

Competitive Search Equilibrium with Asymmetric Information*

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Abstract

This paper explores the behavior of a model economy with search frictions and bilateral asymmetric information. Firms commit to employment contracts in an effort to attract workers. When a worker and firm meet, the worker must decide whether to supply effort to the employment relationship, thereby affecting the distribution of a match-specific productivity shock. Only the worker observes her effort choice and only the firm observes the realization of the shock. We prove that under a standard regularity condition, employment contracts take a simple form: the firm pays a wage w to a worker who is hired and a severance payment b to a worker who is dismissed. The firm hires the worker if her productivity exceeds $w - b$, while the gap between the wage and the severance payment is high enough to ensure that the worker supplies effort to the employment relationship. Asymmetric information unambiguously reduces the vacancy-unemployment ratio and reduces the probability that a meeting results in a match. These results are consistent with earlier findings in the implicit contracts literature, although the precise mechanism is somewhat different. For example, there is no risk-sharing motive in this framework; and mobility restrictions are explicitly caused by a primitive search friction.

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1 Introduction

Asymmetric information is pervasive in employment relationships, a fact that may be important for understanding how the labor market functions. In a series of papers in a special issue of the *Quarterly Journal of Economics*, Azariadis (1983), Chari (1983), Green and Kahn (1983), and Grossman and Hart (1983) developed the earliest models of (implicit) labor contracts in environments with asymmetric information. These authors demonstrated that if workers and firms agree to labor contracts that address the asymmetric information problem in a bilaterally efficient manner, this will generally induce a wedge between the marginal rate of substitution and the marginal rate of transformation of consumption and leisure. In particular, a labor market with long term wage contracts behaves differently than a spot labor market.

In recent years, there has been little work on implicit contract models because of three perceived shortcomings. First, Chari (1983), Cooper (1983), and Green and Kahn (1983) proved that employment is inefficiently low in equilibrium if and only if leisure is an inferior good. In the more natural case where leisure is a normal good, employment is too high.¹ Given that the goal of the implicit contracts literature was to explain inefficiently low levels of employment, this is sometimes seen as a major shortcoming. Second, under similar assumptions unemployed workers are better off than employed workers. For example, suppose workers have preferences that are additively separable in consumption c and leisure ℓ , $u(c) + v(\ell)$, with u and v increasing and u strictly concave. In a symmetric information environment, like the original work by Azariadis (1975) and Baily (1974), employment contracts smooth the marginal utility of consumption, so consumption is independent of leisure; the undesirable result follows immediately. Third, the implicit contracts literature has focused primarily on the relationship between a single worker and firm. In principle, the labor market disciplined the expected utility that a worker derived from the employment relationship, but in practice this was treated as exogenous to the model and constant over the business cycle. Moreover, all of these papers presumed that an unmodelled barrier to mobility prevented the worker from taking another job when doing so would be optimal.² In other words, key features of the labor market were not explicitly modelled.

¹Nosal, Rogerson and Wright (1992) show that in models with home production, employment may be too low even if leisure is a normal good.

²Chari (1983) writes “Once workers and firms have signed their private contracts and the state of nature has been realized, workers cannot move to another firm. This assumption (which is uncomfortably close to involuntary servitude) can to some extent be justified by costs of moving from one location to another.” (p. 110).

This paper attempts to address these shortcomings by embedding a bilateral asymmetric information problem into the competitive search model (Montgomery 1991, Peters 1991, Moen 1997, Shimer 1996, Mortensen and Wright 2002). The competitive search model provides a natural framework for addressing the third issue because it formalizes the barriers to mobility and endogenizes workers' outside option.³ We introduce bilateral asymmetric information through an assumption that firms cannot observe a worker's effort level, in addition to the standard assumption that workers cannot observe a firm's productivity realization. This new source of asymmetric information addresses the second issue, because if firms want to induce workers to supply effort, they must reward workers when the firm is successful, i.e. an employed worker must be made better off than her unemployed peers. At the same time, in an environment with bilateral asymmetric information we can dispense with the usual assumption that workers are more risk averse than firms.⁴ Instead, we show that optimal contracts provide smooth consumption even in the absence of risk-sharing motives. Finally, we prove that in a competitive search equilibrium with asymmetric information, there are too many unemployed workers and too few vacancies in equilibrium, relative to an environment with symmetric or one-sided asymmetric information, thereby addressing the first issue.

The setup of the competitive search model with asymmetric information is as follows: Risk-neutral firms offer employment contracts in an effort to attract workers who have preferences over consumption c and leisure ℓ given by $c + z\ell$ for some $z > 0$. Workers recognize that more lucrative contracts will attract more applicants—the vacancy-unemployment (v-u) ratio will be lower, and so will be harder to get. Firms realize that by being more generous in their wage payments, they are more likely to attract a worker, because the v-u ratio is lower. Once a worker and firm meet, the worker must decide whether to supply effort to the employment relationship, thereby altering the distribution of a match-specific productivity shock x . This leads to two asymmetric information problems: only the worker observes her effort supply, while only the firm observes the match-specific shock. Depending on the incentives generated by the employment contract and on any information that the firm chooses to reveal to the worker, the employment contract then determines whether the worker is hired and the compensation (wage or severance payment) that she receives.

³To my knowledge, there has been little previous effort at using search theory to provide a formal model of barriers to mobility. A notable exception is the effort by Burdett and Mortensen (1980) to use search theory to model labor supply and implicit contract theory to model labor demand.

⁴It is conceptually straightforward to introduce risk-aversion into the competitive search model (see Acemoglu and Shimer 1999), including the model in this paper, but doing so imposes a considerable cost in terms of tractability.

There is a critical contracting problem in this environment: how should a firm best structure an employment contract so as to exploit all of the possible gains from trade given the constraints imposed by asymmetric information? An employment contract can be thought of as a direct revelation mechanism. The firm reports the realization of the match-specific shock x , hires the worker with probability $e(x)$, and pays the worker $c(x)$. The employment probability and compensation must satisfy two restrictions: first, it must induce the worker to supply the desired effort to the employment relationship; and second, it must induce the firm to truthfully reveal the realization of the shock.

Under a plausible condition on the distribution F of the match-specific shock, the best contract is extremely simple: the firm promises to hire the worker and pay her w if x exceeds some threshold x^* , and otherwise does not hire the worker but gives her a positive severance payment b . Incentive compatibility requires that the threshold satisfies $x^* = w - b$, so a firm is indifferent between paying the worker a high wage w and getting output x^* or paying a low severance payment b and getting nothing produced. It also requires that the threshold x^* exceeds the consumption value of the worker's leisure time z , or the worker will be unwilling to supply effort to the employment relationship. This implies that employed workers are strictly better off than unemployed workers, and unemployed workers who find a job but are not hired are better off than unemployed workers who fail to even meet an employer.

More generally, the best contract can have a slightly more complicated characterization: the contract specifies a positive severance payment b , two wages $w_2 > w_1 > b$, and a probability $\pi \in (0, 1)$. When a worker and firm meet, the firm randomly (but publicly) selects a wage, choosing w_1 with probability π and w_2 otherwise, perhaps by forcing the worker to take an uninformative test. It then observes both the wage and the worker's productivity and decides whether to hire her at the selected wage or not to hire her and pay her the severance payment. Incentive compatibility ensures that the firm hires the worker at wage w_i if productivity exceeds $w_i - b$. There is no other contract, no matter how complex, that does better in a competitive search equilibrium with asymmetric information.

The model is sufficiently tractable that we can perform simple comparative statics exercises. We prove first that the presence of bilateral asymmetric information unambiguously reduces vacancy creation and raises the unemployment rate. We then show that an increase in the cost of advertising a vacancy, an increase in workers' cost of providing effort, an increase in the value of leisure, or a worsening of the productivity distribution in the sense of first order stochastic dominance reduces the v-u ratio and therefore raises the unemployment rate.

