A Hedonic Approach to the Quantity-Quality Theory*

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Abstract

This paper develops a hedonic approach to the quantity-quality theory using a classic insight of the assignment literature—when there is negative sorting. Families differ in the number of children they have. We interpret *quality* as a composite commodity that parents use in child rearing. In the equilibrium, children with higher quality are produced in families with smaller size. This setup yields a theory of the implicit cost of child rearing. We extend the literature on the shape and curvature of the cost function for child rearing by establishing that whether the cost function is convex, concave or linear depends on the functional forms of parental preferences, the distribution of family size, and the distribution of the quality of child-rearing resources in the society. We also determine the distribution of the cost of quality of children in the society, which leads to an analysis of inequality in terms of producing higher quality children.

JEL codes: J11, J13.

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1 Introduction

The "quantity-quality hypothesis" states that people with higher earnings have a higher demand for child quality, which raises the cost of quantity, and hence richer families produce fewer children. As a result, there is a negative relation between quantity and quality of children. Early discussions of this hypothesis appear in Becker (1960), Becker and Lewis (1973), Willis (1973) and Becker and Tomes (1976). Although some opposite views exist (see, for example, Angrist et al. (2005)), there is a large empirical literature supporting this hypothesis.[1]

The quantity-quality tradeoff has been used in a strand of literature as an intermediate result to explain the negative fertility-income relationship. However, incorporating this tradeoff into the analysis does not generate a negative relation between fertility and income unless a high elasticity of substitution between consumption and children is assumed, which is not confirmed empirically. Using the standard methods in the economist's toolkit today, there seems to exist little potential for progress in explaining the negative fertility-income relationship by using the quantity-quality tradeoff (see Jones et al. (2008) for an excellent survey of the related literature). Nonetheless, the negative relation between quantity and quality is an important evidence and it has a potential to have valuable implications for understanding the structure of inequality in the society. See Becker and Tomes (1976, 1979, 1986) and Moav (2005) for breakthrough examples. A systematic evaluation of how prices can be used to uncover the link between quantity-quality tradeoff and inequality is missing in the literature.

"Quality of children" is a fairly vague concept both theoretically and empirically. The absence of satisfactory direct observations on child quality has led researchers use many different proxies in their work. Schooling is one of the most popular variables representing child quality in the empirical literature (see, for example, Blake (1981)). However, quality has many other important dimensions. An empirical analysis of quality of children must include parental investment, training, health, nutrition, and even nannies as well as schooling (Becker (1992), Jones et al. (2008)).[2] Following Nerlove et al. (1984) and Razin and Sadka (1995), we interpret

[1] See Leibowitz (1974), Rosenzweig and Wolpin (1980), Knodel and Wongsith (1991), and Hanushek (1992).

[2] In their seminal work, Barro and Becker (1988) develop a dynastic model of altruism. Next generation's utility enters the

quality as a composite commodity, q, that parents use in child rearing. Children raised by using higher quality resources are assumed to be of higher quality. But these resources are expensive. We establish a systematic economic analysis to construct $p(q)$—the implicit price of quality. We assume that children in the same family are of identical quality, so that parental contributions to each must also be the same.[3] The total cost of raising n children each with quality level q is, therefore, $np(q)$.

On the *quantity* side, parents are assumed to differ exogenously in their capacity of child-bearing. We do not discuss the factors determining this variation across families. Instead, we focus on its consequences. In their pioneering work, Cunha and Heckman (2007) establish that early parental influences are key factors in child development. Heckman (2008) argue that the real source of child disadvantage is parenting. An important factor restricting the intensity of parenting is the number of children. The higher the number of children in a family, the lower the intensity of parenting per child. Motivated by these facts and findings, we interpret the quantity dimension as a major source of parenting and we rank parents with respect to the number of children they have.

Our model is closely related to the assignment framework developed by Sattinger (1975, 1979). The objective of our analysis is to obtain explicit pricing functions for child rearing. For the purposes of deriving these functions, we generate the quantity-quality tradeoff by means of negatively assortative matching. The sorting approach to the quantity-quality tradeoff is used in this paper as a modeling device without explicitly accounting for the underlying intra-family microfoundations. In this sense, the sorting mechanism has the same role as other shortcut modeling devices such as aggregate production functions and demand for money functions. See Pissarides (2000) for a philosophical discussion of the role of matching technologies as a modeling device.

