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Matching, Search, and Heterogeneity

Robert Shimer*   Lones Smith†

*Princeton, shimer@uchicago.edu

†University of Michigan, lones@umich.edu

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Abstract

This paper explores the efficiency of decentralized search behavior and matching patterns in a model with ex ante heterogeneity and a constant returns to scale search technology. We show that a linear tax or subsidy on search intensity decentralizes the social optimum. In the absence of the tax, high productivity agents are too willing to match, yet they search too little. Low productivity agents have the opposite behavior. As a result, the equilibrium is always inefficient in the absence of taxes, in contrast to known results on the efficiency of decentralized search models with homogeneous agents. We relate the inefficiencies to thick-market and congestion externalities.

KEYWORDS: search, matching, externality, congestion, thick-market

1   Introduction

Consider the following canonical economic situation: a pair of impatient agents meet and must decide whether to match. The benefit is that their payoff is higher when matched than when unmatched. This is weighed against an opportunity cost, as matching precludes further search for more suitable partners. Examples include the decision of a man and a woman to marry; the decision of a home buyer and home seller to sign a sales contract; and the decision of a worker and firm to enter into an employment relationship.

Economists have long recognized that matching involves an externality, since it alters the meeting opportunities of other agents. In general, two offsetting effects have been identified: when there are more agents looking for partners, it is easier to locate a potential partner (the thick market externality); and when there are more agents looking for partners, competing searchers more rapidly locate and match with one's potential partners (the congestion externality). Special circumstances have been identified in which the correct allocation of property rights ensures optimal search behavior by agents on both sides of the market (Mortensen 1982, Hosios 1990). However, these papers focus on optimal *search* behavior. They sidestep the *matching* dimension of the problem by assuming that agents are *ex ante* identical.[1]

This paper studies a model with both search and matching decisions. Heterogeneous agents choose the intensity of their search activities; and when they meet other agents, they must decide whether to match. Both decisions impose externalities. If agents were homogeneous in our model, these two externalities would cancel out and the decentralized equilibrium would be efficient. This is because the so-called 'Hosios (1990) condition', equating an agent's bargaining power to the elasticity of the matching function, is satisfied in our economy. But with heterogeneous agents, the decentralized equilibrium is never efficient. Thus we show that known results on the efficiency of decentralized search markets do not generalize to an environment with *ex ante* heterogeneity.

A direct comparison of behavior in the decentralized equilibrium and social

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[1]Some papers introduce matching decisions by allowing for *ex post* heterogeneity (Mortensen and Pissarides 1994, Smith 1995). This does not affect the efficiency properties of the standard search model. When agents are *ex ante* homogeneous, one agent's matching decision does not affect another agent's matching opportunities if the search technology has constant returns to scale, since the matching pool always looks the same. The effects we emphasize in this paper require *ex ante* heterogeneity.

optimum does not yield a precise characterization of the nature of this inefficiency, due to the complex interaction between agents. Instead, we characterize the inefficiency by describing a 'Pigouvian' tax scheme that decentralizes the social optimum.[2] A particularly illuminating scheme is a tax on search activity for the least productive agents, and a subsidy for the most. This has two effects: Agents who receive a subsidy increase their search intensity; and they are more reluctant to accept matches, since search is less costly. A search tax creates the opposite incentive, possibly leading some low-productivity agents to forgo search altogether. We conclude that in a decentralized equilibrium without taxes, the most productive agents do not search hard enough and match too frequently, while the opposite holds for the least productive agents.

We can interpret this finding in terms of the standard search externalities. The net effect of a search subsidy on the number of searching agents is ambiguous. It raises search intensity, so agents find potential partners faster, and it raises matching 'choosiness', so agents reject more partners. Nevertheless, the higher search intensity of agents who receive a subsidy ensures that they are better represented in terms of search activity (search intensity times the number of searchers), making it easier to meet them. Put differently, it is too hard to meet the most productive agents in a decentralized equilibrium without taxes. This is precisely the thick-market externality. An agent recognizes that if she does not search, either because she is matched or because she chooses not to search, she cannot meet new partners; however, she fails to internalize the corresponding inability of other agents to meet her. This externality is most important for high productivity agents, since that is whom others would like to meet.