This paper proceeds as follows. The next section reviews the existing literature on search models with asymmetric information. Most have assumed a particular bargaining procedure, rather than the contracting approaches that emerges naturally in a competitive search model. Section 3 describes the competitive search model with asymmetric information and shows that an equilibrium can be described as the solution to a constrained optimization problem. Section 4 uses a standard characterization of incentive compatible contracts to simplify the constrained optimization problem. In particular, we break the problem into two parts that may be solved sequentially: first, the choice of an employment probability function that is nondecreasing in the productivity realization and solves a simple optimization problem; and second the choice of the v-u ratio. Section 5 solves a simplified version of the model when workers do not face a moral hazard problem. In particular, we show that firms' inability to convey their private information to workers is costless in this simpler environment. On the other hand, the optimal contracts that emerge from the simplified model do not give workers an incentive to provide effort, so the moral hazard problem is costly in the full model. Section 6 characterizes the equilibrium contracts in the competitive search model with asymmetric information, proving that an employment contract never involves more than two wages. Section 7 performs the simple comparative statics exercises, proving in particular that asymmetric information unambiguously reduces the v-u ratio. Finally, Section 8 concludes.

2 Related Literature

To my knowledge, only Faig and Jerez (2004) have previously examined a competitive search model with asymmetric information. That paper is much more ambitious than this one, in that the authors build a quantifiable theory of commerce, while we focus on the simplest possible model of search and asymmetric information. This allows me to generalize Faig and Jerez's (2004) findings along some dimensions. In particular, those authors assume that the distribution of productivity F takes a particular functional form (uniform on $[0, 1]$), while we allow for a general distribution; those authors restrict attention to mechanisms that use pure strategies, while we allow for public randomization; and those authors assume that either party may always walk away from a contracting relationship, while we allow for binding contracts.⁵ The last distinction is critical because it forces me to introduce two-sided asymmetric information in order to obtain non-trivial results. Faig and Jerez (2004) look at

⁵Of course a binding contract could effectively permit parties to walk away in some states of the world.

a model with one-sided asymmetric information.⁶

Most previous authors who have examined asymmetric information in search models have presumed that wages are determined by a particular bargaining procedure. For example, Trejos (1999) examines a monetary model of exchange with asymmetric information. The structure of the bargaining game follows Rubinstein (1982): nature randomly selects one party to make an offer to the other; if the offer is rejected, there is a short delay before the random selection procedure is repeated. Trejos (1999) shows that the equilibrium of this game is equivalent to the axiomatic Nash bargaining solution in a model with symmetric information. In particular, all the bilateral gains from trade are exploited. Berentsen and Rocheteau (2003) extend Trejos by allowing for divisible money and divisible goods; however, they maintain his focus on the axiomatic Nash bargaining solution of the related symmetric information model.

The labor economics literature has focused on inefficiencies that may arise in the presence of asymmetric information. Acemoglu (1995) assumes that (uninformed) workers make wage offers to (informed) firms. If an offer is rejected, the worker may make another offer in the following period. This introduces bilaterally inefficient delay in equilibrium, with a worker initially demanding a high wage before gradually reducing her wage demand upon deducing that she is in a low productivity matches. Kennan (2004) and Tawara (2004) allow nature to randomly select one of the parties to make a take-it-or-leave-it offer to the other party. If the worker gets to make the offer, she behaves as in Acemoglu (1995), while if the firm makes the offer, it pays the worker her reservation wage assuming that output exceeds this low threshold. This superficially resembles the optimal contract, since the worker is always employed at sufficiently high productivity realizations, is sometimes employed at intermediate productivity realizations (when the firm makes the offer), and is never employed at low productivity realizations, below the workers's reservation wage.⁷ But despite this, the two thresholds are never optimal. For example, an optimal contract would dictate that a worker producing slightly more than her reservation wage should never be employed, while

⁶In my model with one-sided asymmetric information and no other constraints, firms would optimally provide risk-neutral workers with constant utility, exactly compensating workers for the value of their time when employed. But this cannot be an equilibrium in Faig and Jerez's (2004) world because a firm that chooses not to hire a worker would walk away from the contract, paying the worker nothing. A previous version of this paper explored a model that was closer to Faig and Jerez's (2004): rather than introduce a moral hazard problem, we imposed an additional constraint that $xe(x) - c(x) \geq 0$ for all x , as would be the case if firms always have the option to dismiss workers at no cost.

⁷Kennan (2004) focuses on parameter values such that all meetings result in matches. Tawara (2004) considers this more general case.

the particular bargaining game implies that the worker would be employed if the firm makes the wage offer. This implies that a worker and firm could obtain a Pareto improvement by agreeing to an employment contract of the sort described here before the firm observes the match specific productivity realization.

My finding that there are at most two wages in equilibrium recalls Curtis and Wright's (2003) similar result in a model that superficially looks quite different. They consider a dynamic model in which buyers and sellers meet sequentially. When they meet, the seller sets a price and the buyer privately observes her idiosyncratic valuation for the seller's good. Trade occurs if the buyer's valuation exceeds the seller's price. The authors show that there are generically at most two prices in equilibrium, and under the same regularity condition as the one developed in this paper, the equilibrium price is unique. If there are multiple prices, then trade occurs with probability one if the buyer realizes a sufficiently high valuation, with an intermediate probability when the buyer realizes a valuation between the two prices, and with zero probability at a low valuation, similar to the employment probability function in this paper. But there are some notable differences between the two papers. First, my analysis works in a static model, while Curtis and Wright (2003) require a dynamic framework in order to have a two price equilibrium. Second, we allow sellers (firms) to offer general mechanisms, while Curtis and Wright restrict firms to offer a single price. Third, in a competitive search model, each firm must randomly select a price, while in Curtis and Wright's (2003) random search model, it is enough that a fraction of firms offer a low price and the remaining firms offer a high price. Finally, the link with the implicit contracts literature is clearer in the current environment because firms explicitly commit to contracts in an effort to attract workers.

3 Model

There are a large number of ex ante identical workers and a large number of ex ante identical firms. Each worker is risk-neutral, is endowed with 1 units of time, and gets utility from consumption and leisure. If a worker consumes c and spends time $\ell \in [0, 1]$ enjoying leisure, her realized utility is $c + z\ell$ for some $z > 0$. We call z the value of leisure. Firms are also risk-neutral expected profit maximizers.

At the start of the period, firms can create vacancies at cost k . Each vacancy entitles a firm to post an employment contract, the details of which we discuss below. Workers then observe all the contracts and decide where to apply for a job. Workers and firms anticipate

that each contract is associated with a v-u ratio θ , and that any worker seeking a firm offering such a contract finds one with probability $\mu(\theta)$, while any firm offering the contract finds a worker with probability $\frac{\mu(\theta)}{\theta}$. The matching function $\mu(\theta)$ is nondecreasing and satisfies $0 \leq \mu(\theta) < \min\{1, \theta\}$, so $\mu(\theta)$ and $\frac{\mu(\theta)}{\theta}$ are proper probabilities.

Search is competitive in the following sense: each firm believes that its actions do not affect workers' expected utility, denoted by V , although V is determined endogenously in equilibrium. Obviously $V \geq z$, since the value a worker gets from simply consuming leisure. If an employment contract offers expected utility W to a worker who applies for that contract and succeeds in contacting a firm, the firm anticipates that the v-u ratio will adjust so that a worker applying for that job gets the correct utility, $V = \mu(\theta)W + (1 - \mu(\theta))z$. This implicitly defines θ as a decreasing function of W for fixed V . If a firm offers a value of W below $\lim_{\theta \rightarrow \infty} \frac{V - (1 - \mu(\theta))z}{\mu(\theta)}$, no workers direct their search towards that job.