The theoretical structure of our model is consistent with the following two major points made

utility of the adult in the current period. It is possible to interpret the utility of children as a "quality" argument in an altruistic setup. But we abstract from such considerations.

[3]We ignore the birth-order effects and child spacing.

in the early literature: (*i*) the analysis in this paper does not rely on the assumption that substitutability between quantity and quality is high. We show that, depending on the cost structure, even supermodular payoff functions (i.e., complementarity between quantity and quality) can generate quantity-quality tradeoff. The Cobb-Douglas example we provide in Section 3 illustrates this point; (*ii*) the cost of child rearing in our model is based on the idea that the cost function is supermodular. In other words, an increase in quantity increases the cost of raising the quality of children, and *vice versa* (see Becker and Lewis (1973) for a discussion of how this idea is used to explain that the observed income elasticity of demand for quality is high and the observed quantity elasticity is low and often even negative).

There seems to exist little consensus in the empirical literature on the shape and curvature of the child rearing cost function (see Lazear and Michael (1980), Espenshade (1984), Hotz and Miller (1988) and Tertilt (2005)). Most of the theoretical literature impose assumptions on the shape of the cost function and a common assumption is, not surprisingly, linearity. Tertilt (2005) assumes a convex cost function to limit the child-bearing capacity of women. We extend this literature by establishing that the shape and curvature of the cost function for child rearing can be derived from an explicit economic model. We show that whether the cost function is convex, concave or linear depends on the functional forms of parental utility, the distribution of family size, and the distribution of the quality of child-rearing resources in the society. We also determine the distribution of the cost of quality of children in the society, which enables us analyze inequality in terms of producing higher quality children.

In this paper, we present a simple framework to provide some theoretical insights into modeling the cost of child rearing and its effects on inequality. We leave the actual task of implementing this model empirically to future research (see Tumen (2009)). Our goal in this paper is to describe a model that is complementary to the strand of literature built on quantity-quality tradeoff. The model is presented in Section 2. Section 3 provides some examples and discusses the model's implications for inequality. Section 4 concludes.

2 Model

Hedonic models simplify the world a lot and provide useful tools for the analysis of heterogeneity. The solution of a hedonic model consists of a systematic pricing rule combining price and quality pairs in the equilibrium. We develop a model with two-sided heterogeneity: heterogeneity in quantity and quality dimensions.[4] Conceptually, the hedonic model assumes that the agents on one side of the market take the other side's actions or characteristics as given when making their decisions. Parents on the "quantity" side take as given the "quality" of resources they use in child rearing and choose a quantity-quality combination. Prices provide an interface for the quantity-quality interaction. This is a brief description of how the cost of child rearing is jointly determined by quantity and quality in our model.

Following Nerlove et al. (1984) and Razin and Sadka (1995), we assume that parents use a composite commodity, q, in child rearing. q is a rival commodity with a bounded support $\left[\underline{q}, \overline{q}\right] \subset \mathbb{R}$.[5] Children raised by using more of this commodity are assumed to be of higher quality. Let $n \in [\underline{n}, \overline{n}] \subset \mathbb{R}$ be the number of children a family can have. For the sake of simplicity, we treat n as a continuous variable. $\psi : [\underline{n}, \overline{n}] \times \left[\underline{q}, \overline{q}\right] \to \mathbb{R}$ describes the payoff function for a family raising n children each with quality level q. We ignore heterogeneity in the payoff structures across parents and assume that $\psi(n, q)$ is a homogeneous "output". ψ is continuous, twice differentiable in its arguments and $\psi_n > 0$, $\psi_q > 0$, $\psi_{nn} < 0$, and $\psi_{qq} < 0$.

Let $F(n)$ be the cumulative distribution of families in the society with respect to the number of children they have and $G(q)$ be the cumulative distribution of the quality levels of the resources available for parents' use. Both densities are monotone, strictly increasing and have positive support. Data on the distribution of family size in the society is readily accessible. The distribution of quality seems harder to imagine. However, it is well-documented in the literature that socially advantaged students typically have greater access to high quality resources than

[4] The sources of variation in both dimensions are exogenous and are not affected by parents' actions. We also abstract from the general equilibrium effects.

