period of time when the colonial period and the hierarchy as an extractive process ends by 0. The initial level of assets that each individual has, corresponds to the level of income at the moment that the colonial period ends, which is given by:

$$b(0)_k = \phi s_k \tag{22}$$

Hence, each individual in the colony maximizes:

$$U_k = \int_0^\infty u_k e^{-\rho t} dt \tag{23}$$

subject to (21) and (22). And subject to the usual No Ponzi game condition.

The Hamiltonian for this problem is given by:

$$H_k = (c_k - \phi) e^{-\rho t} + \lambda \left[\frac{\tau}{s_{k+1}} b_{k+1} + r_k \left(1 - \frac{\tau}{s_k}\right) b_k - c_k + w_k \right] \tag{24}$$

we have omitted the time index to save on notation, but it is clear that $b_k, r_k, c_k, w_k, \lambda$ all depend on time.

Notice that given the linearity of the utility function in consumption, the agent will be willing to allow swings in his consumption in order to maximize utility. In fact the hamiltonian is not concave in consumption. The necessary conditions for maximum are, that consumption maximizes the hamiltonian in any moment of time, which we can express as:

$$c = \begin{cases} c_{max} & \text{if } e^{-\rho t} > \lambda \\ \in [0, c_{max}] & \text{if } e^{-\rho t} = \lambda \\ 0 & \text{if } e^{-\rho t} < \lambda \end{cases}$$

and:

$$\lambda \left[r_k \left(1 - \frac{\tau}{s_k} \right) \right] = -\dot{\lambda}$$

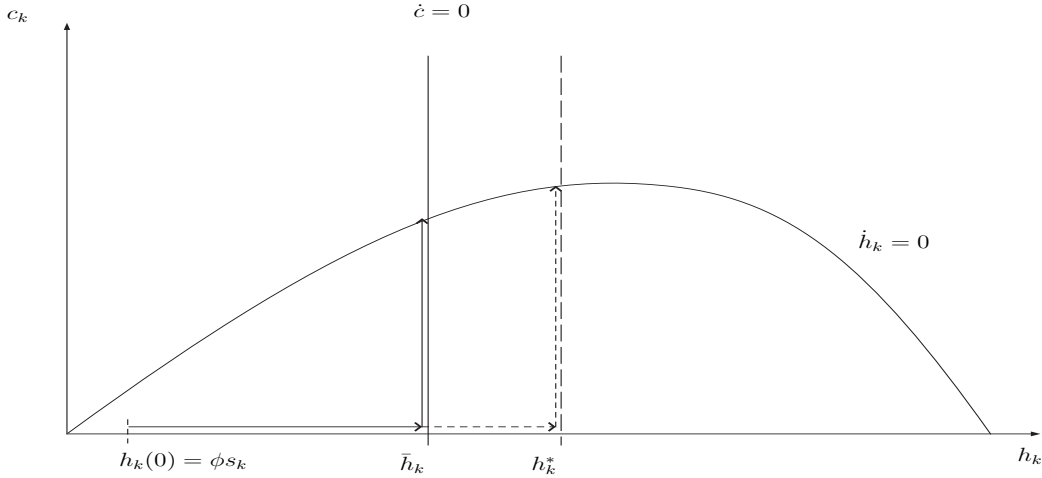
In each tier the equilibrium condition in the asset market will imply: $b_k = h_k$, where h_k is the per-capita level of capital. Also we have that¹²: $f'(h_k) = r_k + \delta$ and $f(h_k) - f'(h_k)h_k = w_k$.

The solution to the problem imply that consumption will be zero, and capital will accumulate in each tier until the point where:

$$(f'(h_k) - \delta) \left(1 - \frac{\tau}{s_k} \right) = \rho \quad (25)$$

At this point capital will be constant and consumption will jump to its steady state value.

Heuristically, we can draw the behavior of the variables under the solution of the problem as it is shown in the following figure:



In the figure, we can see that the solution implies that consumption will be zero, and all resources will be accumulated until condition (25) is satisfied. At that moment consumption will jump towards its steady state value. This is only an approximate description of the solution of the problem, because when agents are accumulating resources

¹²This follows because by assumption for the households, both savings and buying claims on capital holdings are perfect substitutes. If R is the rental rate of capital that firms pay, then it must be the case that: $(f'(h_k) - \delta)(1 - \tau/s_k) = R$, and hence $f'(h_k) - \delta = r_k$.

they also receive resources from the lower tier, and hence the problem has not an standard growth model solution.