When a worker and firm meet, the worker must decide whether to supply effort to form a good employment relationship. Supplying effort uses a portion α of the worker's time endowment, and so reduces the worker's utility from leisure by αz . A firm cannot directly observe whether a worker supplies effort. Instead, it observes a match-specific productivity realization x . If the worker supplies effort, x is distributed according to $F(x)$, with $F(0) = 0$ and $F(\bar{x}) = 1$ for some $\bar{x} \in (z, \infty]$. Otherwise, the match-specific productivity realization is always equal to zero.⁸ The match-specific productivity realization, which is observed only by the firm, determines the amount of output that the worker-firm pair produce if the match is consummated. Next, the firm decides whether to employ the worker, thereby using the remaining $(1 - \alpha)z$ of the worker's time endowment, or to fire the worker, allowing her to consume leisure but producing nothing. Finally, production occurs and payments are made as dictated by the employment contract. We assume there is common knowledge about the economic environment, although only the worker can observe her effort level and only the firm can observe the realization of the productivity shock.

We return now to a discussion of the employment contracts. We look for an equilibrium in which the firm induces the worker to supply effort to the employment relationship.⁹ In addition, we restrict attention to incentive compatible direct revelation mechanisms: if a

⁸This assumption can be relaxed to allow $x \sim G$ when the worker does not provide effort, where F first order stochastically dominates G ; see footnote 10. Since the results are qualitatively unaffected by this assumption, we focus on the simpler case in this paper.

⁹If a firm does not induce a worker to supply effort, the output from a match is always equal to zero, and so the firm will be unwilling to hire a worker at a positive wage. This outcome occurs if there is no equilibrium in which the firm induces the worker to supply effort. See Proposition 1 below.

worker contacts a firm, the firm announces the productivity level $x \in [0, \bar{x}]$, makes a payment $c(x)$, and hires the worker, using the worker's remaining time endowment, with probability $e(x) \in [0, 1]$. The restriction to incentive compatible direct revelation mechanisms is without loss of generality for the standard reasons (Myerson 1982).

A firm's payoff is zero if it fails to meet a worker and $xe(y) - c(y)$ if it meets a worker, realizes productivity x , and tells the worker that the productivity realization is y . Incentive compatibility requires that its payoff is higher if it truthfully announces its productivity realization is x rather than falsely report some other realization y ,

$$xe(x) - c(x) \geq xe(y) - c(y),$$

for all $\{x, y\} \in [0, \bar{x}]^2$. Incentive compatibility for a worker requires that she is willing to supply effort to the employment relationship. Her expected payoff is $c(x) + z(1 - \alpha)(1 - e(x))$ if she meets a firm, supplies effort, and the firm announces a productivity realization x ; and $c(x) + z(1 - (1 - \alpha)e(x))$ if she meets a firm, does not supply effort, and the firm announces productivity realization x . Using the fact that the firm will truthfully announce its productivity realization in equilibrium, the worker supplies effort if

$$\int_0^{\bar{x}} (c(x) + z(1 - \alpha)(1 - e(x))) dF(x) \geq c(0) + z(1 - (1 - \alpha)e(0)).$$

To summarize, in a competitive search equilibrium with asymmetric information, each firm chooses incentive compatible employment probability and compensation functions in order to maximize its expected profits, taking as given that the v-u ratio θ will adjust so that a worker seeking this contract receives some market-determined level of utility V . For interesting parameter values, the contract induces the worker to supply effort. Moreover, the maximized level of profit must equal the sunk cost of a vacancy k . This can be represented

as a constrained optimization problem:

$$\begin{aligned}
k &= \max_{\theta, e, c} \frac{\mu(\theta)}{\theta} \int_0^{\bar{x}} (xe(x) - c(x)) dF(x) \\
\text{subject to } V &= \mu(\theta) \int_0^{\bar{x}} (c(x) + z(1 - \alpha)(1 - e(x))) dF(x) + (1 - \mu(\theta))z \\
&\int_0^{\bar{x}} (c(x) + z(1 - \alpha)(1 - e(x))) dF(x) \geq c(0) + z(1 - (1 - \alpha)e(0)) \\
&xe(x) - c(x) \geq xe(y) - c(y) \text{ for all } x \text{ and } y \\
&e(x) \in [0, 1] \text{ for all } x,
\end{aligned}$$

It is easier to work directly with the dual problem. Then a competitive search equilibrium with asymmetric information is a tuple $\{V, \theta, e, c\}$ solving

$$\begin{aligned}
V &= \max_{\theta, e, c} \mu(\theta) \int_0^{\bar{x}} (c(x) + z(1 - \alpha)(1 - e(x))) dF(x) + (1 - \mu(\theta))z & \text{(P1)} \\
\text{subject to } &\frac{\mu(\theta)}{\theta} \int_0^{\bar{x}} (xe(x) - c(x)) dF(x) = k \\
&\int_0^{\bar{x}} (c(x) + z(1 - \alpha)(1 - e(x))) dF(x) \geq c(0) + z(1 - (1 - \alpha)e(0)) \\
&xe(x) - c(x) \geq xe(y) - c(y) \text{ for all } x \text{ and } y \\
&e(x) \in [0, 1] \text{ for all } x.
\end{aligned}$$

It is clear that if there are multiple competitive search equilibria with asymmetric information, workers must get the same expected utility in each.

4 Characterization of Incentive Compatible Contracts

This constrained optimization problem appears to be complicated to solve because of the continuum of incentive constraints. Fortunately, a standard trick permits us to simplify the problem considerably, yielding something like a ‘first order approach’ (Mirrlees 1971, Laffont and Maskin 1980, Rogerson 1985, Milgrom and Segal 2002):

Lemma 1. *The following two conditions are equivalent:*

$$A: \quad xe(x) - c(x) \geq xe(y) - c(y) \text{ for all } \{x, y\} \in [0, \bar{x}]^2$$

$$B: \quad c(x) = c_0 + xe(x) - \int_0^x e(y)dy \text{ and } e(x) \text{ is a nondecreasing function.}$$

Proof. We first prove that condition A implies condition B. If $xe(x) - c(x) \geq xe(y) - c(y)$ for all x and y , then this true in particular for y close to x . This implies the generalized first order condition $dc(y) = yde(y)$. Integrate this condition using the boundary condition $c(0) = c_0$ and integration-by-parts to get

$$\int_0^x dc(y) = \int_0^x yde(y) \Rightarrow c(x) = c_0 + xe(x) - \int_0^x e(y)dy.$$

This is half of condition B. To prove monotonicity of the employment probability, sum the incentive compatibility constraints $\{x, y\}$ and $\{y, x\}$:

$$xe(x) - c(x) + ye(y) - c(y) \geq xe(y) - c(y) + ye(x) - c(x) \Rightarrow (e(x) - e(y))(x - y) \geq 0.$$

This proves $e(x) \geq e(y)$ when $x > y$.