On the other hand, low productivity agents search too much and match too infrequently. This is due to the congestion externality. An agent does not internalize that her search activity reduces contacts between other pairs of agents. All searchers exert this negative influence on search markets; however, the congestion externality is dominated by the thick market externality for high productivity agents, while the thick market externality is negligible for low productivity agents, since no one benefits much from meeting them. Only with homogeneous agents do the two externalities cancel out. As long as the decision of an agent to match or to search harder alters the distribution of

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[2]This is not a public finance exercise, and so we make little effort to discuss whether these taxes could be implemented in practice. However, we point out in Section 5 that the regressive nature of the search tax is inessential for decentralizing the social optimum.

meetings for other agents, the decentralized equilibrium will be inefficient.

A few other papers have shown that the decentralized equilibrium of a search economy is always inefficient when agents are *ex ante* heterogeneous. Davis (1995) and Acemoglu and Shimer (1999) look at holdup problems in search economies where agents must make *ex ante* investments, and find that underinvestment will generally prevail. Davis (1995) studies a one-shot game, which ensures that agents always accept a match. Thus the effect of one's search and matching behavior on the future opportunities of other agents, a central part of our analysis, is missing from Davis's. Instead, there is underinvestment in his model because in the bargaining game that occurs after matching, an agent is able to reap some of the returns from her partner's *ex ante* investment, a classic holdup problem. Acemoglu and Shimer (1999) show that the holdup problem depends on the random search assumption. In a 'directed search' environment, where agents can choose whom to meet, the equilibrium may still be efficient (see also Shi 2001). The key difference is that if an agent matches in an economy with directed search, it is no more difficult to contact other similar agents. One agent's matching behavior does not affect other agents' matching opportunities.

Section 2 describes our model of search and matching with heterogeneous agents. We solve for the decentralized equilibrium in Section 3 and the social optimum in Section 4. We compare these allocations using Pigouvian taxes in Section 5. We conclude by discussing the extent to which our results generalize to other search processes in Section 6.

2   Model

We develop a continuous time model in which a continuum of agents search for partners. We focus on steady states, and comment on the importance of this assumption at the end of this section.

All agents are risk-neutral, infinitely-lived, and discount the future at rate $r > 0$. They are identified by their immutable type $i \in \{1, \ldots, N\}$.[3] Type i agents enter the market at an exogenous flow rate $\ell_i > 0$ (mass per unit time), and exit the market for exogenous reasons at rate $\delta > 0$. Let u_i be the

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[3]This model does not distinguish between two 'sides' of the search market, e.g. men and women or workers and firms. Instead, any agents may meet and match. This is purely for expositional convenience. All of our results carry through to a two-sided environment, at the cost of carrying around twice as many variables and equations.

endogenous mass of type i agents in the market. This may differ from ℓ_i if different agents find matches at different rates. The effect of search intensity and matching decisions on the searching population u_i is central to our analysis.

Each agent i searches for partners with endogenous intensity ρ_i at cost $c(\rho_i)$, a continuous and convex function, with $c(0) = 0$ and $\lim_{\rho \to \infty} c(\rho) = \infty$. Such search frictions may be rationalized by the following story: Agents meet in crowded locations ('bars'). An agent's search intensity determines how frequently she visits a bar. When she visits one, she is guaranteed to meet one other agent, randomly selected from those at the bar. As a result, the rate at which she meets other agents depends only on her own search intensity. However, the potential partner is drawn randomly from the population, with likelihood proportional to his search intensity. Thus an agent who searches with intensity ρ meets a type j agent according to a Poisson process with arrival rate $\rho \rho_j u_j / \sum_{k=1}^{N} \rho_k u_k$. Note that this is an example of a *linear search technology*: The mass of matches is linearly proportional to the mass of searching agents.[4] We discuss an alternative specification of the search technology in Section 6.

When two agents i and j meet, they must decide whether to match or to remain unmatched, producing nothing, but continuing to search for partners. If they match, they bargain over the division of output $f_{ij} \geq 0$, strictly increasing in each of its arguments, produce the output if they reach an agreement, and exit the market. That higher types produce more is the only source of *ex ante* heterogeneity in this model. Output is divided according to the symmetric Nash bargaining solution, with the threat of continued search following disagreement. At the end of Section 5, we explain why our main result, the failure of the Hosios condition to generalize to an environment with *ex ante* heterogeneity, carries over to other bargaining environments.