In any given tier k , the process of accumulation of capital will be given by:

$$\dot{h}_k = \frac{\tau}{s_{k+1}}h_{k+1} + f(h_k) \left[1 - \frac{\tau}{s_k} \frac{f'(h_k)h_k}{f(h_k)} \right] - \delta \left(1 - \frac{\tau}{s_k} \right) h_k$$

To simplify the analysis, we further assume that the production function is given by: $f(h_k) = (h_k)^\alpha, 0 < \alpha < 1$. Then the solution of the problem will be given by the solution of the following system of differential equations:

$$\begin{aligned} \dot{h}_0 &= \frac{\tau}{s_1}h_1 + h_0^\alpha - \delta h_0 \\ \dot{h}_1 &= \frac{\tau}{s_2}h_2 + h_1^\alpha \left(1 - \alpha \frac{\tau}{s_1} \right) - \delta \left(1 - \frac{\tau}{s_1} \right) h_1 \\ \dot{h}_2 &= \frac{\tau}{s_3}h_3 + h_2^\alpha \left(1 - \alpha \frac{\tau}{s_2} \right) - \delta \left(1 - \frac{\tau}{s_2} \right) h_2 \\ \dot{h}_3 &= \frac{\tau}{s_4}h_4 + h_3^\alpha \left(1 - \alpha \frac{\tau}{s_3} \right) - \delta \left(1 - \frac{\tau}{s_3} \right) h_3 \\ &\cdot = \cdot \\ &\cdot = \cdot \\ &\cdot = \cdot \\ \dot{h}_K &= h_K^\alpha \left(1 - \alpha \frac{\tau}{s_K} \right) - \delta \left(1 - \frac{\tau}{s_K} \right) h_K \end{aligned}$$

This system is complicated to solve because of its recursive nature. We follow the approach of solving it numerically. The following values for the parameters were used: $\alpha = 0.4, \theta = 1, \phi = 0.37, \delta = 0.05, \rho = 0.02, \tau = 0.1$. The next figure shows the rate of growth of each tier, and the rate of growth of the economy as a whole.

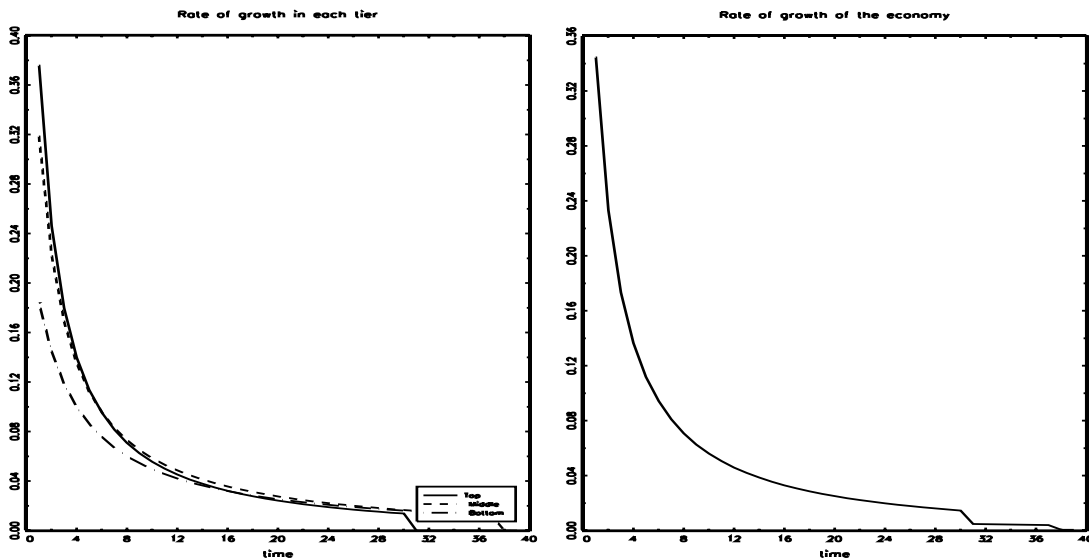


Figure 4. The curves were computed with the parameter values: $\theta = 1, \phi = 0.307, \alpha = 0.4, \delta = 0.05, \rho = 0.02, \tau = 0.1$.