Now we prove that condition B implies condition A. Under the proposed compensation scheme, for any $x < y$,

$$xe(x) - c(x) - xe(y) + c(y) = \int_x^y (e(y) - e(x'))dx'.$$

The integrand, and hence the integral, is nonnegative because e is nondecreasing, proving $xe(x) - c(x) \geq xe(y) - c(y)$. The proof when $x > y$ is symmetric. \square

Use Lemma 1 to eliminate the compensation from constrained optimization problem (P1):

$$V = \max_{\theta, e, c_0} \mu(\theta) \left(\int_0^{\bar{x}} \left(xe(x) + z(1 - \alpha)(1 - e(x)) - \int_0^x e(y)dy \right) dF(x) + c_0 \right) + (1 - \mu(\theta))z$$

subject to $\frac{\mu(\theta)}{\theta} \left(\int_0^{\bar{x}} \left(\int_0^x e(y)dy \right) dF(x) - c_0 \right) = k$

$$\int_0^{\bar{x}} \left(xe(x) + z(1 - \alpha)(1 - e(x)) - \int_0^x e(y)dy \right) dF(x) \geq z(1 - (1 - \alpha)e(0))$$

$$0 \leq e(x) \leq e(y) \leq 1 \text{ for all } x < y.$$

Next, eliminate c_0 from the optimization problem using the free entry condition:

$$\begin{aligned}
V &= \max_{\theta, e} \mu(\theta) \int_0^{\bar{x}} (xe(x) + z(1 - \alpha)(1 - e(x))) dF(x) + (1 - \mu(\theta))z - \theta k \\
\text{subject to } & \int_0^{\bar{x}} \left(xe(x) + z(1 - \alpha)(1 - e(x)) - \int_0^x e(y) dy \right) dF(x) \geq z(1 - (1 - \alpha)e(0)) \\
& 0 \leq e(x) \leq e(y) \leq 1 \text{ for all } x < y.
\end{aligned}$$

Integration-by-parts implies $\int_0^{\bar{x}} \left(\int_0^x e(y) dy \right) dF(x) = \int_0^{\bar{x}} e(x)(1 - F(x)) dx$, which allows for rewriting workers' incentive constraint:

$$\begin{aligned}
V &= \max_{\theta, e} \mu(\theta) \int_0^{\bar{x}} (xe(x) + z(1 - \alpha)(1 - e(x))) dF(x) + (1 - \mu(\theta))z - \theta k & \text{(P2)} \\
\text{subject to } & \int_0^{\bar{x}} (xe(x) + z(1 - \alpha)(1 - e(x))) dF(x) - \int_0^{\bar{x}} e(x)(1 - F(x)) dx \\
& \geq z(1 - (1 - \alpha)e(0)) \\
& 0 \leq e(x) \leq e(y) \leq 1 \text{ for all } x < y.
\end{aligned}$$

This problem is linear in the employment probability function, suggesting that a bang-bang solution is optimal, although the monotonicity constraint makes it difficult to use this expression to provide a precise characterization. Instead, perform integration-by-parts again to get an expression in terms of $de(x)$:

$$\begin{aligned}
V &= \max_{\theta, e} \mu(\theta) \left(\int_0^{\bar{x}} (y - (1 - \alpha)z) dF(y) e(0) \right. \\
& \quad \left. + \int_0^{\bar{x}} \int_x^{\bar{x}} (y - (1 - \alpha)z) dF(y) de(x) - \alpha z \right) + z - \theta k \\
\text{subject to } & \int_0^{\bar{x}} (x - (1 - \alpha)z)(1 - F(x)) de(x) \geq \alpha z \\
& e(0) \geq 0, de(x) \geq 0 \text{ for all } x, \text{ and } e(\bar{x}) \leq 1.
\end{aligned}$$

It is straightforward to prove that $e(0) = 0$. For suppose to the contrary that in the solution to this optimization problem $\{\theta, e\}$, $e(0) > 0$ and consider an alternative policy $\{\theta, \tilde{e}\}$ defined as $\tilde{e}(x) = e(x) - e(0)$ if $x < (1 - \alpha)z$ and $\tilde{e}(x) = e(x)$ if $x \geq (1 - \alpha)z$. This policy satisfies

all of the constraints and it raises the value of the objective function since

$$\int_0^{\bar{x}} (y - (1 - \alpha)z) dF(y) < \int_{(1-\alpha)z}^{\bar{x}} (y - (1 - \alpha)z) dF(y).$$

By changing θ , one could do better still. This permits a further simplification of the optimization problem:¹⁰

$$\begin{aligned} V = \max_{\theta, e} \mu(\theta) & \left(\int_0^{\bar{x}} \int_x^{\bar{x}} (y - (1 - \alpha)z) dF(y) de(x) - \alpha z \right) + z - \theta k & \text{(P3)} \\ \text{subject to} & \int_0^{\bar{x}} (x - (1 - \alpha)z)(1 - F(x)) de(x) \geq \alpha z \\ & e(0) = 0, de(x) \geq 0 \text{ for all } x, \text{ and } e(\bar{x}) \leq 1. \end{aligned}$$

The the v-u ratio does not enter the constraint set, and so one can break this problem into two pieces. First, the employment probability function is chosen to solve

$$\begin{aligned} W^* = \max_e & \int_0^{\bar{x}} \int_x^{\bar{x}} (y - (1 - \alpha)z) dF(y) de(x) - \alpha z & \text{(P4)} \\ \text{subject to} & \int_0^{\bar{x}} (x - (1 - \alpha)z)(1 - F(x)) de(x) \geq \alpha z \\ & e(0) = 0, de(x) \geq 0 \text{ for all } x, \text{ and } e(\bar{x}) \leq 1. \end{aligned}$$

W^* is the expected net output from a meeting. If a worker and firm consummate a match with productivity x , they realize net output $x - (1 - \alpha)z$ from the match, so the expected output from a meeting is $\int_0^{\bar{x}} e(x)(x - (1 - \alpha)z) dF(x) = \int_0^{\bar{x}} \int_x^{\bar{x}} (y - (1 - \alpha)z) dF(y) de(x)$. From this it is necessary to subtract the worker's initial effort expenditure αz , which is incurred regardless of whether the match is ultimately formed. So the employment probability function e is chosen to be incentive compatible and maximize the expected net output from a meeting.

Second, the v-u ratio is selected to maximize $W^* \mu(\theta) - \theta k$, which can be interpreted as expected net output per unemployed worker. Each worker finds a firm with probability $\mu(\theta)$, in which case the relationship produces W^* on average. Each vacancy costs k , and there

¹⁰Suppose that when a worker does not supply effort to the employment relationship, productivity is not always equal to zero, but instead is drawn from some distribution G that is first order stochastically dominated by F , so $F(x) \leq G(x)$ for all x . Then problem (P3) is unchanged except for the incentive constraint, which becomes $\int_0^{\bar{x}} (x - (1 - \alpha)z)(G(x) - F(x)) de(x) \geq \alpha z$.

are θ vacancies per unemployed worker, so the per capita vacancy cost is just the product of these two numbers.

Using a similar methodology and Lemma 1, one can show that the compensation function must satisfy

$$c(x) = \mu(\theta) \int_0^{\bar{x}} e(y)(1 - F(y))dy - \theta k + xe(x) - \int_0^x e(y)dy \quad (1)$$

for all x . A tuple $\{\theta, e, c\}$ is a competitive search equilibrium with asymmetric information if and only if $\{\theta, e\}$ solves problem (P3) and c satisfies equation (1). This problem is relatively easily solved.

Before proceeding further, it is worth noting one circumstance in which problem (P3) has no solution:

Proposition 1. *Assume $\max_x (x - (1 - \alpha)z)(1 - F(x)) < \alpha z$. Then there are no job vacancies and no production in a competitive search equilibrium with asymmetric information.*

Proof. Under the assumption in the statement of the proof, the constraint set in optimization problem (P3) is empty. This implies that regardless of the contract that the firm offers, workers will not supply effort to the employment relationship and so firms do not create any vacancies. \square

In the remainder of the paper we assume $\max_x (x - (1 - \alpha)z)(1 - F(x)) \geq \alpha z$, so there are vacancies and production in a competitive search equilibrium with asymmetric information. Still, the borderline case when $\max_x (x - (1 - \alpha)z)(1 - F(x)) = \alpha z$ is interesting. Feasibility implies that $de(x)$ is positive only at points in $\arg \max_x (x - (1 - \alpha)z)(1 - F(x))$, while problem (P4) shows that optimally $de(x)$ is positive only at the smallest such point, x^* , so

$$W^* = \int_{x^*}^{\bar{x}} (y - (1 - \alpha)z)dF(y) - \alpha z = \int_{x^*}^{\bar{x}} (y - x^*)dF(y)$$

Since $F(x^*) < 1$, this expression is strictly positive and, depending on the properties of the matching function μ , the v-u ratio, $\theta^* \in \arg \max W^* \mu(\theta) - \theta k$, may also be strictly positive. It follows that an arbitrarily small change in parameter values may shut down an active market by eliminating all incentive feasible employment probability functions. This suggests that the v-u ratio could be very sensitive to market conditions.