In order to facilitate comparison of the decentralized equilibrium and the social optimum, we introduce a search tax: Each agent pays a tax $\tau_i \rho_i$ in proportion to her search intensity. In a decentralized equilibrium without taxes, $\tau_i = 0$ for all i. One might imagine that decentralizing the social optimum would require a more complex tax schedule. Section 5 shows that there is a

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[4]From a purely theoretical perspective, this is the most interesting case. Diamond (1982a) shows that if the search technology exhibits increasing returns to scale, there is no congestion externality and so optimality demands a search subsidy to internalize the thick-market externality even with homogeneous agents. One can show that this inefficiency carries over to models with heterogeneous agents. Moreover, from an empirical perspective, there is scant evidence for increasing returns to scale (Blanchard and Diamond 1989).

linear search tax that decentralizes the social optimum; the N different tax rates τ_i ensure that the N search intensities and the $N(N+1)/2$ matching decisions are optimal.

An agent i's strategy consists of her search intensity ρ_i and the probability $\tilde{\alpha}_{ij} \in [0,1]$ that she accepts a match with agent $j \in \{1, \ldots, N\}$ should they meet. The match (i,j) is consummated with chance $\alpha_{ij} = \alpha_{ji} = \tilde{\alpha}_{ij}\tilde{\alpha}_{ji}$. Note that existence of an equilibrium may require the use of mixed matching strategies, $\tilde{\alpha}_{ij} \in (0,1)$. An equilibrium (defined formally in the next section) is a profile of strategies, one for each type, that maximizes the utility of each agent, taking as given all other strategies. Moreover, we rule out the use of weakly dominated strategies, whereby a pair does not match solely because each expects the other to reject the match. This trivial coordination problem does not seem central to search theory, and so we impose $\tilde{\alpha}_{ij} = 1$ if there are gains from matching. This allows us to focus on the joint determination of α_{ij} by i and j. Further reference to $\tilde{\alpha}_{ij}$ is not needed, and so is suppressed.

Finally, we shall insist upon a steady-state. To understand this restriction, it will be convenient to define the expected value of any type-specific variable $\phi_1, \ldots, \phi_N$ drawn randomly from the population:

$$E_j\phi_j \equiv \frac{\sum_{j=1}^{N} \rho_j u_j \phi_j}{\sum_{j=1}^{N} \rho_j u_j},$$

where j is an arbitrary index. Then the measure of type i agents u_i is in steady state if

$$(1) \qquad\qquad (\rho_i E_j \alpha_{ij} + \delta)u_i = \ell_i.$$

Each type i agent meets another agent at rate ρ_i. The probability that a meeting results in a match is $E_j\alpha_{ij}$. In addition, an agent exogenously exits the market at rate δ. Multiplying this by the mass u_i of such agents gives the rate at which type i agents find new matches. In steady state, this must equal ℓ_i, the entry rate of new type i agents.

In a decentralized equilibrium, steady-state is a significant but reasonable restriction. In particular, if all agents use time-invariant strategies and the state variables u_i are constant as well, an individual's best response set must always include a time-invariant strategy. This logic does not carry over to the social optimum, however, since optimality requires that there be no gain to changing *all* strategies simultaneously. A hypothetical social planner can

always consider using time-varying strategies. This is not merely a technical concern. In Shimer and Smith (2000b), we develop a related model in which matches persist after they are created, as is standard in the labor search literature, and show that nontrivial limit cycles are optimal for a generic class of parameterizations. The social planner periodically destroys a subset of matches, discretely changing the composition of the searching population. Later, the planner permits those matches to form again, before going through another cycle of destruction and creation. In contrast, in this paper, exchanges are 'once-and-for-all', as is the case in the housing market. Once a buyer and seller trade, the exchange cannot be negated. This moderates the advantage of using time-varying policies, since it is impossible to generate discrete jumps in the composition of the searching population. Simulations suggest that socially optimal matching strategies are in fact stationary in this environment.

A second type of nonstationary strategy could arise in a social optimum, however. The planner could use time-varying search intensities to coordinate meetings. For example, he can prevent two types of agents from meeting by having them search with positive intensity at different times, possibly 'chattering' between short periods when each type searches. Allowing for such strategies would violate the spirit of the random matching literature, that it is impossible to decide whom you want to meet; they are trivially ruled out by our restriction to stationary strategies.

3   Equilibrium

We characterize an equilibrium via a recursive equation. Let w_i denote the expected value of future payoffs to an agent i who is optimally searching for a partner, defined implicitly by

$$(2) \qquad rw_i = \max_{\rho, \alpha_j} -\tau_i \rho - c(\rho) + \rho E_j \alpha_j (W_{i|j} - w_i) - \delta w_i,$$

where $W_{i|j}$ is the value to i of matching with an agent j. An agent optimally chooses her search effort ρ and matching behavior α_j. Her flow payoff comes from the search tax and search cost, $-\tau_i \rho - c(\rho)$. The probability of meeting a new potential partner is ρ. If she matches with a type j partner, she enjoys a capital gain $W_{i|j} - w_i$; otherwise there is no capital gain. Taking expectations over the population from which potential partners are drawn, the expected capital gain upon meeting an agent drawn at random is $E_j \alpha_j (W_{i|j} - w_i)$.