[5] An alternative formulation could assume an arbitrary number, say B, of different resources that families use in child rearing. Each of these resources are ranked from the highest to the lowest quality. $\boldsymbol{q} \in Q \subset \mathbb{R}^B$ is a B-dimensional vector of quality levels for B different resources. There is a relation, $\mathcal{Q} : Q \to \left[\underline{q}, \overline{q}\right]$, that maps the quality levels of those B resources into a single measure of quality, q, with bounded support, $\left[\underline{q}, \overline{q}\right] \subset \mathbb{R}$. We drop this more complicated setup for notational convenience.

do socially disadvantaged students (see, for example, Raudenbush et al. (1998)). A recent report, UNESCO (2009), documents that "one in three children in developing countries (193 million in total) reaches primary school age having had their brain development and education prospects impaired by malnutrition." There are many children in the schooling system not receiving quality education. Early disadvantages in life lead future exclusion from meaningful participation in the economic, social, political and cultural life of communities. Even the developed countries are plagued with similar problems (see Cunha and Heckman (2009) for a detailed analysis of the sources of disadvantages in US). Cunha and Heckman (2007) argue that the gap between advantaged and disadvantaged children open up early and persists over the life cycle. In the empirical implementation of Cunha and Heckman (2007), Cunha et al. (2009) demonstrate that inequality in early investment in children nicely links to the literature on the expanding inequality and polarization in the US society (see, for example, Card and Lemieux (2001), Eckstein and Nagypál (2004), and Autor et al. (2006)). In this paper, to discuss the implications of our model for inequality, we presume that there exists some $G(q)$ describing the distribution of the quality of child-rearing resources in the economy and focus on theory only. In a companion paper, Tumen (2009) establishes econometric identification of our theoretical framework, estimates $G(q)$, and presents empirical findings regarding inequality in the allocation of resources in child-rearing .

We assume $L_q > L_n$, where L_n is the total number of families in the population and L_q represent the number of resources available for parents' use. Heckman (2008) argue that parenting is the scarce resource in child rearing. To highlight this idea, we assume that total number of families are smaller than total number of different varieties of quality resources so that rents accrue on parents in the assignment problem.

For the purpose of deriving the implicit price of quality, $p(q)$, we first match quantity-quality pairs to obtain a functional sorting rule in the equilibrium. Our primary assumption—negative sorting—implies that families with smaller size are assigned to higher quality commodities.

Therefore, in the equilibrium, the condition

$$L_n \int_{\underline{n}}^{n} dF(x) = L_q \int_{q}^{\overline{q}} dG(z) \tag{2.1}$$

must hold. In other words, in order the family with n children to be assigned to the quality level q, the number of families ranked from $\underline{n}$ to n has to be equal to the number of commodities with quality levels from q to $\overline{q}$ as long as negative sorting holds. Thus, negative sorting defines a relationship $q(n)$ (or $n(q)$) in equilibrium. This will be a strictly decreasing function unless there are intermediate levels for n or q such that $F'(n)$ and $G'(q)$ are zero. But this possibility is ruled out by assumption.

A benevolent social planner solves the maximization problem

$$\max_q \big[\psi(\tilde{n}, q) - \tilde{n}p(q)\big], \tag{2.2}$$

where $\tilde{n}$ is given. Net return of rearing $\tilde{n}$ children each with quality level q is residually determined, $\pi(\tilde{n}, q) = \psi(\tilde{n}, q) - \tilde{n}p(q)$. The first and the second order conditions for this problem are

$$\psi_q(\tilde{n}, q) = \tilde{n}p'(q) \quad \text{and} \quad \psi_{qq}(\tilde{n}, q) - \tilde{n}p''(q) < 0, \tag{2.3}$$

respectively, where q is the equilibrium quality level corresponding to $\tilde{n}$. In other words, $\tilde{n} = \varphi(q)$. To put more structure on this equilibrium relationship, we totally differentiate the first order condition to get:

$$\psi_{qq}(\tilde{n}, q)\big|_{\tilde{n}=\varphi(q)} - \tilde{n}\big|_{\tilde{n}=\varphi(q)} p''(q) = \left[p'(q) - \frac{\partial^2 \psi(\tilde{n}, q)}{\partial n \partial q}\Big|_{\tilde{n}=\varphi(q)} \right] \frac{d\tilde{n}}{dq}. \tag{2.4}$$