5 Economy without Moral Hazard

Before characterizing the solution to constrained optimization problem (P3), it is useful to characterize the economy without the moral hazard problem, as would be the case if firms could observe whether workers' supply effort. One can verify directly that the only effect of this is to eliminate the workers' incentive constraint from the problem. From constrained optimization problem (P2), the v-u ratio θ and employment probability function e solve

$$V = \max_{\theta, e} \mu(\theta) \int_0^{\bar{x}} (xe(x) + z(1 - \alpha)(1 - e(x)))dF(x) + (1 - \mu(\theta))z - \theta k$$

subject to $0 \leq e(x) \leq e(y) \leq 1$ for all $x < y$

with $c(x)$ satisfying equation (1). The solution is obviously to set $e(x) = 1$ if $x > (1 - \alpha)z$ and $e(x) = 0$ otherwise,¹¹ with θ chosen to solve

$$V = \max_{\theta} \mu(\theta) \left(\int_{(1-\alpha)z}^{\bar{x}} (x - z(1 - \alpha))dF(x) - \alpha z \right) - \theta k + z \quad (2)$$

The unique incentive compatible compensation function that gives firms zero profits and permits this employment probability function is

$$c(x) = \begin{cases} \mu(\theta) \int_{(1-\alpha)z}^{\bar{x}} (1 - F(y))dy - \theta k + (1 - \alpha)z & \text{if } x \geq (1 - \alpha)z \\ \mu(\theta) \int_{(1-\alpha)z}^{\bar{x}} (1 - F(y))dy - \theta k & \text{if } x < (1 - \alpha)z \end{cases}$$

The higher compensation for employed workers ensures that firms truthfully reveal the productivity realization. But for this to occur, workers must receive a constant utility whenever they contact a firm, independent of the productivity realization. If a worker could secretly shirk, she would obtain additional leisure αz without changing these calculations, creating a clear incentive problem. Instead, it is necessary to characterize the equilibrium when the incentive constraint in problem (P3) binds.

¹¹The argument here presumes that $dF(x) > 0$ for all $x \in (0, \bar{x})$. Otherwise other policies may be optimal. As an extreme example, suppose the productivity distribution F is degenerate at a single point $\bar{x} > z$. Then any monotone employment probability function satisfying $e(\bar{x}) = 1$ and $e(z) = 0$ solves the constrained optimization problem (P4) and delivers match value $W = \bar{x} - z$. In this case, the moral hazard problem is costless because there is only one-sided asymmetric information.

6 Equilibrium Contracts

This section characterizes equilibrium employment contracts. We represent the constrained optimization problem (P3) as a Lagrangian with nonnegative multiplier $\mu(\theta)\psi$ on the binding incentive constraint. Suppressing the requirements that $e(0) = 0$, $de(x) \geq 0$, and $e(\bar{x}) \leq 1$, this can be expressed as

$$L(\theta, de) = \mu(\theta) \left(\int_0^{\bar{x}} \phi(x) de(x) - (1 + \psi)\alpha z \right) + z - \theta k, \quad (3)$$

where

$$\phi(x) = \int_x^{\bar{x}} (y - (1 - \alpha)z) dF(y) + \psi(x - (1 - \alpha)z)(1 - F(x)). \quad (4)$$

The Lagrangian (3) implies $de(x) > 0$ only if $x \in \arg \max_y \phi(y)$. Moreover, since $\psi \geq 0$, $\phi(x) > 0$ for all $x \geq (1 - \alpha)z$, and so optimality dictates that $e(\bar{x}) = 1$. Since we have already argued that $e(0) = 0$, it remains to resolve how $e(x)$ increases from 0 to 1 as x goes from 0 to \bar{x} . The following proposition provides part of the answer:

Proposition 2. *Assume $\max_x (x - (1 - \alpha)z)(1 - F(x)) \geq \alpha z$. For generic cumulative distribution functions F and value of leisure z the employment probability function e is a step function with at most two discontinuities.*

Proof. There is only a single endogenous variable, the multiplier ψ , in the definition of ϕ , equation (4), so for generic F and z and for arbitrary (not necessarily generic) values of ψ , the function ϕ has at most two maxima. The result then follows from the Lagrangian (3), which implies that $e(x)$ is increasing only at $x \in \arg \max_y \phi(y)$ \square

Proposition 2 puts a tight limit on the generic behavior of the employment probability function. One might conjecture that the limits are too loose; there is an equilibrium in which workers are employed if and only if their productivity exceeds some threshold x^* . The following proposition verifies this intuition if the productivity distribution F satisfies a regularity condition:

Proposition 3. *Assume $\max_x (x - (1 - \alpha)z)(1 - F(x)) \geq \alpha z$. Also assume F is differentiable and $h(x) \equiv \frac{x - (1 - \alpha)z F'(x)}{1 - F(x)}$ is an increasing function when $(1 - \alpha)z < x < \bar{x}$. Define the employment threshold x^* via*

$$x^* \equiv \min_x \text{ subject to } (x - (1 - \alpha)z)(1 - F(x)) \geq \alpha z$$

and let $W^* \equiv \int_{x^*}^{\bar{x}} (x - x^*) dF(x)$. Then there is a competitive search equilibrium with asymmetric information in which the v-u ratio satisfies $\theta^* \in \arg \max_{\theta} W^* \mu(\theta) - \theta k$ and the employment probability and compensation functions satisfy

$$e(x) = \begin{cases} 0 \\ 1 \end{cases} \quad \text{and } c(x) = \begin{cases} W^* \mu(\theta^*) - \theta^* k \\ W^* \mu(\theta^*) - \theta^* k + x^* \end{cases} \quad \text{if } \begin{cases} x \in (0, x^*) \\ x \in (x^*, \bar{x}) \end{cases}.$$

Proof. We first prove that for any value of ψ , $\phi(x)$ defined in equation (4) is single-peaked. Since $\phi'(x) = -(1 + \psi)(x - (1 - \alpha)z)F'(x) + \psi(1 - F(x))$, ϕ is increasing if and only if $h(x) < \frac{\psi}{1 + \psi}$. Monotonicity of h then yields the desired result. From the Lagrangian (3), it follows that for any ψ there exists an \tilde{x} increasing in ψ and satisfying $h(\tilde{x}) = \frac{\psi}{1 + \psi}$, such that $de(\tilde{x}) = 1$ and $de(x) = 0$ otherwise.

The next step is to deduce the equilibrium value of \tilde{x} , i.e. the threshold x^* . Taking advantage of the threshold characterization, rewrite the constrained optimization problem (P4) as

$$W^* = \max_{\tilde{x}} \int_{\tilde{x}}^{\bar{x}} (y - (1 - \alpha)z) dF(y) - \alpha z$$

subject to $(\tilde{x} - (1 - \alpha)z)(1 - F(\tilde{x})) \geq \alpha z$.

The objective function is decreasing in $\tilde{x} > (1 - \alpha)z$, so the solution is to choose the smallest \tilde{x} satisfying the constraint, i.e. x^* as defined above. Substituting the binding constraint into the objective function and simplifying gives $W^* = \int_{x^*}^{\bar{x}} (x - x^*) dF(x)$. Finally, the v-u ratio solves $\theta^* \in \arg \max_{\theta} W^* \mu(\theta) - \theta k$ and one can compute the compensation function using (1). \square

In words, if $F(x)$ satisfies a regularity condition discussed in the next paragraph, the implementation of a competitive search equilibrium is simple. A firm pays worker it contacts $W^* \mu(\theta) - \theta k \geq 0$ and gives a bonus x^* to any worker it hires. This simple contract ensures that the firm hires the worker if and only if her productivity exceeds x^* . Moreover, the bonus x^* is set so as to give the worker enough incentive to provide effort.