Finally, she is forced to exit the market with flow probability δ, in which case she loses her value w_i.

Nash bargaining imposes that that match surplus $W_{i|j} - w_i = W_{j|i} - w_j$ is shared equally, and that output is fully utilized, $W_{i|j} + W_{j|i} = f_{ij}$. Combining these equations yields

$$W_{i|j} - w_i = \frac{f_{ij} - w_i - w_j}{2}.$$

Substitute this into equation (2) to produce a recursive expression for w_i alone:

$$(3) \qquad (r + \delta)w_i = \max_{\rho, \alpha_j} -\tau_i \rho - c(\rho) + \rho E_j \alpha_j \frac{f_{ij} - w_i - w_j}{2}.$$

Using the preferences embedded in (3), we now impose the restrictions from optimizing behavior. First, matches with positive surplus are accepted, while negative surplus matches are declined:

$$(4) \qquad \alpha_{ij} = \left\{ \begin{array}{c} 1 \\ 0 \end{array} \right. \quad \text{if } f_{ij} - w_i - w_j \gtrless 0.$$

No restriction holds in the event of equality. Thus, the surplus created by a meeting between i and j is $\alpha_{ij}(f_{ij} - w_i - w_j) = \max\langle f_{ij} - w_i - w_j, 0\rangle$.

Second, search effort must maximize the right hand side of (3):

$$(5) \qquad \rho_i = \arg\max_{\rho \geq 0} -\tau_i \rho - c(\rho) + \rho E_j \max\left\langle \frac{f_{ij} - w_i - w_j}{2}, 0 \right\rangle.$$

Finally, the value function is defined recursively by

$$(6) \qquad (r + \delta)w_i = -\tau_i \rho_i - c(\rho_i) + \rho_i E_j \max\left\langle \frac{f_{ij} - w_i - w_j}{2}, 0 \right\rangle.$$

These expressions permit a precise definition of a search equilibrium:

Definition 1. *An* equilibrium *is a tuple* (w, ρ, α, u) *where: w solves* (6) *given* (ρ, u)*; ρ solves* (5) *given* (w, u)*; α is optimal given w, i.e. it solves* (4)*; and u solves the steady state equation* (1) *given* (ρ, α).

Shimer and Smith (2000a) prove the existence of an equilibrium in a closely related model. That proof can be extended to this environment using

Kakutani's fixed point theorem, since the values w contain all the information in the model. In particular, we can construct a non-empty, convex-valued, and upper-hemicontinuous mapping from the compact space of values $[0, f_{NN}/2]^N$ into itself.

4 Optimum

A social optimum maximizes the present value of output net of search costs in the economy, subject to the same search frictions as face the decentralized economy, the usual criterion in the search literature. The social objective can be expressed in terms of the measures u_i:

$$(7) \qquad \max \int_0^\infty e^{-rt} \sum_{i=1}^N \left(\tfrac{1}{2}\rho_i(t)u_i(t)E_j\alpha_{ij}(t)f_{ij} - c(\rho_i(t))u_i(t) \right) dt.$$

The first term is the gross production in the economy (being careful not to double-count), while the second term subtracts the search costs. The social planner's problem is to choose time paths of α and ρ so as to maximize (7) subject to a sequence of dynamic constraints

$$(8) \qquad \dot{u}_i(t) = \ell_i - \left(\rho_i(t)E_j\alpha_{ij}(t) + \delta \right)u_i(t).$$

The increase in the measure of type i agents is the difference between the entry rate of new agents and the matching rate of the stock of searchers.