The left-hand side in equation (2.4) is negative by the second order condition which means that the right-hand side must also be negative. To generate the quantity-quality tradeoff, $\frac{d\tilde{n}}{dq} < 0$, the following condition has to hold:

$$p'(q) > \frac{\partial^2 \psi(\tilde{n}, q)}{\partial n \partial q}\Big|_{\tilde{n}=\varphi(q)} \tag{2.5}$$

where $p'(q)$ is positive. This proves our claim that the sign of the cross partial ψ_{qn} can be positive or negative as long as the condition (2.5) holds. In other words, our solution does not rely on any specific assumption on substitutability or complementarity between quantity and quality. This solution gives us a framework in which quantity and quality are jointly determined, and quantity is negatively related to quality in the equilibrium. With the assumption that the density has no mass points or holes, the functional sorting rule $\varphi(\cdot)$ has an inverse.

Becker and Lewis (1973) and Becker and Tomes (1976) argue that the cost of an additional child, $p(q)$, holding q fixed, is greater the higher q is, and, similarly, the cost of a unit increase in q, $np'(q)$, holding n fixed, is greater the higher n is. In other words, it has to be the case that $p'(q) > 0$. The economic interpretation of this condition is that a unit increase in quality is more expensive if the number of children is higher because the increase has to apply to more units. In the literature, the negative relationship between quantity and quality is generated through specific assumptions—such as greater than average substitution between quantity and quality—on preferences (or, equivalently, on production technologies: see Jones et al. (2008)). Becker and Lewis (1973) emphasize the importance of $p'(q) > 0$ rather than imposing specific assumptions on the payoff functions. Our model is consistent with these observations.

The assumption $L_q > L_n$ implies that the least productive resources to produce quality are unemployed. From the first order condition,

$$p'(q) = \frac{1}{\varphi(q)} \frac{\partial \psi(\varphi(q), q)}{\partial q}. \tag{2.6}$$

Equation (2.6) defines the slope of the hedonic line with a continuum of quality levels. Let $\hat{q} > \underline{q}$ denote the marginal quality level. $\hat{q}$ is the quality level produced in the family with size

$\overline{n}$. Therefore, it has to be true that

$$p'(\hat{q}) = \frac{1}{\varphi(\hat{q})}\frac{\partial\psi(\varphi(\hat{q}),\hat{q})}{\partial q}. \tag{2.7}$$

Quality levels lower than $\hat{q}$ are not employed. By the assumption of competitive exhaustion of resources for quality, we obtain $p(\hat{q}) = p_R$, where p_R is the reserve price for the resources used to produce quality. Following Sattinger (1979), the hedonic pricing function for quality, $p(q)$, is obtained by integrating equation (2.6) over the interval $[\hat{q}, q]$:

$$p(q) = \int_{\hat{q}}^{q} \frac{1}{\varphi(z)}\frac{\partial\psi(\varphi(z),z)}{\partial z}dz + p_R, \tag{2.8}$$

where the reserve price, p_R, is the constant of integration.[6]

Density of the cost of quality is determined by inverting $p(q)$. Invertibility is ensured by our initial assumptions, and hence, letting $p(q) = \eta(q)$, we get $q = \eta^{-1}(p)$. Therefore, the density for the cost of quality per child can be described by

$$g(\eta^{-1}(p))\frac{d\eta^{-1}(p)}{dp}, \tag{2.10}$$

where $g(\cdot) = G'(\cdot)$. Expression (2.10) leads to an important question about inequality. Does the distribution of the quality of resources in the society resemble the distribution of the cost of quality? This idea is analogous to the Pigou's problem. We discuss this link over a simple example in Section 3.

The model presented in this section postulates a systematic framework by which we can determine the implicit cost of the quality of children (and therefore the total cost of child rearing) and the distribution of this cost in the society. A detailed empirical implementation of this framework, among other things, is carried out by Tumen (2009) in an ongoing work.