Is it possible to see that, agents in the upper tiers display a higher rate of growth than agents in lower tiers. Notice that the economy converges towards the steady state with zero growth. The graphic shows the rate of growth during the transition.

It is possible to explore the consequences on the rate of growth of the economy of an increase in the level of expropriation τ in the next table we show the consequences of increase the rate of expropriation to $\tau' = 0.7$.

Avg. rate of growth	$\tau = 0.1$	$\tau' = 0.7$
tier 0	0.0408	0.0604
tier 1	0.0516	0.0648
tier 2	0.0499	0.0494
aggregate	0.0496	0.0562

It is interesting, and perhaps surprising that the rate of growth is increased in the top and middle tier, and in fact the overall rate of growth of the economy increases with the higher level of expropriation!. This result is present in spite of the fact that the wealthiest agents belong to the upper tiers, because one would expect that the higher the level of assets, the decreasing returns to capital would hamper growth. What is behind this result is that the amount of expropriation comes as a windfall gain for agents in the upper tiers, effectively increasing the amount that can be saved for the next period. The structure of the problem in this particular example, yields the result that the gain in growth offsets the detrimental effect of expropriation in the rate of growth.

The next graphic shows the behavior of the rate of growth in time for the three tiers and the economy in general.

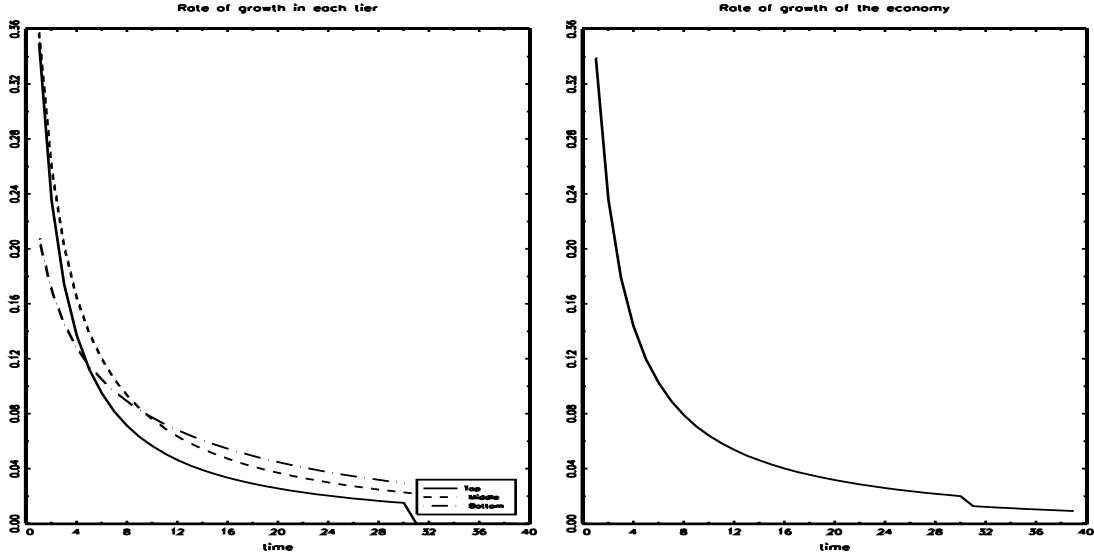


Figure 5. The curves were computed with the parameter values: $\theta = 1, \phi = 0.307, \alpha = 0.4, \delta = 0.05, \rho = 0.02, \tau' = 0.7$.

We can see how the rate of growth can change among the tiers in non monotone ways when the rate of expropriation changes. This shows the rather intricate mechanisms that underlie the solution of the system.

Another feature that we can analyze is the behavior of wealth. In the following graphic we plot the wealth of a single individual in tier 1 and tier 2, relative to the wealth of the individual in the top of the hierarchy.