To solve the social planner's problem, write down the current-valued Hamiltonian with multiplier v_i on the rate of change of the measure u_i, and suppress the time dependence of variables:

$$H = \sum_{i=1}^N \left(\tfrac{1}{2}\rho_i u_i E_j\alpha_{ij}f_{ij} - c(\rho_i)u_i + v_i\ell_i - v_i\left(\rho_i E_j\alpha_{ij} + \delta\right)u_i \right).$$

Expand the expectations operators and group terms:

$$H = \frac{\sum_{i=1}^N \sum_{j=1}^N \rho_i\rho_j u_i u_j \alpha_{ij}(f_{ij} - v_i - v_j)}{2\sum_{k=1}^N \rho_k u_k} - \sum_{i=1}^N c(\rho_i)u_i + v_i(\ell_i - \delta u_i).$$

There are three types of necessary first order conditions. First is the steady

state costate equation, that the discount rate r times the multiplier v_i equals the partial derivative of the Hamiltonian with respect to u_i:

$$(9) \qquad rv_i = -c(\rho_i) + \rho_i E_k \left(\alpha_{ik} \sigma_{ik} - E_l \alpha_{kl} \sigma_{kl}/2 \right) - \delta v_i,$$

where we introduce the notation $\sigma_{ik} \equiv f_{ik} - v_i - v_k$ for the *surplus* of an (i, k) match.[5] Observe that if agents are homogeneous, $N = 1$, equation (9) reduces to $(r + \delta)v = -c(\rho) + \rho\alpha(f/2 - v)$. The value of an agent is the chance that she meets someone and has the opportunity to produce her half of f, at the cost of then exiting the market v, minus the search costs. More generally, the social value v_i consists of three terms. First, search by agent i is costly. Second, by searching, she may meet a type k agent, drawn randomly from the population, and create a new match with surplus σ_{ik}. The fact that we count the full surplus in computing this term represents the thick market externality. In a decentralized equilibrium, an individual only internalizes half the surplus (equation 6). The thick-market effect is increasing in the expected surplus created by agent i. The third term represents the congestion externality, since i's search activity makes it more difficult for type k and l agents to meet. The social cost of congestion depends on k and l's values from a match.

The next first order condition is the optimality condition for matching. The linearity of the problem ensures that matching probabilities α will typically be driven to the boundaries:

$$(10) \qquad \alpha_{ij} = \left\{ \begin{array}{l} 1 \\ 0 \end{array} \right. \quad \text{if } \sigma_{ij} \equiv f_{ij} - v_i - v_j \gtrless 0.$$

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[5]Consider the costate equation $rv_1 - \dot{v}_1 = \partial H/\partial u_1$. Since we are looking at steady states, the time derivative of the costate variable $\dot{v}_1$ is zero. Also,

$$\frac{\partial H}{\partial u_1} = \frac{\sum_{j=1}^N \rho_1 \rho_j u_j \alpha_{1j}(f_{1j} - v_1 - v_j)}{2\sum_{k=1}^N \rho_k u_k} + \frac{\sum_{i=1}^N \rho_i \rho_1 u_i \alpha_{i1}(f_{i1} - v_i - v_1)}{2\sum_{k=1}^N \rho_k u_k}$$
$$- \rho_1 \frac{\sum_{i=1}^N \sum_{j=1}^N \rho_i \rho_j u_i u_j \alpha_{ij}(f_{ij} - v_i - v_j)}{2\left(\sum_{k=1}^N \rho_k u_k\right)^2} - c(\rho_1) - \delta v_i.$$

The first two terms are each equal to $\rho_1 E_k \alpha_{1k}(f_{1k} - v_1 - v_k)/2$, while the third term is equal to $\rho_1 E_k E_l \alpha_{kl}(f_{kl} - v_k - v_l)/2$. Rewriting this for a generic agent i yields equation (9).

Plugging this into (9) yields an implicit equation for the shadow price v:

$$(11) \qquad (r+\delta)v_i = -c(\rho_i) + \rho_i E_k\big(\max\langle\sigma_{ik},0\rangle - E_l\max\langle\sigma_{kl}/2,0\rangle\big).$$

The final optimality condition is that search intensity is optimally chosen:

$$(12) \qquad \rho_i = \arg\max_{\rho\geq0} -c(\rho) + \rho E_k\big(\max\langle\sigma_{ik},0\rangle - E_l\max\langle\sigma_{kl}/2,0\rangle\big).$$

This leads to a concise characterization of a social optimum, symmetric to the definition of a search equilibrium:

Lemma 1. *If a tuple (v,ρ,α,u) is a* social optimum*: v solves* (11) *given (ρ,u); ρ solves* (12) *given (v,u); α is optimal given v, i.e. it solves* (10)*; and u solves the steady state equation* (1) *given (ρ,α).*

Recall that the production function f is strictly increasing in its arguments, the only source of heterogeneity in the model. This implies that higher types have a higher social value v:

Lemma 2. *In a social optimum (v,ρ,α,u), the value v is nondecreasing.*

Proof. Take $i>j$ and suppose $E_k\max\langle\sigma_{jk},0\rangle > E_k\max\langle\sigma_{ik},0\rangle$. Then

$$\begin{aligned}(r+\delta)v_j &\geq -c(\rho_i) + \rho_i E_k\big(\max\langle\sigma_{jk},0\rangle - E_l\max\langle\sigma_{kl},0\rangle/2\big)\\ &\geq -c(\rho_i) + \rho_i E_k\big(\max\langle\sigma_{ik},0\rangle - E_l\max\langle\sigma_{kl},0\rangle/2\big) = (r+\delta)v_i\end{aligned}$$

where the first inequality uses (12), i.e. ρ_j maximizes the right side of (11) for type j; the second uses the assumption that $E_k\max\langle\sigma_{jk},0\rangle > E_k\max\langle\sigma_{ik},0\rangle$; and the final equality uses (11). But $v_j \geq v_i$ implies $\sigma_{jk} \equiv f_{jk} - v_j - v_k < f_{ik} - v_i - v_k \equiv \sigma_{ik}$ for all k, hence $E_k\max\langle\sigma_{jk},0\rangle \leq E_k\max\langle\sigma_{ik},0\rangle$, a contradiction. Replicating this logic with the valid inequality $E_k\max\langle\sigma_{ik},0\rangle \geq E_k\max\langle\sigma_{jk},0\rangle$, we establish that $v_i \geq v_j$. □

Note that it is not possible to make useful statements about α and u without additional strong assumptions. For example, Shimer and Smith (2000a) examine conditions under there is positively assortative matching, i.e. replacing any pair of matches with a match between the higher pair and a match between the lower pair raises output. This requires making a number of additional supermodularity assumptions on the production function f.

5 Search Externalities

We are now in a position to characterize the search externalities in this economy. Our characterization relies on the following Pigouvian tax:

Proposition 1. *Let there be a type-specific search tax*

(13) $$\tau_i^* \equiv E_k E_l \max \langle \sigma_{kl}/2, 0 \rangle - E_k \max \langle \sigma_{ik}/2, 0 \rangle ,$$

where expectations are taken with respect to the search intensities ρ_i and populations u_i prevailing at the social optimum. Then there is a decentralized equilibrium that is socially optimal. The optimal tax τ_i^* is nonincreasing in i.

Proof. The optimality of the tax is immediately confirmed by substituting for τ in (5) and (6), imposing $w_i = v_i$, and comparing with (12) and (11). To show τ^* is nondecreasing, recall that we in fact proved in Lemma 2 that for all $i > j$, $E_k \max\langle \sigma_{ik}, 0 \rangle \geq E_k \max\langle \sigma_{jk}, 0 \rangle$. The result then follows immediately from the definition of τ^*. □

We do not claim that this search tax can be implemented in practice. Instead, it is informative about the nature of search externalities.

With only one agent type, $N = 1$, the expectations operator collapses, and equation (13) yields $\tau_i^* = 0$. This is just a restatement of the Hosios (1990) condition. Although each agent only captures half the surplus from a match — a hallmark of underinvestment in the search and matching literature — this disincentive to search internalizes the fact that if one agent searches harder, it becomes more difficult for other agents to find their partners. The congestion and thick-market externalities cancel.

In contrast, if the type distribution is not degenerate, an equilibrium is not socially optimal without taxes. The Hosios condition does not generalize to this environment. Since $E_k \max\langle \sigma_{ik}, 0 \rangle$ is nondecreasing in i, it is less than its expected value $E_k E_l \max\langle \sigma_{kl}, 0 \rangle$ for small i and greater for large i. More precisely:

Proposition 2. *If $N \geq 2$, then $\tau_N^* < \tau_1^*$ and $0 \in [\tau_N^*, \tau_1^*]$. In particular, an equilibrium is not socially optimal in the absence of taxes.*

Proof. We proved in Lemma 2 that $E_k \max\langle \sigma_{Nk}, 0 \rangle \geq E_k \max\langle \sigma_{1k}, 0 \rangle$. Suppose, for a contradiction, that this holds as an equality. Denote the expected value by x. Equation (11) reduces to $(r + \delta)v_i = -c(\rho_i) + \rho_i x/2$, $i = 1, N$. This then implies that $v_1 = v_N$.