[6] The net return to parents is residually determined and can be formulated as

$$\pi(n,\varphi^{-1}(n)) = \int_{n}^{\overline{n}} \frac{\partial\psi(x,\varphi^{-1}(x))}{\partial x}dx + k \tag{2.9}$$

where k is an arbitrary constant. From the condition, $d\psi = \psi_n dn + \psi_q dq$, we get output exhaustion locally. Thus, if we know k, we pin down p_R. Using the assumption $L_q > L_n$, the feasibility condition must satisfy $\pi(\hat{q},\overline{n}) \geq k$.

3 A Cobb-Douglas Example

For the rest of the discussion, we specialize the payoff function $\psi(n, q)$ to the Cobb-Douglas form

$$\psi(n, q) = n^{\alpha} q^{\theta}, \tag{3.1}$$

where $\alpha > 0$ and $\theta > 0$. We choose a Cobb-Douglas example for the purposes of simplicity and demonstrating that the model does not require $\psi_{qn} < 0$ to generate the salient features of the basic quantity-quality hypothesis. However, multiplicatively separable payoff functions rule out the idea of "comparative advantage". Using a more general CES form, which will induce comparative advantage, is a natural but an algebraically cumbersome exercise. We briefly mention the implications of using a CES function, but the task of performing a detailed analysis is left to our future work. For the ease of exposition, we presume a Pareto distribution for quality levels of the resources, i.e.,

$$g(q) = hq^{-\sigma} \tag{3.2}$$

where $\sigma > 2$, and $q \geq 1$ (to ensure finite variance). The higher σ, the more equal is the distribution of q. Pareto distributions have been used by economists to discuss inequality in the distribution of earnings, income, education, and wealth among individuals and families. We follow Sattinger (1979) in using Pareto distributions to discuss the implications of the sorting framework for inequality. We assume a uniform distribution on family size, where $n \in [0, \overline{n}]$. For simplicity, we shut down the parametric implications of the distributional impacts of family size by assuming uniformity. However, a full characterization of our analysis must contain the interplay between $F(n)$ and $G(q)$. More realistic assumptions on functional forms are used and tested empirically in our future work. From equation (2.1), the following sorting condition has to be true in the equilibrium:

$$L_n \frac{\varphi(q)}{\overline{n}} = L_q \int_q^{\infty} hz^{-\sigma} dz, \tag{3.3}$$

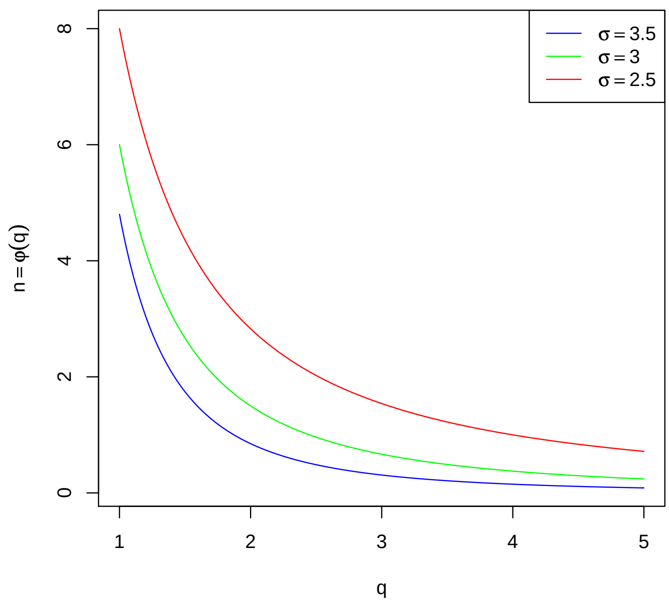


Figure 3.1: The hedonic line.

which gives us the function

$$\tilde{n} = \varphi(q) = \frac{L_q}{L_n} \frac{\overline{n}h}{\sigma - 1} q^{1-\sigma} \tag{3.4}$$

that we use to define the relationship between quantity and quality along the hedonic line. Figure (3.1) describes the hedonic line and demonstrates the quantity-quality tradeoff generated by our framework. The higher the inequality in q (the lower σ), the stronger the tradeoff.