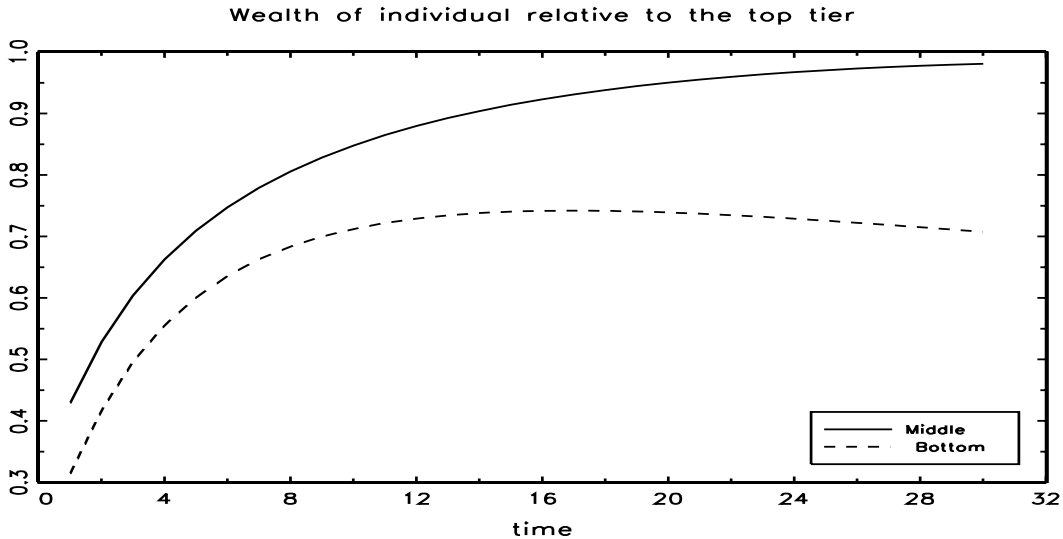


Figure 6. The curves were computed with the parameter values: $\theta = 1, \phi = 0.307, \alpha = 0.4, \delta = 0.05, \rho = 0.02, \tau = 0.1$.

The next graphic shows the same computation with a higher level of expropriation.

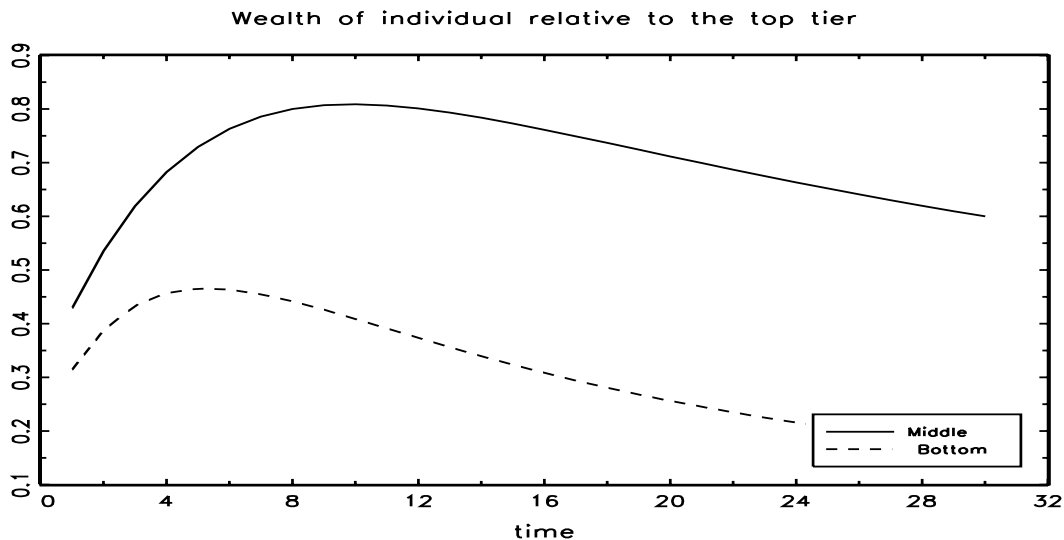


Figure 7. The curves were computed with the parameter values: $\theta = 1, \phi = 0.307, \alpha = 0.4, \delta = 0.05, \rho = 0.02, \tau' = 0.7$.

The results in the above figure are interesting. Recall that we have found that the overall rate of growth of the economy can increase towards the steady state when the rate of expropriation is higher. Figure 7 shows that the distribution of wealth in the transition to the steady state becomes more skewed through time. Initially it appears that the distribution of wealth is converging, but then towards the steady state the gap with respect to the agent in the top tier widens. Hence we could have an economy with an increasingly skewed distribution of wealth towards the steady state but with relatively high rates of growth.

It is not surprising that the level of capital in the steady state, and welfare is lower when expropriation is higher, as the following two tables show.