Now suppose $\sigma_{1k} = 0$ for all k. Then (12) implies $\rho_1 = 0$, yielding maximal value $v_1 = 0$ by (11). Since $v_N = v_1$ as well, $\sigma_{1N} = f_{1N} - v_1 - v_N = f_{1N}$. This is strictly positive, since $f_{11} \geq 0$ and f is strictly increasing. This contradicts our assumption $\sigma_{1k} = 0$ for all k.

So take any j with $\sigma_{1j} > 0$. Since $f_{Nj} > f_{1j}$ and $v_N = v_1$, $\sigma_{Nj} > \sigma_{1j} > 0$. As this is true for all j with $\sigma_{1j} > 0$, we obtain $E_k \max\langle \sigma_{Nk}, 0 \rangle > E_k \max\langle \sigma_{1k}, 0 \rangle$, contrary to our original supposition that the two terms are equal. This proves $E_k \max\langle \sigma_{Nk}, 0 \rangle > E_k \max\langle \sigma_{1k}, 0 \rangle$.

Apply this to the definition of τ^* to obtain $\tau_N^* < \tau_1^*$. At the same time, (13) implies $E_i \tau_i^* = 0$, which precludes both extreme values negative or both positive: i.e. $0 \in [\tau_N^*, \tau_1^*]$. Note, however, that if $N = 2$ and $\rho_1 = 0$, the expectations operator degenerates yielding $\tau_2^* = E_i \tau_i^* = 0$.   □

Thus the most productive agents receive a search subsidy, while the least productive agents pay a search tax.

The nondegenerate search taxes shed light into the nature of externalities. Take an agent i who should receive a subsidy, i.e. $\tau_i^* < 0$. Such a transfer has two effects on her behavior. Most obviously, she searches harder than she would without the subsidy. But in addition, the subsidy makes her less willing to match. One way to see this is to temporarily assume that she does not change her search intensity in response to the subsidy. Then it simply raises the value of being searching, eliminating the surplus in previously marginal matches. Conversely, a search tax $\tau_i^* > 0$ reduces search intensity and makes agents more willing to match.

Put differently, in a decentralized equilibrium without search subsidies, the most productive agents do not search hard enough and are too willing to accept a match when they meet another agent. This is the effect of the thick-market externality: When a productive agent searches harder or rejects a match, she confers a benefit on other agents who have an opportunity to meet her. A search subsidy internalizes this effect.

The opposite inefficiency characterizes the least productive agents, who impose a congestion externality on others. When an unproductive agent searches harder or rejects a match, the benefits of search are diminished for other agents, who must waste some of their time meeting her. A search tax forces her to internalize this effect.

We have focused on search subsidies and taxes because these provide a particularly compact representation of the search externalities. But there are typically other taxes that achieve the same result, and that do not necessarily

have the regressive structure described here. Consider the following example: $N = 2$, $f_{22} = 1$, $f_{11} = \varepsilon$, and $f_{12} = \varepsilon(1+\varepsilon)$, with a convex search cost function c satisfying $c(0) = c'(0) = 0$. If ε is sufficiently small, it is not optimal for type 1 agents to search at all, $\rho_1 = 0$, since any search activity by type 1 agents makes it much more difficult for type 2 agents to meet each other.[6] Plugging $\rho_1 = 0$ into Proposition 1 yields a simple formula for the optimal search tax, $\tau_1^* = \sigma_{22}/2$ and $\tau_2^* = 0$. It is only necessary to tax search by type 1 agents. A 'welfare' system, with payments for inactivity, can achieve the same result without this regressive character, by giving low productivity agents a payment in return for not searching. Moreover, such a system can screen agents, rather than relying on a social planner to set type-contingent taxes. High productivity agents choose to forgo the welfare payment and take the higher return on market activity.[7]

Mortensen (1982) and Hosios (1990) showed that the social optimum can be decentralized through an appropriate allocation of property rights in models with *ex ante* homogeneity. Our Proposition 2 reaches a very different conclusion when agents are heterogeneous. However, one still might think that efficiency can be restored through some clever generalization of the bargaining