The planning problem is

$$\max_q \left[\tilde{n}^\alpha q^\theta - \tilde{n} p(q) \right] \tag{3.5}$$

for a given $\tilde{n}$. The resulting first order condition is

$$p'(q) = \theta \tilde{n}^{\alpha-1} q^{\theta-1}. \tag{3.6}$$

Parallel to the solution outlined in Section 2 and using equation (3.4), we substitute $\tilde{n} = \varphi(q)$

into (3.6) to reach

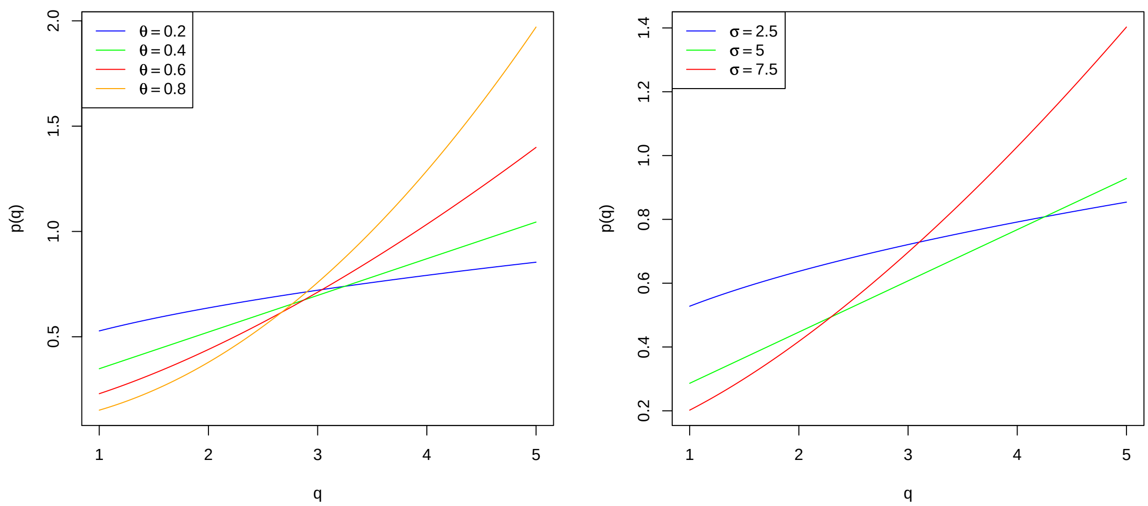
$$p'(q) = \theta \left[\frac{L_q}{L_n} \frac{\overline{n}h}{\sigma - 1} \right]^{\alpha - 1} q^C, \tag{3.7}$$

where $C = (\alpha - 1)(1 - \sigma) + \theta - 1$ is a constant. To make sure that we get negative sorting in the equilibrium, we need to verify whether the condition (2.5) is satisfied or not. Our simplifying assumptions in terms of functional forms pay here and, after trivial algebra, we conclude that the condition (2.5) is satisfied if $\alpha < 1$. Using the hedonic pricing formula (2.8), we get

$$p(q) = \left[\frac{\theta \left[\frac{L_q}{L_n} \frac{\overline{n}h}{\sigma - 1} \right]^{\alpha - 1}}{(\alpha - 1)(1 - \sigma) + \theta} \right] q^{(\alpha - 1)(1 - \sigma) + \theta} + p_R, \tag{3.8}$$

where p_R is a constant of integration. Obviously, $(\alpha - 1)(1 - \sigma) + \theta > 0$, and hence the implicit price of quality of children increases with q, i.e., $p'(q) > 0$ is satisfied. Whether the hedonic pricing function, $p(q)$, is convex or concave in q depends on the sign of the exponent, C, in equation (3.7). We also need to pin down p_R.

Suppose now that the payoff function has constant returns to scale. Plugging $\alpha + \theta = 1$ into the constant C yields $C = \sigma(1 - \alpha) - 1 = \sigma\theta - 1$. If $C > 0$, then $p(q)$ is convex. The condition for convexity, therefore, is $\sigma\theta > 1$ (for concavity, the condition becomes $\sigma\theta < 1$). We get an interplay of two objects: (i) the degree of equality in the distribution of the resources that determine the quality of children, and (ii) the share (importance) of quality in parental preferences. In other words, the curvature of the hedonic pricing function, $p(q)$, is determined by the interaction between the dispersion of resources determining quality and how parents weigh quality. Intuitively, if σ is higher, i.e., if quality is more equally distributed, there is greater demand for top quality which induces the cost function to be convex. Specific curvature assumptions made in the literature arise as special cases in our general framework. However, given that $\sigma > 2$ by the Pareto distribution, if $\theta > 0.5$, then the implicit price of quality is always a convex function of q, which supports the convexity claim in Tertilt (2005). If the quality is weighted more than quantity in parental preferences, implicit price of quality is



(a) $\sigma = 2.5$ (b) $\theta = 0.2$

Figure 3.2: The implicit price of quality.

always convex. Figure (3.2) shows some examples of implicit prices as a function of σ and θ.