The condition that $h(x)$ is increasing is related to some more familiar conditions. First, the condition is equivalent to the requirement that the elasticity of $1 - F(x)$ with respect to x must be decreasing. A stronger but more familiar requirement is the monotone hazard rate condition that $\frac{F'(x)}{1 - F(x)}$ is nondecreasing. This last condition is identical to the requirement that $1 - F(x)$ is log-concave. It is satisfied by a broad class of standard distributions,

including any distribution with a nondecreasing density p (e.g. the uniform), the normal distribution truncated at zero, the log normal distribution, and the exponential distribution.

Of course, not all distributions satisfy this restriction on F , notably any discrete distribution. We next turn to characterizing an equilibrium with two discontinuities in the employment probability function. From Proposition 2, we know that generically this is the most complex possibility. In fact, the next proposition shows that even if there is a competitive search equilibrium with three or more discontinuities or with $e(x)$ strictly increasing on some interval, it is possible to find another (payoff equivalent) equilibrium with only two discontinuities:

Proposition 4. *Assume $\max_x (x - (1 - \alpha)z)(1 - F(x)) \geq \alpha z$. Let $x_1^* < x_2^*$, e^* , and W^* solve*

$$W^* \equiv \max_{e \in [0,1], x_1, x_2} e \int_{x_1}^{\bar{x}} (y - x_1) dF(y) + (1 - e) \int_{x_2}^{\bar{x}} (y - x_2) dF(y) \quad (\text{P5})$$

subject to $e(x_1 - (1 - \alpha)z)(1 - F(x_1)) + (1 - e)(x_2 - (1 - \alpha)z)(1 - F(x_2)) = \alpha z$.

Then there is a competitive search equilibrium with asymmetric information in which the v - u ratio satisfies $\theta^ \in \arg \max_{\theta} W^* \mu(\theta) - \theta k$ and the employment probability and compensation functions satisfy*

$$e(x) = \begin{cases} 0 \\ e^* \\ 1 \end{cases} \quad \text{and } c(x) = \begin{cases} W^* \mu(\theta^*) - \theta^* k & x \in (0, x_1^*) \\ W^* \mu(\theta^*) - \theta^* k + e^* x_1^* & \text{if } x \in (x_1^*, x_2^*) \\ W^* \mu(\theta^*) - \theta^* k + e^* x_1^* + (1 - e^*) x_2^* & x \in (x_2^*, \bar{x}) \end{cases} .$$

Proof. We start by showing that a competitive search equilibrium need only use at most three employment probabilities, 0, e^* , and 1. The proof can be broken into two steps. First, suppose there is an interval of points, say $[y_1, y_2]$, with $e(y)$ strictly increasing on the interval. From the Lagrangian (3), this can be the case only if $y \in \arg \max \phi(x)$ for all $y \in [y_1, y_2]$, which implies $\phi'(y) = -(1 + \psi)(y - (1 - \alpha)z)F'(y) + \psi(1 - F(y)) = 0$ for all $y \in (y_1, y_2)$, and in particular implies F is differentiable, hence continuous, on this interval. Then the intermediate value theorem implies that there is a point $y^* \in [y_1, y_2]$ with

$$(y^* - (1 - \alpha)z)(1 - F(y^*)) = \frac{\int_{y_1}^{y_2} (x - (1 - \alpha)z)(1 - F(x)) de(x)}{\int_{y_1}^{y_2} de(x)}$$

This allows us to define an alternative policy \tilde{e} that is feasible and achieves the same value

as the original policy e :

$$\tilde{e}(x) = \begin{cases} e(x) \\ e(y_1) \\ e(y_2) \\ e(x) \end{cases} \quad \text{if} \quad \begin{cases} x \in [0, y_1) \\ x \in [y_1, y^*) \\ x \in [y^*, y_2) \\ x \in [y_2, \bar{x}] \end{cases}$$

This implies that a competitive search equilibrium with asymmetric information never requires an interval of points on which the employment probability function increases continuously.

Next, suppose there are (at least) three points, x_1 , x_2 , and x_3 , with $de(x_i) > 0$ in a competitive search equilibrium with asymmetric information. From the Lagrangian, we know that $\phi(x_1) = \phi(x_2) = \phi(x_3)$. Also, we may assume without loss of generality that

$$(x_1 - (1 - \alpha)z)(1 - F(x_1)) \leq (x_2 - (1 - \alpha)z)(1 - F(x_2)) \leq (x_3 - (1 - \alpha)z)(1 - F(x_3)).$$

Once again, it is possible to remove probability mass from x_2 to x_1 and x_3 , leaving the expected value of $(x_i - (1 - \alpha)z)(1 - F(x_i))$ unchanged. This implies that only two discontinuity points x_1^* and x_2^* are needed to support a competitive search equilibrium with asymmetric information.

To characterize the two discontinuity points and the probability e^* , note that these three numbers must solve problem (P4), yielding the optimization problem (P5) in the statement of this proposition. Finally, the v-u ratio and compensation functions are computed as in the proof of Proposition 3. \square

The implementation of this general contract is only slightly more complex than the implementation in the case when $h(x)$ is increasing. After a worker chooses her effort level, the firm selects a wage randomly, equally to $W\mu(\theta^*) - \theta^*k + x_1^*$ and $W\mu(\theta^*) - \theta^*k + x_2^*$ with probability e^* and $1 - e^*$, respectively. The firm then observes the worker's productivity and decides whether to hire her and pay the random wage or dismiss her and pay $W\mu(\theta^*) - \theta^*k \geq 0$. If her productivity exceeds x_2^* , she is always hired, receiving an average wage of $W\mu(\theta^*) - \theta^*k + e^*x_1^* + (1 - e^*)x_2^*$. If her productivity lies between x_1^* and x_2^* , she is hired only if the wage is low, with probability e^* , and then paid the low wage $W\mu(\theta^*) - \theta^*k + x_1^*$. For still lower productivity realizations, the worker is not hired but instead gets the dismissal payment $W\mu(\theta^*) - \theta^*k$.

Using optimization problem (P5), one can verify that if F is differentiable, the disconti-

nuity points must satisfy

$$\frac{(x_1^* - (1 - \alpha)z)F'(x_1^*)}{1 - F(x_1^*)} = \frac{(x_2^* - (1 - \alpha)z)F'(x_2^*)}{1 - F(x_2^*)} = \frac{\int_{x_1^*}^{x_2^*} (x - (1 - \alpha)z)dF(x)}{\int_{x_1^*}^{x_2^*} (1 - F(x))dx}. \quad (5)$$

Given these points, the employment probability function must satisfy the constraint in problem (P5), so

$$e^* = \frac{(x_2^* - (1 - \alpha)z)(1 - F(x_2^*)) - \alpha z}{(x_2^* - (1 - \alpha)z)(1 - F(x_2^*)) - (x_1^* - (1 - \alpha)z)(1 - F(x_1^*))}. \quad (6)$$

This defines a proper probability if $(x_1^* - (1 - \alpha)z)(1 - F(x_1^*)) \leq \alpha z \leq (x_2^* - (1 - \alpha)z)(1 - F(x_2^*))$ or both inequalities are reversed. Condition (5) illustrates two other important points. First, if $h(x) = \frac{(x - (1 - \alpha)z)F'(x)}{1 - F(x)}$ is strictly increasing, the condition immediately implies that there cannot be multiple discontinuity points. Second, for arbitrary h , if there were a third discontinuity point x_3^* , a similar logic would yield two more equations in this single unknown, which is generically impossible, consistent with the conclusion in Proposition 2.