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[6]Suppose $\rho_1 = 0$, so the expectations operator degenerates: $E_k \max\langle \sigma_{ik}, 0\rangle = \max\langle \sigma_{i2}, 0\rangle$ and $E_k E_l \max\langle \sigma_{kl}/2, 0\rangle = \sigma_{22}$. Then (11) and (12) reduce to

$$(r + \delta)v_2 = \max_{\rho \geq 0} -c(\rho) + \rho(f_{22} - 2v_2)/2,$$

implicitly defining v_2 and ρ_2. Note in particular that v_2 and $\sigma_{22} \equiv f_{22} - 2v_2$ are strictly positive and independent of f_{11} and f_{12}. This ensures that the unique solution to

$$(r + \delta)v_1 = \max_{\rho \geq 0} -c(\rho) + \rho(\max\langle \sigma_{12}, 0\rangle - \sigma_{22}/2)$$

is indeed $\rho_1 = v_1 = 0$ for $f_{12} < \frac{1}{2}$: either $\sigma_{12} < 0$, in which case the last term is maximized by setting $\rho_1 = 0$; or $\sigma_{12} - \sigma_{22}/2 = f_{12} - v_1 - \frac{1}{2} < 0$, where the inequality follows because the possibility of setting $\rho_1 = 0$ ensures $v_1 \geq 0$. Formally, this proves that setting $\rho_1 = 0$ is consistent with the conditions for a steady state optimum, but does not prove that it is the optimum. For it to be one, the steady state measure of type 1 agents in the population, ℓ_i/δ, must not be too large relative to their productivity, which is of order ε.

[7]This example also illustrates that our notion of optimality is stronger than the Pareto criterion. In a decentralized equilibrium, type 1 agents search and match with each other, yielding a positive value w_1. This is inefficient if they are sufficiently unproductive. Output is higher if they don't search at all, although it is type 2 agents who enjoy all the gains. The welfare scheme gives an idea of one might redistribute output to obtain a Pareto improvement.

solution — for example, an asymmetric Nash bargaining solution with i receiving a surplus share $\beta_{ij} = 1 - \beta_{ji}$ when bargaining with j (so $\beta_{ii} = 1/2$). The example in the previous paragraph shows why that cannot be true in general. Even though it is optimal for type 1 agents not to search, they will prefer to search for each other, albeit at a low intensity, as long as the productivity in a $(1,1)$ match is positive and the marginal cost of the first unit of search is zero. Some additional manipulation, such as a search tax, is required to keep them from searching at all, as the social planner finds desirable.

6 Discussion

An important question is to what extent our results depend on our specification of the search process. While our model of anonymous search seems natural to us, the literature offers at least one alternative linear search technology (Diamond 1982b, Mortensen 1982). All agents register with a central matching agency. An agent's search intensity determines how frequently she calls the agency. When she calls, she is given information on how to contact one other agent, selected randomly from the population registered at the agency. Conversely, another agent may search and be given her contact information. Thus this model distinguishes meetings between two agents according to who initiates contact. An agent with search intensity ρ contacts a type j agent at rate $\rho u_j / \sum_{k=1}^{N} u_k$. Regardless of her own search activity, the agent is also contacted by a type j agent at rate $\rho_j u_j / \sum_{k=1}^{N} u_k$.

Mortensen (1982) shows that with this search technology and homogeneous agents, equilibrium search behavior is optimal if an agent keeps all the surplus from the matches that she initiates. An agent's search intensity does not affect the ability of other agents to contact her, and so she should not receive any of the benefits from those meetings. However, she incurs the full cost of initiating contact with other agents, and so must get the full benefit if she is to search efficiently.

This finding does not carry over to an environment with heterogeneous agents, however.[8] When an agent decides to match, she alters the distribution of searching agents. This changes the returns to search for other agents, an external effect that is not internalized by Mortensen's (1982) proposed allocation of property rights. In particular, when a high productivity agent matches, the

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[8]The formal proof of these results has a similar structure to the analysis in the paper.

searching pool becomes less attractive. Thus from a social perspective, these agents are generally too willing to match, and a search subsidy is desirable. Conversely, low productivity agents are too reluctant to match. This is consistent with the findings from our model.

We conclude that a necessary condition for efficiency of a search equilibrium with heterogeneous agents, is that one agent's behavior must not alter the opportunities available to others. Since a fundamental characteristic of these models is that matching decisions affect search opportunities by altering the distribution of searching agents, this is a stringent requirement. In general, high productivity agents are too willing to match, because they ignore the benefit they impart on the searching population. Similarly, low productivity agents fail to internalize the congestion that they impose on search markets. These external effects can only be appreciated in a model with heterogeneous agents and a nontrivial matching decision.

7   Colophon

Contact Information:

Robert Shimer
Department of Economics
Princeton University
204 Fisher Hall
Princeton, NJ 08544, USA
shimer@princeton.edu
www.princeton.edu/~shimer/

Lones Smith
Department of Economics
University of Michigan
611 Tappan Street
Ann Arbor, MI 48109, USA
lones@umich.edu
www-personal.umich.edu/~lones/

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