Now we determine the distribution of $p(q)$. The implicit price of quality is assumed to be determined competitively, and hence, $p_R = p(\hat{q})$. From equation (3.4) and using the fact $\overline{n} = \varphi(\hat{q})$, we get

$$\hat{q} = \left(\frac{L_q}{L_n} \frac{h}{\sigma - 1} \right)^{\frac{1}{\sigma - 1}} \tag{3.9}$$

which is a constant. We directly pin down the reserve price p_R by plugging $\hat{q}$ into (3.8). The density of "translated" $p(q)$ is derived using the formula (2.10) we provide in section 2:

$$\xi(p - p_R) = \frac{h}{(\alpha - 1)(1 - \sigma) + \theta} K^{\left(\frac{\sigma - 1}{(\alpha-1)(1-\sigma)+\theta}\right)} (p - p_R)^{-\left(1 + \frac{1-\sigma}{(1-\alpha)(1-\sigma)-\theta}\right)} \tag{3.10}$$

where $\xi(\cdot)$ is the probability density function for $p - p_R$ and $K = \left[\frac{\theta \left[\frac{L_q}{L_n} \frac{\overline{n} h}{\sigma - 1} \right]^{\alpha - 1}}{(\alpha-1)(1-\sigma)+\theta} \right] > 0$. The Pareto coefficient in this distribution is $1 + \frac{1-\sigma}{(1-\alpha)(1-\sigma)+\theta}$. Thus, the distribution of the implicit price of the quality is also Pareto, but its shape is different. The implicit price of quality is more unequally distributed than the quality itself, if

$$\sigma > 1 + \frac{1-\sigma}{(1-\alpha)(1-\sigma)-\theta} \tag{3.11}$$

or, equivalently,

$$1 < \frac{1}{(\alpha-1)(1-\sigma)+\theta} \tag{3.12}$$

By the constant returns to scale assumption, the condition simplifies to $\sigma\theta < 1$, which is familiar (the condition for $p(q)$ to be concave). Therefore, if the implicit price of quality is a concave function of q, then price of quality is more unequally distributed than the resources. Similarly, if the implicit price of quality is convex, then price of quality is more equally distributed than the resources.

This section provides some examples and we do not claim empirical relevance at this stage. The idea of comparative advantage might also be interesting to consider within this framework. A natural comparative advantage argument could be the following: smaller size families have a comparative advantage in using higher quality resources. To establish this link, one can use a CES preference relation, i.e., $\left[an^{\phi} + bq^{\phi}\right]^{1/\phi}$, which, unlike Cobb-Douglas, recognizes the idea of comparative advantage. Cobb-Douglas preferences and Pareto distributions are particularly amenable to analysis. To get more realistic results, one has to choose more general functional forms and distributions that have empirical counterparts. In his original paper, Sattinger (1979) uses a CES production function with truncated normal densities for skills and capital. See (Tumen (2009)) for an empirical analysis of the model presented in this paper.

4 Concluding Remarks

This paper links the assignment and quantity-quality literatures to establish a formal theory of the implicit cost of child rearing. We make two contributions: (1) we derive the curvature of the cost of child rearing rather than imposing functional forms *a priori*, and (2) we develop a framework potentially useful in studying distributional aspects of the quantity-quality theory.

Using the technical properties of the child cost function we derive in Section 2, we postulate a link between the distribution of resources that families use in raising their children and inequality. We argue that the distribution of the resources determining child quality and the distribution of the cost of child rearing are closely connected but may differ in shape. At this stage, many of the variables that are likely to be important in empirical analysis of child rearing are left out. Empirical implementation of the model and further econometric concerns, i.e., identification and estimation, are discussed in our ongoing work.

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