Finally, an example illustrates that a competitive search equilibrium may require using public randomization, so Proposition 4 is not vacuous. Let $F'(x) = 0.1$ for $x \in [0, 1]$ and $F'(x) = 3(x - 1)(2 - x)^2(3 - x)$ for $x \in [1, 3]$. Also assume $(1 - \alpha)z = 1$. There is a single solution to condition (5), $x_1^* = 1.280$ and $x_2^* = 2.191$. If $0.184 < \alpha < 0.468$, $e^* \in [0, 1]$, while for lower (higher) values of α , there is a single threshold equilibrium, as in Proposition 3, with the threshold lying below x_1^* (above x_2^*).

To complete the example, suppose $\alpha = 0.3$. Then equation (6) implies $e^* = 0.592$ so $W^* = 0.421$. If instead one artificially restricted attention to strategies that do not involve public randomization, as in the statement of Proposition 3, one finds that $x^* = 1.708$, giving $W^* = 0.368$, a lower value from a meeting and hence a lower value for workers. It follows that in this example, contracts that use public randomization would drive out contracts that do not take advantage of this possibility because public randomization provides workers with more utility at a given level of firm profits.

7 Comparative Statics

Simple comparative statics help illustrate how the model economy functions and what role asymmetric information plays. First, compare an economy without moral hazard (Section 5)

to a competitive search model with bilateral asymmetric information:

Proposition 5. *The presence of bilateral asymmetric information weakly reduces the v-u ratio and weakly reduces the fraction of meetings that result in matches.*

Proof. In an economy without moral hazard (symmetric information), the v-u ratio maximizes $W^s\mu(\theta) - \theta k$ where

$$W^s = \max_e \int_0^{\bar{x}} \int_x^{\bar{x}} (y - (1 - \alpha)z) dF(y) de(x) - \alpha z$$

subject to $e(0) = 0, de(x) \geq 0$ for all x , and $e(\bar{x}) \leq 1$.

In an economy with moral hazard (asymmetric information), the v-u ratio maximizes $W^*\mu(\theta) - \theta k$ where W^* solves problem (P4). Clearly $W^* \leq W^s$, since the symmetric information problem relaxes the constraint set in the asymmetric information problem.

Next, we prove that the associated v-u ratios satisfy $\theta^s \geq \theta^*$. Since each v-u ratio maximizes $W\mu(\theta) - \theta k$ with W taking the appropriate value, it follows that

$$W^s\mu(\theta^s) - \theta^s k \geq W^s\mu(\theta^*) - \theta^* k \quad \text{and} \quad W^*\mu(\theta^*) - \theta^* k \geq W^*\mu(\theta^s) - \theta^s k.$$

Sum these inequalities and simplify to get $(W^s - W^*)(\mu(\theta^s) - \mu(\theta^*)) \geq 0$. $\theta^s \geq \theta^*$ follows because μ is increasing.

To prove that fewer meetings result in matches when there is asymmetric information, note that with symmetric information, meetings result in matches if and only if $x \geq z(1 - \alpha)$. From equation (4), it is clear that $\phi'(x) = -(1 + \psi)(x - (1 - \alpha)z)dF(x) + \psi(1 - F(x))$ is positive when $x \leq (1 - \alpha)z$, and in particular such $x \notin \arg \max \phi(x)$. Then the Lagrangian (3) implies $de(x) = 0$ for such x . It follows that the employment probability function under asymmetric information is less than the employment probability function under symmetric information for each x . □

The remainder of this section focuses here on the impact of the vacancy cost k , the extent of moral hazard α , the distribution of productivity F , and the value of leisure z on the v-u ratio:

Proposition 6. *An increase in the cost of a vacancy k weakly reduces the v-u ratio.*

Proof. Let θ_1 and θ_2 denote the v-u ratios associated with two different costs k_1 and k_2 . Recall that the v-u ratio maximizes $W\mu(\theta) - \theta k$ where W solves problem (P4). Since the

vacancy cost does not enter this problem, it does not affect W , so

$$W\mu(\theta_1) - \theta_1 k_1 \geq W\mu(\theta_2) - \theta_2 k_1 \quad \text{and} \quad W\mu(\theta_2) - \theta_2 k_2 \geq W\mu(\theta_1) - \theta_1 k_2.$$

Summing these inequalities and simplifying gives $(\theta_1 - \theta_2)(k_2 - k_1) \geq 0$, the desired result. \square

We use the following Lemma to prove the remainder of the Propositions:

Lemma 2. *A change in the extent of moral hazard α , the distribution of productivity F , or the value of leisure z weakly increases the v-u ratio if and only if it weakly increases W defined in problem (P4).*

Proof. None of these variables directly enter the problem of choosing the v-u ratio conditional on W , $\theta^* \in \arg \max W\mu(\theta) - \theta k$, and so their only impact comes indirectly through W . Moreover, take W_1 and W_2 and the associated v-u ratios θ_1 and θ_2 . Then

$$W_1\mu(\theta_1) - \theta_1 k \geq W_1\mu(\theta_2) - \theta_2 k \quad \text{and} \quad W_2\mu(\theta_2) - \theta_2 k \geq W_2\mu(\theta_1) - \theta_1 k.$$

Summing these inequalities and simplifying gives $(W_1 - W_2)(\mu(\theta_1) - \mu(\theta_2)) \geq 0$. The result follows since μ is increasing. \square

Proposition 7. *An increase in the time cost of providing work effort α weakly reduces the v-u ratio.*

Proof. Take $\alpha_1 < \alpha_2$ and consider the solution to problem (P4). Let e_1 and e_2 denote the associated employment probability functions. The incentive constraint in problem (P4) can be expressed as

$$\int_0^{\bar{x}} (x - z)(1 - F(x)) de_i(x) \geq \alpha_i z \left(1 - \int_0^{\bar{x}} (1 - F(x)) de_i(x) \right).$$

In particular, since $\alpha_1 < \alpha_2$, policy e_2 is feasible with the low cost of work effort α_1 (but not vice versa). It follows that this policy must give a lower value of the objective function:

$$\int_0^{\bar{x}} \int_x^{\bar{x}} (y - (1 - \alpha_1)z) dF(y) de_1(x) - \alpha_1 z \geq \int_0^{\bar{x}} \int_x^{\bar{x}} (y - (1 - \alpha_1)z) dF(y) de_2(x) - \alpha_1 z.$$

Since furthermore $\alpha_1 < \alpha_2$, simple algebra shows that

$$\int_0^{\bar{x}} \int_x^{\bar{x}} (y - (1 - \alpha_1)z) dF(y) de_2(x) - \alpha_1 z \geq \int_0^{\bar{x}} \int_x^{\bar{x}} (y - (1 - \alpha_2)z) dF(y) de_2(x) - \alpha_2 z.$$

This proves that $W_1 \geq W_2$, and so Lemma 2 implies $\theta_1 \geq \theta_2$. \square

Next turn to the value of leisure. As in most search models, an increase in the value of leisure discourages vacancy creation:

Proposition 8. *An increase in z weakly reduces the v-u ratio.*

Proof. Take $z_1 < z_2$ and consider the solution to problem (P4). Let e_1 and e_2 denote the associated employment probability functions. The incentive constraint in problem (P4) can be expressed as

$$\int_0^{\bar{x}} x(1 - F(x))de(x) \geq z \left(\alpha + (1 - \alpha) \int_0^{\bar{x}} (1 - F(x))de(x) \right).$$