Table 6

Capital in S.S.	$\tau = 0.1$	$\tau = 0.7$
tier 0	18.3	18.3
tier 1	18.1	16.74
tier 2	18	16.11

Table 7

Welfare	$\tau = 0.1$	$\tau = 0.7$
tier 0	2.28	2.28
tier 1	2.27	2.24
tier 2	2.26	2.23

Conclusions

The formation of the hierarchy delivers results over different variables, relevant for the extractive economy of the colonial period that we analyze. We have found that the span of control is decreasing down the tiers. Also, the wages are decreasing while the number of workers is increasing down the tiers. This generates an skewed distribution of income, technological improvements induce more income and more welfare for all individuals in the hierarchy, but the distribution of income becomes more unequal. These are predictions that match, at least qualitatively some empirical features of actual extractive economies during the colonial period.

If the hierarchy has access to enough number of workers that don't decrease over time, then the hierarchy is stationary over time, and the resource is depleted linearly.

When the availability of workers decreases over time the hierarchy also shrinks. In finite time, the hierarchy would collapse to a single agent. The rate of extraction up to that point will be tied to the rate at which available workers decrease. A positive feature of the shrinking hierarchy is that the distribution of income becomes more egalitarian.

When we analyze the consequences of the formation of the hierarchy in the long run behavior of the economy, we find interesting results. These results depend on two assumptions. First, we assume that the hierarchy remains from the colonial period, not as a labor relationship but as a social structure, that will enable agents in the different tiers of the hierarchy to involve in expropriation activities. The second assumption is that expropriation only takes place from agents in the upper tiers to agents in the lower tiers, and expropriation takes place in a hierarchical fashion, an agent in the middle tier will be subject to expropriation from an upper tier, and also will expropriate resources of agents in the lower tier.

We found that expropriation can yield to a higher overall rate of growth of the economy. This result is related to the fact that expropriation helps to avoid diminishing returns to capital accumulation in the production process of agents who expropriate, even though expropriation will also have negative effects on growth from agents subject to it, we found that for some values of the parameters, the net effect can yield to a higher overall growth than without expropriation.

In the model, growth eventually ceases, which means that even though a higher growth of the economy can be achieved with expropriation, in terms of welfare the result is unambiguous, expropriation creates a distortion which causes little capital accumulation in the steady state and low levels of consumption.

Interestingly we also found that high growth rates toward the steady state can be achieved under expropriation, but with a high and increasing level of wealth inequality.

References

- Acemoglu D.; Johnson S. and Robinson J. "Reversal Of Fortune: Geography and Institutions in the Making of the Modern World Income Distribution". The Quarterly Journal of Economics, November (2002).
- Acemoglu D. "A Simple Model of Inefficient Institutions". May 2006, forthcoming, Scandinavian Journal of Economics.
- Keren M. and Levhari D. "The Optimal Span of Control in a Pure Hierarchy". Management Science, 25 (1979).
- Calvo, G. and Wellisz, S. "Supervision, Loss of Control and the Optimal Size of the Firm". (1978), Journal of Political Economy, 86,943-952.
- Calvo, G. and Wellisz, S. "Hierarchy, Ability and Income Distribution". (1979), Journal of Political Economy, 87,991-1010.
- Klein H. "Historia General de Bolivia". Editorial "Juventud" La Paz-Bolivia (1985).
- Olson M. "Power and Prosperity: Outgrowing Communists and Capitalists Dictatorships" (2000) New York, NY: Basic Books.
- North D. and Weingast B. "Constitutions and Commitment: Evolution of Institutions Governing Public Choice in Seventeenth Century England". Journal of Economic History, XLIX(1989).
- North D. and Thomas R. "The Rise of the Western World: A New Economic History". Cambridge, UK: Cambridge University Press (1973).
- Qian Y. "Incentives and Loss of Control in an Optimal Hierarchy". The Review of Economic Studies (1994).
- Randall L. "A Comparative Economic History of Latin America 1500-1914". (1977) Institute of Latin American Studies. Columbia University.
- A. Seierstad, K. Sydsæter. "Optimal Control Theory with Economic Applications (Advanced Textbooks in Economics)". North Holland; 3rd reprint 2002 edition (February 1, 1987).
- Williamson, O. "Hierarchical Control and Optimal Firm Size". (1967) Journal of Political Economy, 75, 123-138.