In particular, since $z_1 < z_2$, policy e_2 is feasible with the low value of leisure z_1 (but not vice versa). It follows that this policy must give a lower value of the objective function:

$$\int_0^{\bar{x}} \int_x^{\bar{x}} (y - (1 - \alpha)z_1)dF(y)de_1(x) - \alpha z_1 \geq \int_0^{\bar{x}} \int_x^{\bar{x}} (y - (1 - \alpha)z_1)dF(y)de_2(x) - \alpha z_1.$$

Since furthermore $z_1 < z_2$,

$$\int_0^{\bar{x}} \int_x^{\bar{x}} (y - (1 - \alpha)z_1)dF(y)de_2(x) - \alpha z_1 > \int_0^{\bar{x}} \int_x^{\bar{x}} (y - (1 - \alpha)z_2)dF(y)de_2(x) - \alpha z_2.$$

This proves that $W_1 > W_2$, and so Lemma 2 implies $\theta_1 \geq \theta_2$. \square

Finally we consider changes in the distribution of productivity. As one might expect, drawing from a better productivity distribution raises the v-u ratio:

Proposition 9. *An increase in F in the sense of first order stochastic dominance weakly raises the v-u ratio.*

Proof. Using integration-by-parts on the objective function, problem (P4) may be rewritten as

$$\begin{aligned} & \max_e \int_0^{\bar{x}} \left((x - (1 - \alpha)z)(1 - F(x)) + \int_x^{\bar{x}} (1 - F(y))dy \right) de(x) \\ & \text{subject to } \int_0^{\bar{x}} (x - (1 - \alpha)z)(1 - F(x))de(x) \geq \alpha z \\ & e(0) = 0, \quad de(x) \geq 0 \text{ for all } x, \text{ and } e(\bar{x}) \leq 1. \end{aligned}$$

Take F_1 and F_2 with $F_1(x) < F_2(x)$ for all x and the associated employment probability functions e_1 and e_2 . Stochastic dominance implies that policy e_2 is feasible with the better productivity distribution F_1 and yields a higher value of the objective function:

$$\int_0^{\bar{x}} \left((x - (1 - \alpha)z)(1 - F_1(x)) + \int_x^{\bar{x}} (1 - F_1(y))dy \right) de_2(x) \geq \int_0^{\bar{x}} \left((x - (1 - \alpha)z)(1 - F_2(x)) + \int_x^{\bar{x}} (1 - F_2(y))dy \right) de_2(x).$$

By definition, the actual employment probability function is also feasible and so yields still more utility,

$$\int_0^{\bar{x}} \left((x - (1 - \alpha)z)(1 - F_1(x)) + \int_x^{\bar{x}} (1 - F_1(y))dy \right) de_1(x) \geq \int_0^{\bar{x}} \left((x - (1 - \alpha)z)(1 - F_2(x)) + \int_x^{\bar{x}} (1 - F_2(y))dy \right) de_2(x).$$

This proves $W_1 \geq W_2$, hence $\theta_1 \geq \theta_2$ by Lemma 2. \square

In many search models, second order stochastic dominance is associated with a decrease in profitability and hence the v-u ratio because of the option of throwing away bad productivity draws. For example, in the symmetric information environment of Section 5, it is clear from equation (2) that an increase in the sense of second order stochastic dominance from F_1 to F_2 (so $\int_z^{\bar{x}} (x - (1 - \alpha)z)dF_1(x) \geq \int_z^{\bar{x}} (x - (1 - \alpha)z)dF_2(x)$ for all z) reduces the term multiplying $\mu(\theta)$ and hence reduces the v-u ratio.

This is not necessarily the case in the model with asymmetric information. Fix $z = 1$ and suppose F_1 puts all its mass on $x = 2$, while $F_2(x) = \frac{x}{4}$ with support $[0, 4]$, i.e. a uniform distribution with the same mean. Without the moral hazard problem, equation (2) implies that all matches with productivity exceeding $(1 - \alpha)z$ are consummated, so the v-u ratio under F_1 is chosen to maximize $\mu(\theta) - \theta k$, while under F_2 it maximizes $(1 + \frac{1}{8}(1 - \alpha)^2) \mu(\theta) - \theta k$. The ability to throw away bad matches raises the value of sampling from the more disperse distribution and hence raises the v-u ratio.

But if the moral hazard problem, measured by the magnitude of α , is sufficiently severe, this can change. Under F_1 , Proposition 3 implies $x^* = 1$, independent of α , so $W_1 = 1$. But under F_2 , $x^* = \frac{1}{2}(5 - \alpha - \sqrt{9 - 10\alpha + \alpha^2}) > 1$, from which one can compute that $W_2 = \frac{1}{16}(9 - 2\alpha + \alpha^2 + (3 + \alpha)\sqrt{9 - 10\alpha + \alpha^2})$. When $\alpha > \sqrt{2} - 1 \approx 0.414$, $W_2 < W_1$ and hence the $v - u$ ratio is lower under the distribution function F_2 than under F_1 .

One can even construct examples in which an increase in the dispersion of productivity causes the market to break down completely. For example, if one raises the value of leisure to $z = \frac{3}{2}$, $W_1 = \frac{1}{2}$ under F_1 but the market breaks down completely under F_2 if $\alpha \geq \frac{1}{3}(11 - 4\sqrt{6}) \approx 0.401$. There is no way to induce workers to supply effort and firms to truthfully reveal the productivity realization. We conclude that the effect of productivity dispersion on the v-u ratio is ambiguous in this model because the increase in dispersion makes the moral hazard problem more acute, offsetting the usual truncation argument.

8 Conclusion

This paper has constructed a simple static competitive search model with asymmetric information. Firms commit to employment contracts in an effort to attract workers. Under a standard regularity condition, the employment contracts take a simple form: the firm pays a wage w to a worker who is hired and a severance payment b to a worker who is dismissed. The firm hires the worker if her productivity exceeds $w - b$, while the gap between the wage and the severance payment is high enough to ensure that the worker supplies effort to the employment relationship. Asymmetric information unambiguously reduces the vacancy-unemployment ratio and reduces the probability that a meeting results in a match. These results are consistent with earlier findings in the implicit contracts literature, although the precise mechanism is somewhat different. For example, there is no risk-sharing motive in this framework; and mobility restrictions are explicitly caused by a primitive search friction.

The next step is to analyze a dynamic version of this model. This is important for several reasons. First, one may view the employment relationship as consisting of two phases: an initial screening period in which the worker supplies effort that determines the productivity of the match; and a later productive period in which output is actually produced. In that world, optimal employment contracts will likely consist of an ‘up-or-out’ decision at the end of the screening period. Firms will commit to an inefficiently high separation threshold in an effort to induce workers to supply effort in the initial phase.

Second, in a static model it is sometimes necessary to use public randomization to separate workers into high productivity (always hired), medium productivity (hired with probability e^* at a low wage), and low productivity (never hired) groups. In a dynamic model, the same objective could be achieved by hiring medium productivity workers on a fixed-term contract at a low wage, an outcome that seems empirically more plausible.

Finally, in a dynamic model one can think seriously about the quantitative behavior of

vacancies and unemployment. Shimer (2004) argues that neither the standard competitive search model nor the textbook search and matching model (Mortensen and Pissarides 1994, Pissarides 2000) is capable of explaining the observed business-cycle-frequency fluctuations in the vacancy-unemployment ratio in response to shocks of a plausible magnitude. Introducing asymmetric information may improve the performance of the model. For example, a small change in productivity may make some previously inactive markets active by reversing the sign of $\max(x - (1 - \alpha)z)(1 - F(x)) - \alpha z$. Even when markets are active, the result that there are gains from trade in marginal matches, i.e. matches at the productivity threshold x^* , may be important for the behavior of the model economy. A small change in parameters will induce a small change in the productivity threshold, but this can lead to a first order change in the gains from trade. An exploration of the quantitative importance of this channel remains a topic for future research.

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